GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2000
Problem Set #7

Assigned: 18-Feb-00
Due Date: Week of 28-Feb-00

There will be a lab quiz at the beginning of Lab #6 (21-24 Feb).

Quiz #2 on 3-March (Friday). Coverage is Homeworks #4, #5, #6 and #7; as well as Chapters 4, 5 and 6 in *DSP First* plus the Fourier Series notes.

Reading: In *DSP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response.*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 7.1*:

A discrete-time system is defined by the input/output relation

$$y[n] = x[n] + 3x[n-1] + x[n-2]. \qquad (1)$$

(a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal. Explain your answers.

(b) Obtain an expression for the frequency response of this system.

(c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency. *Hint: Use symmetry to simplify your expression before determining the magnitude and phase.*

(d) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

Express your answer in terms of cosine functions.

(e) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

Hint: use the linearity and time-invariance properties.

PROBLEM 7.2*:

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^2. \tag{2}$$

(a) Determine whether or not the system defined by (2) is (i) linear; (ii) time-invariant; (iii) causal.

(b) For the system of Equation (2), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any squared powers of cosine functions in your answers.

(c) For the system of Equation (2), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

(d) This system produces output contain frequencies that are not present in the input signal. Explain how this system might cause "aliasing" of sinusoidal components of the input.

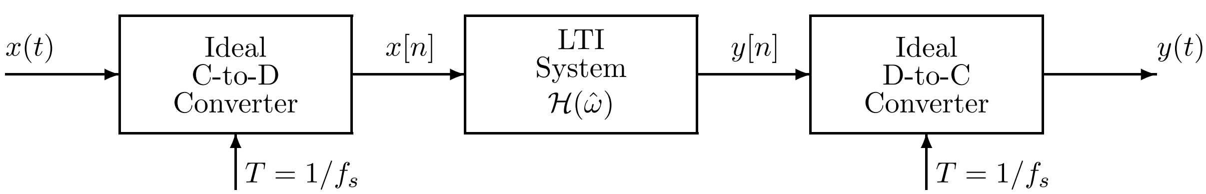
PROBLEM 7.3*:

The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 5\cos(4000\pi t) + 6\cos(6000\pi t - \pi/2)$$

The frequency response for the digital filter (LTI system) is $\mathcal{H}(\hat{\omega}) = \dfrac{\sin(3\hat{\omega})}{6\sin(\frac{1}{2}\hat{\omega})}e^{-j2.5\hat{\omega}}$

The sampling frequency is $f_s = 12000$ samples/second.



(a) For the *Dirichlet* function: $\tilde{\mathcal{D}}(\hat{\omega}, 6) = \dfrac{\sin(3\hat{\omega})}{6\sin(\frac{1}{2}\hat{\omega})}$. make a plot of $\tilde{\mathcal{D}}(\hat{\omega}, 6)$ over the range $-2\pi \le \hat{\omega} \le +2\pi$. Label all the zero crossings.

(b) Determine the period of $\tilde{\mathcal{D}}(\hat{\omega}, 6)$. Is it equal to 2π; why, or why not?

(c) Find the maximum value of the function $\tilde{\mathcal{D}}(\hat{\omega}, 6)$.

(d) Determine an expression for $y(t)$, the output of the D-to-C converter (as a sum of sinusoids).

In MATLAB consult help on `diric` for more information about computing the *Dirichlet* function; also there is a function called `dirich()` on the web site.

PROBLEM 7.4*:

The intention of the following MATLAB program is to filter a sinusoid via the `conv` function. However, the cosine signal has a starting point at $n = 0$; so we assume that it is zero for $n < 0$.

```
omegahat = pi/2;
nn = [ 0:4000 ];
xn = 6*cos(omegahat*nn - pi/2);
bb = ones(1,6)/6;
yn = conv( bb, xn );
```

(a) Determine a formula for $\mathcal{H}(\hat{\omega})$ for this FIR filter.

(b) Make a plot of the magnitude of $\mathcal{H}(\hat{\omega})$ and label *all* the frequencies where $|\mathcal{H}(\hat{\omega})|$ is zero. Use `freqz(bb, 1, ww)` in MATLAB, where `ww` is a vector of frequencies that defines a dense grid for $\hat{\omega}$.

(c) Use convolution to determine a formula (or table) for $y[n]$, the signal contained in the vector `yn`. Give the individual values for $n = 0, 1, 2, 3, 4, 5$, and then provide a general formula for $y[n]$ that is correct for $6 \le n \le 4000$. This formula should give numerical values for the amplitude, phase and frequency of $y[n]$. Hint: the formula is a sinusoid for $n \ge 6$.

(d) Give at least one different value of `omegahat` such that the output is guaranteed to be zero, for $n \ge 6$.

PROBLEM 7.5*:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1: \quad y_1[n] = x_1[n] + x_1[n-2]$$

$$\mathcal{S}_2: \quad y_2[n] = 7x_2[n-5] + 7x_2[n-6]$$

$$\mathcal{S}_3: \quad \mathcal{H}_3(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the difference equation for $\mathcal{S}_3$.

(b) Determine the frequency response of the first two systems: $\mathcal{H}_i(\hat{\omega})$ for $i = 1, 2$.

(c) Determine the frequency response of the overall cascaded system.

(d) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.