GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #8

Assigned: 15 October 99
Due Date: 22 October 99 (FRIDAY)

Reading: In *DSP First*, Chapter 7 on *The z-Transform* and Chapter 8 on *IIR Filters.*

Remember that the second quiz will be given Monday, October 25. It will cover material through this problem set. **Be sure to check out exams from previous years.**

No lab next week. Turn in lab reports at next lab meeting for your lab.

Monday and Tuesday Recitations attend recitation (in VanLeer 361) at your Lab hour.

$\Longrightarrow$ The three **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Similar problems (with solutions) can be found on the CD-ROM or WebCT.

PROBLEM 8.1*:

We have developed several concepts that are useful in solving problems involving LTI systems. The main concepts are the *difference equation*, the *impulse response*, the *system function*, and the *frequency response* function. You need to be able to go back and forth among these different mathematical representations of the LTI system because, as simple as it seems, the z-transform is *not* always the best tool for solving these problems. Indeed for a specific problem, one of these representations may be more convenient than the others, or we may need to use more than one of these representations in solving a given problem. The following is a simple problem that might be posed about an LTI system:

> *Given the input sequence $x[n]$ find the output sequence $y[n]$ of a 5-point running average filter for all values of n.*

The following is a partial list of possible approaches to solving this problem:

1. Use the difference equation representation of the system to compute the output $y[n]$ for all required values of n.

2. Multiply the z-transform of the input by the system function and determine $y[n]$ as the inverse z-transform of $Y(z)$.

3. Break the input into a sum of complex exponential signals, use the frequency response function to determine the output due to each complex exponential signal separately, and finally, add the individual outputs together to get $y[n]$.

In each of these solution methods you would use one or more of the basic representations of the 5-point running average filter. Which method is easiest will have a lot to do with the nature of the input signal. This may require that you convert a given representation of the system into one of the other forms. For example, if you are given the difference equation and you want to use approach #2, you will have to determine the system function $H(z)$ from the difference equation coefficients.

Now in each of the following cases, the input will be given. In each case, determine which representation of the system and which of the above approaches will lead to the easiest solution of the problem, and detail the steps in using that approach to solve the problem. For example, if you choose approach #2 to solve the problem, your answer should be something like the following:

Step 1 Find $X(z)$, the z-transform of $x[n]$.

Step 2 Find $H(z)$, the system function of the 5-point running averager.

Step 3 Multiply $X(z)H(z)$ to get $Y(z)$.

Step 4 Take the inverse z-transform of $Y(z)$ to get $y[n]$.

Now here are some possible inputs. In each case, state which of the above (#1, #2, or #3) approaches you would use. There may not be a clear cut answer. Give the approach that you *think* will yield the solution with least effort. Outline your approach to solving the problem of finding the output of the 5-point moving averager. **You do not have to actually find the output—just tell how you would solve it in a step-by-step procedure described as illustrated above.**

(a) $x[n]$ is a sampled speech signal. It is represented by a vector of 10000 numbers.

(b) $x[n] = u[n]$.

(c) $x[n] = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi)$ for $-\infty < n < \infty$.

(d) $x[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

(e) $x[n] = 10\delta[n - 50]$.

PROBLEM 8.2*:

A linear time-invariant filter is described by the difference equation

$$y[n] = \frac{1}{5}\{x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\} = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$$

(a) Find the output $y_1[n]$ when the input is

$$x_1[n] = 10\delta[n-50].$$

(b) Find the output $y_2[n]$ when the input is

$$x_2[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

(c) Find the output $y_3[n]$ when the input is

$$x_3[n] = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi) \qquad \text{for } -\infty < n < \infty.$$

(d) Use the concept of linearity to find the output $y_4[n]$ when the input is

$$x_4[n] = 10\delta[n-50] + 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi) \qquad \text{for } -\infty < n < \infty$$

PROBLEM 8.3*:

A linear time-invariant system is described by the difference equation

$$y[n] = 0.8y[n-1] + x[n] + x[n-1].$$

(a) Suppose that the input is the unit step sequence, i.e., $x[n] = u[n]$. The output signal will be infinitely long. Determine by iterating the difference equation, the output of this system for the range $0 \le n \le 10$.

(b) Suppose that the input is the pulse sequence

$$x[n] = u[n] - u[n-6] = \begin{cases} 1 & 0 \le n \le 5 \\ 0 & \text{otherwise.} \end{cases}$$

Once again, the output signal will be infinitely long. Determine the output of this system for $0 \le n \le 10$. *You can do this either by iterating the difference equation as you did in part (a), or you may apply linearity and time invariance to the answer that you computed in part (a). Do the problem either or both ways.*

In reviewing for Quiz #2 on October 25, you should look at old exams under "Word From Previous Quarters" on WebCT.