GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #5

Assigned: 4-Feb-00
Due Date: Week of 14-Feb-00

Reading: The notes on *Fourier Series*. In *DSP First*, Chapter 4 on *Sampling*.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 5.1*:

A periodic signal is represented by the Fourier Series synthesis formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j2400\pi kt} \qquad \text{where} \quad a_k = \begin{cases} \dfrac{1}{4+j2k} & \text{for } k = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{for } |k| > 3 \end{cases}$$

(a) Determine a formula for the signal $x(t)$ and a sum of sinusoids, using the cosine form.

(b) Determine the minimum sampling rate f_s (in Hz) such that $x(t)$ can be reconstructed from its samples, $x(n/f_s)$.

PROBLEM 5.2*:

In the rotating disk and strobe demo described in Chapter 4 of *DSP First,* we observed that different flashing rates of the strobe light would make the spot on the disk stand still.

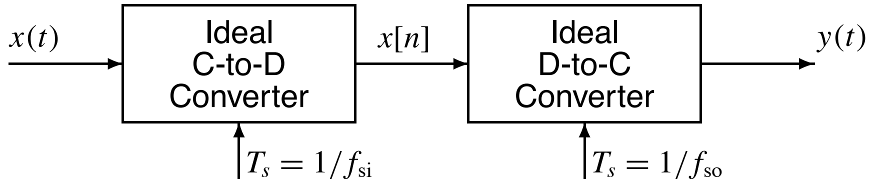
(a) Assume that the disk is rotating in the counter-clockwise direction at a constant speed of 12 revolutions per second. Express the movement of the spot on the disk as a rotating complex exponential $e^{j\omega t}$ (a.k.a. a rotating phasor).

(b) If the strobe light can be flashed at a rate of n flashes *per second* where n is an integer greater than zero, determine all possible flashing rates such that the disk can be made to stand still.
NOTE: the only possible flashing rates are integers: 1 per second, 2 per second, 3 per second, etc.

(c) Now assume that the flashing rate is fixed, so that the interval between flashes is 100 millisec. Explain how the spot will move and write a complex phasor that gives the position of the spot at each flash.

(d) Draw a spectrum plot of the discrete-time signal in part (c) to explain your answer.

PROBLEM 5.3*:

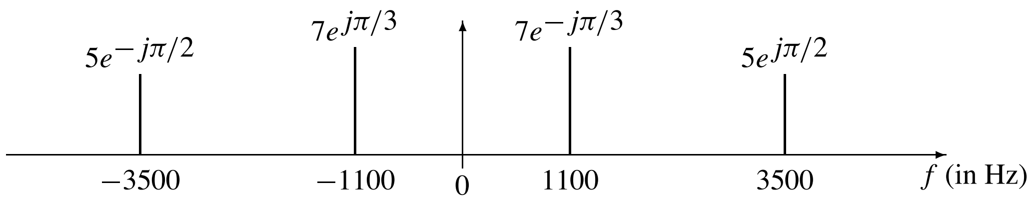


(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos(0.25\pi n + 3\pi/4)$$

If the sampling rate of the C-to-D converter is $f_{si} = 4000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 4000 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_{si}) = x_2(nT_{si})$ if $T_{si} = 1/4000$.

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the *discrete-time* spectrum for $x[n]$ when $f_{si} = 4000$ samples/sec. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.

(c) Using the discrete-time spectrum from part (b), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{so} = 8000$ Hz. In other words, the sampling rates of the two converters are different.

PROBLEM 5.4*:

Suppose that a MATLAB function has been written to calculate a sum of discrete-time sinusoids, e.g., something similar to the function that was written for the lab. Here is the actual function:

```
function  xn = makedcos(omegahat,XX,Length)
%MAKEDCOS  make a discrete-time sinusoid for x[n]
%
xn = real( exp( j*(0:Length-1)'*omegahat(:)' ) * XX(:));
```

(a) Write an equation for $x[n]$, the discrete-time signal that is created by this MATLAB function, when the following function call is used:

```
xn = makedcos(pi*[0,0.25,0.75,1.75],[1,1-1i,-7i,2i],200001)
```

Your equation should be in terms of cosine functions. To do this you must figure out how the matrix multiplications and `exp( )` in the MATLAB statement defining `xn` work.[1]

(b) Draw a plot of the discrete-time spectrum (vs. $\hat{\omega}$) of the discrete-time signal defined by this MATLAB operation. Make sure that you include all the spectrum components in the $-\pi$ to $+\pi$ interval.

PROBLEM 5.5*:

This is a direct continuation of Problem 5.4*. Use your results from Problem 5.4(a) and (b) in this problem. The following MATLAB commands are used to make an output sound:

```
xn = makedcos(pi*[0,0.25,0.75,1.75],[1,1-1i,-7i,2i],200001)
soundsc(xn,8000)
```

Since we can listen to the sound produced by the `soundsc()` function, we can regard the `soundsc()` function as a D-to-C converter whose input is `xn`, and whose output is the analog signal that we hear.

(a) Draw a plot of the (idealized) continuous-time spectrum (vs. f in Hz) of the continuous-time signal that would be created at the output of an ideal D-to-C converter (approximately realized by the `soundsc()` function).

(b) Write an equation for $x(t)$, the continuous-time signal that is created at the output of the ideal D-to-C converter.

(c) What is the duration (in seconds) of the continuous-time signal $x(t)$?

[1]For this part, ignore the fact that the total length of the signal `xn` is finite.