GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #5

Assigned: 24 September 99
Due Date: 1 October 99 (FRIDAY)

Reading: In *DSP First*, Chapter 5 on *FIR Filters*.

An on-line survey is to be done on WebCT. Also, there will be an on-line homework on WebCT.

$\Longrightarrow$ The five **STARRED** problems (5.1, 5.2, 5.4, 5.6 and 5.7) will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 5.1*:

Suppose that a MATLAB function has been written to calculate a sum of discrete-time sinusoids, e.g., something similar to the `sinus( )` that was written for the lab. Here is the actual function:

```
function  xn = makedcos(omegahat,XX,Length)
%MAKEDCOS  make a discrete-time sinusoid for x[n]
%
xn = real( exp( j*(0:Length-1)'*omegahat(:)' ) * XX(:));
```

(a) Write an equation for $x[n]$, the discrete-time signal that is created by this MATLAB function, when the following function call is used:

```
x = makedcos(pi*[0,0.5,0.8,1.2],[1,-2i,1i,1-1i],100001)
```

Your equation should be in terms of cosine functions. To do this you must figure out how the matrix multiplications and `exp( )` in the MATLAB statement defining `xn` work.

(b) Draw a plot of the discrete-time spectrum (vs. $\hat{\omega}$) of the discrete-time signal defined by this MATLAB operation. Make sure that you include all the spectrum components in the $-\pi$ to $+\pi$ interval.

PROBLEM 5.2*:

This is a direct continuation of Problem 5.1*. Use your results from Problem 5.1(a) and (b) in this problem. The following MATLAB commands are used to make an output sound:

```
x = makedcos(pi*[0,0.5,0.8,1.2],[1,-2i,1i,1-1i],100001)
soundsc(x,2000)
```

(a) Draw a plot of the (idealized) continuous-time spectrum (vs. f in Hz) of the continuous-time signal that would be created at the output of an ideal D-to-C converter (approximately realized by the `soundsc()` function).

(b) Write an equation for $x(t)$, the continuous-time signal that is created at the output of the ideal D-to-C converter.

(c) What is the duration (in seconds) of the continuous-time signal $x(t)$?

PROBLEM 5.3:

A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.1$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} 1 & 0 \le n \le 4 \\ (0.5)^{(n-4)} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus n.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

PROBLEM 5.4*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

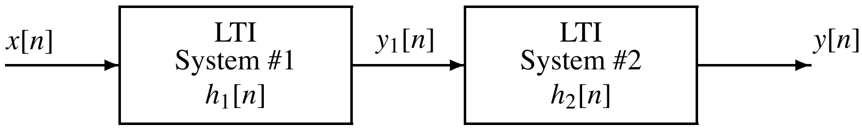


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a "blurring" filter described by the difference equation

$$y_1[n] = \sum_{k=0}^{6} \beta^k x[n-k],$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n] - \beta\delta[n-1],$$

where β is a real number. Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(b) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = \frac{1}{2}$.

PROBLEM 5.5:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n]\cos(.3\pi n)$

(b) $y[n] = |x[-n]|$

PROBLEM 5.6*:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] - \beta x[n-1]$$

(a) When the input to this system is

$$x[n] = \begin{cases} 0 & n < 0 \\ \beta^n & n = 0, 1, 2, 3, 4, 5, 6 \\ 0 & n > 6 \end{cases}$$

Use convolution to compute the values of $y[n]$, over the range $0 \le n \le 10$. Give a general formula in terms of β, and also show that most of the output values are equal to zero.

(b) Use the results from the previous part and plot both $x[n]$ and $y[n]$ for the case where $\beta = \frac{1}{2}$.

PROBLEM 5.7*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the text.

(b) Determine the impulse response $h[n]$ for this system.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \leq n \leq 10$.

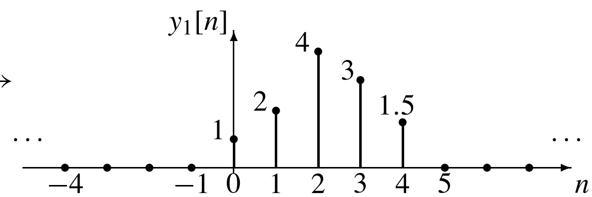
PROBLEM 5.8:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

(a) When tested with an input signal that is a shifted impulse, $x_1[n] = \delta[n-1]$, the observed output from the filter is the signal $h[n]$ shown below:

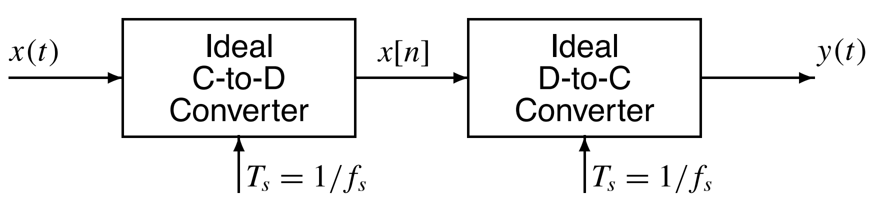
$$\delta[n-1] = \begin{cases} 1 & \text{for } n = 1 \\ 0 & \text{for } n \neq 1 \end{cases} \longrightarrow$$



Use linearity and time-invariance to solve the following problem. Determine the output when the input to the LTI system is $x_2[n] = \delta[n] - \delta[n-2]$. Give your answer as a plot of $y_2[n]$ versus n, or a list of values for $-\infty < n < \infty$.

(b) Define the property of *causality*. Is this system *causal?*

PROBLEM 5.9:



(a) If the input to the ideal C/D converter is a sinusoid with frequency of 700 Hz, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the frequency of the output.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(400\pi t^2) \qquad \text{for } 0 \leq t \leq 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`.