GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #7

Assigned: 8 October 99
Due Date: 15 October 99 (FRIDAY)

Reading: In *DSP First*, Chapter 7 on *The z-Transform.*

Remember that the second quiz will be given Monday, October 25.

$\Longrightarrow$ The five **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM or WebCT, especially the "unstarred" problems.

PROBLEM 7.1*:

We now have four ways of describing an LTI system: the difference equation; the impuluse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = \frac{1}{4}(x[n] - x[n-4])$.

(b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$.

(c) $H(e^{j\hat{\omega}}) = [2 + 2\cos(\hat{\omega})]e^{-j\hat{\omega}2}$.

(d) $H(z) = z^{-3} + z^{-6} + z^{-9}$.

PROBLEM 7.2:

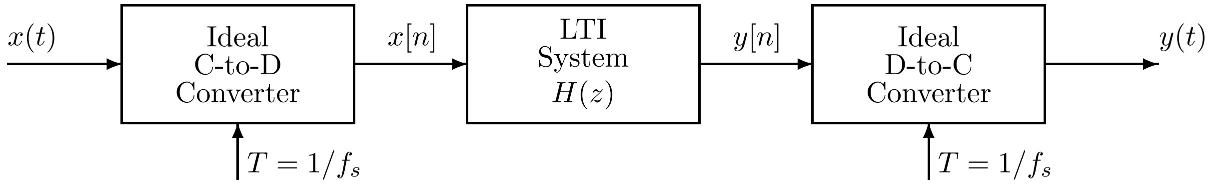
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 2\cos(6000\pi t - \pi/4) + 11\cos(12000\pi t - \pi/3)$$

The system function for the LTI system is

$$H(z) = \frac{1}{4}(1 - z^{-4})$$

If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



This is Problem 7.3 of Spring 1999. Try working it before you consult the answer.

PROBLEM 7.3*:

Consider the following MATLAB program:

```
nn = 0:16000;
xx = 3 + 2*cos(0.75*pi*nn-pi/4) + 11*cos(1.5*pi*nn-pi/3);
yy = conv([1,0,0,0,-1]/4,xx);
soundsc(yy,8000)
```

(a) What is the system function $H(z)$ of the system that is implemented by the `conv( )` statement?

(b) What is the frequency response of the system?

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement. *Hint: The result of Problem 7.2 should be useful here.*

PROBLEM 7.4*:

A linear time-invariant filter is described by the difference equation

$$y[n] = \frac{1}{5}\{x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\} = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$$

(a) What is the impulse response, $h[n]$, of this system?

(b) Determine the system function $H(z)$ for this system.

(c) Plot the poles and zeros of $H(z)$ in the complex z-plane. *Hint: Remember the N-th roots of unity.*

(d) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of this system.

(e) Show that your answer in (d) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{5\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}}.$$

(f) Sketch the frequency response (magnitude and phase) as a function of frequency from the formula above (or plot it using `freqz( )`).

(g) Suppose that the input is

$$x[n] = 5 + 4\cos(0.1\pi n) + 3\cos(0.4\pi n - \pi/4) \qquad \text{for} \quad -\infty < n < \infty$$

Obtain an expression for the output in the form $y[n] = A + B\cos(\hat{\omega}_0 n + \phi_0)$. (In other words, one of the sinusoids is removed by the filter.)

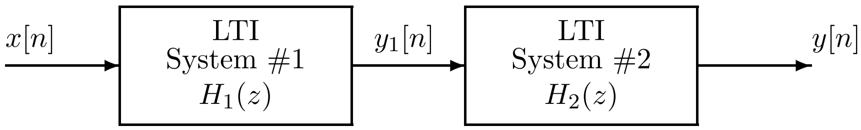


Figure 1: Cascade connection of two LTI systems.

PROBLEM 7.5*:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system. In Figure 1, assume that both systems are 3-point moving averagers; i.e.,

$$y_1[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2]) \quad \text{and} \quad y[n] = \frac{1}{3}(y_1[n] + y_1[n-1] + y_1[n-2]).$$

(a) Determine the system function $H(z) = H_1(z)H_2(z)$ for the overall system.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) Use multiplication of z-transform polynomials to determine the impulse response $h[n]$ of the overall system in Figure 1.

(d) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of the overall cascade system.

(e) Use your result from (d) as an aid in sketching the frequency response (magnitude and phase) functions of the overall cascade system for $-\pi \le \hat{\omega} \le \pi$.

PROBLEM 7.6*:

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 - z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})(1 - 0.5e^{j\pi/3}z^{-1})(1 - 0.5e^{-j\pi/3}z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. Make sure that all the filter coefficients $\{b_k\}$ in your difference equation are purely real.

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = \delta[n-2] + 2\delta[n-4] - \delta[n-5].$$

(d) Plot the poles and zeros of $H(z)$ in the z-plane.

(e) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of this system.

(f) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of frequency $\hat{\omega}$ will the output signal be zero for all n (i.e., $y[n] = 0$) ? Find all possible frequencies in the range $-\pi \le \hat{\omega} \le \pi$. *Hint: Take a look at the locations of the zeros of $H(z)$ as plotted in part (d).*