GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2000
Problem Set #4

Assigned: 16-Sept-00
Due Date: Week of 25-Sept-00

Reading: The notes on *Fourier Series.* In *DSP First*, Chapter 4 on *Sampling.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 4.1*:

Let $x(t)$ be the periodic signal

$$x(t) = \Big[2 + 3\cos(150\pi t - 0.2\pi)\Big]\cos(500\pi t)$$

(a) What is the fundamental frequency of $x(t)$?

(b) A periodic signal may be expanded in a Fourier series expansion as

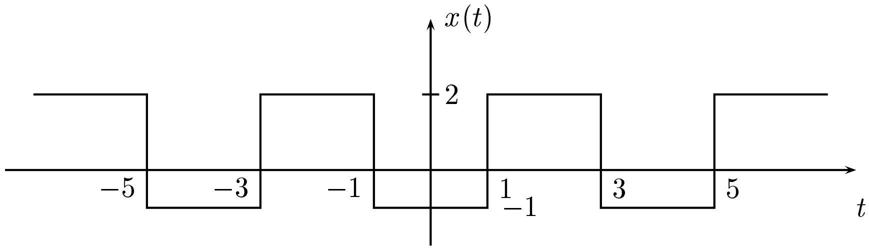
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t}$$

Find the Fourier series coefficients a_k for the signal above.

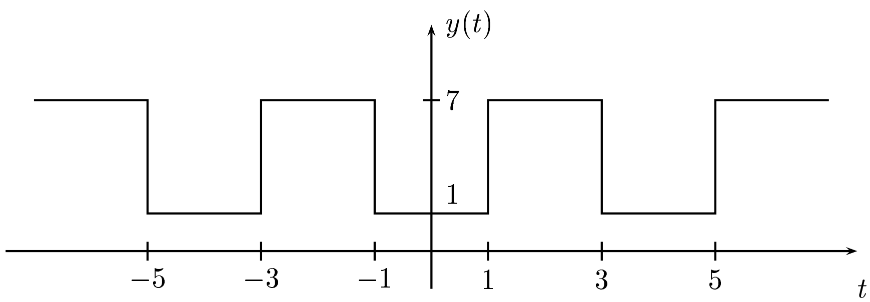
(c) Plot the coefficients a_k versus k.

PROBLEM 4.2*:

Let $x(t)$ be the periodic signal shown in the figure below, with Fourier series coefficients, a_k.



(a) Consider the signal shown below,

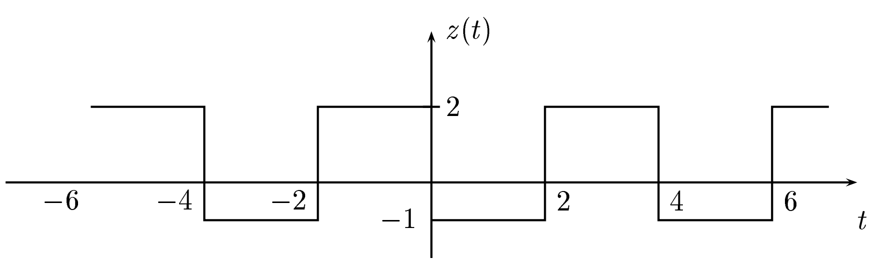


which is related to $x(t)$ by

$$y(t) = 2x(t) + 3$$

Express the Fourier series coefficients for this signal, b_k, in terms of the coefficients a_k for $x(t)$. Hint: This is a simple relationship, and finding it should not require that you compute any coefficients explicitly.

(b) Consider the signal shown below,



which is related to $x(t)$ by

$$z(t) = x(t-1)$$

Express the Fourier series coefficients for this signal, b_k, in terms of the coefficients a_k for $x(t)$. Hint: Again, this is a simple relationship, and finding it should not require that you compute any coefficients explicitly.

PROBLEM 4.3*:

A periodic signal $x(t) = x(t + T_0)$ is described *over one period*, $0 \le t \le T_0$, by the equation

$$x(t) = t \qquad -T_0/2 \le t \le T_0/2$$

(a) Make a sketch of the periodic function $x(t)$.

(b) Determine the D.C. coefficient of the Fourier Series, a_0.

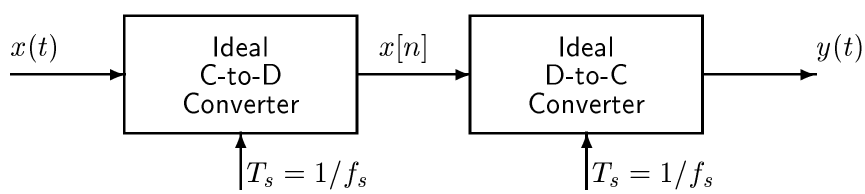
(c) Use the Fourier *analysis* integral[1]

$$a_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\omega_0 t}dt$$

to determine a general formula[2] for the Fourier Series coefficients a_k. Your final result for a_k should depend on k. Note: the frequency ω_0 would be given in rads/sec, but it does not have a specific value. However, you can simplify your formulas by using the identity $\omega_0 T_0 = 2\pi$.

(d) Use the Fourier Series coefficients to sketch the spectrum of $x(t)$ for the case $\omega_0 = 2\pi(\frac{1}{4})$ rad/sec and $t_c = \frac{1}{2}T_0$. Include *only* those frequency components corresponding to $k = 0, \pm1, \pm2, \pm3$. Label each component with its frequency and its complex amplitude (i.e., numerical values of magnitude and phase).

PROBLEM 4.4*:



Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period T_s to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [15 + 30\sin(250\pi t)]\cos(1000\pi t)$$

(a) Sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. *Hint: Recall the AM spectrum from a previous homework set.*

(b) Is this waveform periodic? If so, what is the period?

(c) What is the minimum sampling rate f_s that can be used in the following system so that $y(t) = x(t)$?

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[1]The Fourier integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

[2]The Fourier integral requires integration by parts—an opportunity to use your calculus skills.

PROBLEM 4.5*:

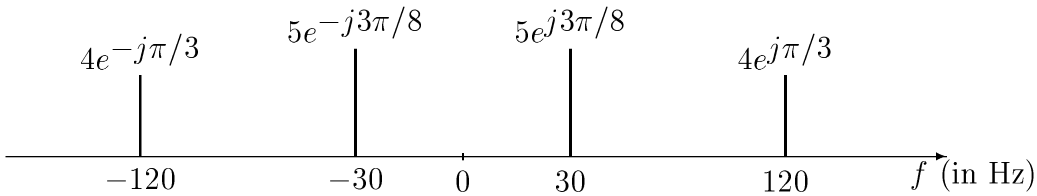
Again consider the ideal C-to-D converter and ideal D-to-C converter shown in previous problem.

(a) Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 4\cos(0.125\pi n + \pi/8)$$

If the sampling rate of the C-to-D converter is $f_s = 2000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 2000 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/2000$ secs.

(b) Now if the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the spectrum for $x[n]$ when $f_s = 120$ samples/sec. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.