8.1 (a) This problem illustrates one particular advantage of Z-transforms.

$$y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k] \qquad H(z) = \frac{1}{5}\sum_{k=0}^{4} z^{-k}$$

$$x_1[n] = 10\,\delta[n-50] \qquad X_1(z) = 10\,z^{-50}$$

$$Y_1(z) = X(z)\cdot H(z) = \frac{1}{5}\left(z^{0} + z^{-1} + z^{-2} + z^{-3} + z^{-4}\right)\left(10\,z^{-50}\right)$$

$$= 2\left(z^{-50} + z^{-51} + z^{-52} + z^{-53} + z^{-54}\right)$$

$$y_1[n] = 2\left(\delta[n-50] + \delta[n-51] + \delta[n-52] + \delta[n-53] + \delta[n-54]\right)$$

8.1 (b) $x_2[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$

$$Y_2(z) = X_2(z)\cdot H(z)$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2										
		.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2									
			.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2								
				.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2							
					.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2						
$y_2[n]$	.2	.4	.6	.8	1	1	1	1	1	1	1	.8	.6	.4	.2	0	0	0	0	0	.....

8.1(c) $x_3[n] = 4\cos\left(\frac{\pi n}{10} + \pi/2\right) + 3\cos\left(\frac{4\pi}{10} - \pi\right)$

Use the frequency domain. From $y[n] = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$ we have

$$H(e^{j\hat{\omega}}) = \frac{1}{5}\sum_{k=0}^{4} e^{-jk\hat{\omega}}$$

$$H(\pi/10) = \frac{1}{5}\left(1 + e^{-j\pi/10} + e^{-j2\pi/10} + e^{-j3\pi/10} + e^{-j4\pi/10}\right) = 0.9e^{-j0.63}$$

$$H\left(\frac{4\pi}{10}\right) = \frac{1}{5}\left(1 + e^{-j.4\pi} + e^{-j.8\pi} + e^{-j1.2\pi} + e^{-j1.6\pi}\right) = 0$$

$$y_3[n] = 4(0.9)\cos\left(\frac{\pi n}{10} + \pi/2 - 0.63\right) + 0$$

$$y_3[n] = 3.6\cos\left(\frac{\pi n}{10} + 0.94\right)$$

8.2(a) $y[n] = 4x[n] - 4x[n-4]$

$$H(z) = 4 - 4z^{-4}$$

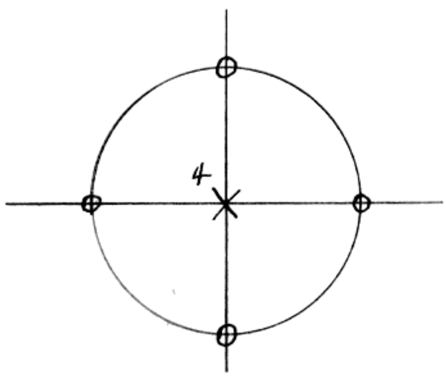
$$\frac{z^4}{z^4}H(z) = \frac{4}{z^4}(z^4 - 1) \qquad 4 \text{ poles at } z = 0$$

zeros at roots of $z^4 = 1 = e^{j0}$ $\quad z = 1$

$e^{j2\pi}$ $\quad z = e^{j\pi/2}$

$e^{j4\pi}$ $\quad z = e^{j\pi}$

$e^{j6\pi}$ $\quad z = e^{j3\pi/2}$



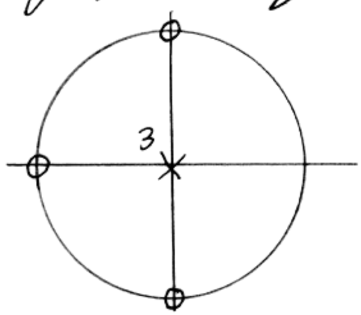
8.2(b) $h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$

$H(z) = -(1 + z^{-1} + z^{-2} + z^{-3})$

$\frac{z^3 H(z)}{z^3} = \frac{-1}{z^3}(1 + z + z^2 + z^3)$ 3 poles at $z=0$

The zeros of $z^3 + z^2 + z + 1$ can be found by factoring. $(z+1)(z^2+1)$

$z=-1 \quad z = \pm j = e^{\pm j\pi/2}$



8.2(c) $H(e^{j\hat{\omega}}) = [4j\sin(4\hat{\omega})]e^{-j5\hat{\omega}}$

Using Euler, $H(e^{j\hat{\omega}}) = \frac{4j}{2j}(e^{j4\hat{\omega}} - e^{-j4\hat{\omega}})e^{-j5\hat{\omega}}$

$= 2[e^{-j\hat{\omega}} - e^{-j9\hat{\omega}}] \Longrightarrow 2[z^{-1} - z^{-9}] = \frac{2}{z^9}(z^8-1)$

There is a 9th order pole at $z=0$. The zeros are found as below. $z^8 = 1 = e^{j0}$ $\quad z=1$

$= e^{j2\pi/8} \quad z = e^{j\pi/4}$

$= e^{j4\pi/8} \quad z = e^{j\pi/2}$

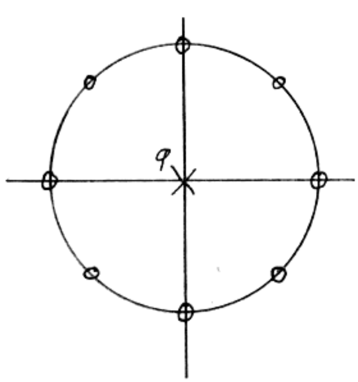
$= e^{j6\pi/8} \quad z = e^{j\frac{3\pi}{4}}$

$= e^{j8\pi/8} \quad z = e^{j\pi}$

$= e^{j10\pi/8} \quad z = e^{j\frac{5\pi}{4}}$

$= e^{j12\pi/8} \quad z = e^{j\frac{6\pi}{4}}$

$= e^{j\frac{14\pi}{8}} \quad z = e^{j\frac{7\pi}{4}}$



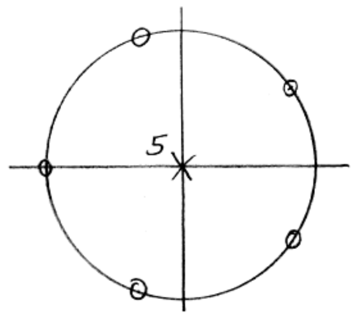
8.2 (d) $h[n] = \delta[n] + \delta[n-5]$

$$H(z) = 1 + z^{-5} \qquad \frac{z^5}{z^5} H(z) = \frac{1}{z^5}\left(z^5 + 1\right)$$

5th order pole at $z=0$

5 zeros at

$$z^5 = -1 = e^{j\pi} \qquad z = e^{j\pi/5}$$
$$e^{j3\pi} \qquad z = e^{j\frac{3\pi}{5}}$$
$$e^{j5\pi} \qquad z = e^{j\pi}$$
$$e^{j7\pi} \qquad z = e^{j\frac{7\pi}{5}}$$
$$e^{j9\pi} \qquad z = e^{j\frac{9\pi}{5}}$$

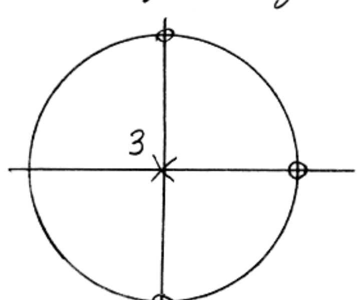


8.3(a) $y_1[n] = x[n] + x[n-2] \qquad y[n] = y_1[n] - y_1[n-1]$

$$H_1(z) = 1 + z^{-2} \qquad H_2(z) = 1 - z^{-1}$$
$$H(z) = (1+z^{-2})(1-z^{-1}) = 1 - z^{-1} + z^{-2} - z^{-3}$$

(b) $\frac{z^3}{z^3} H(z) = \frac{1}{z^3}\left(z^3 - z^2 + z - 1\right)$ 3 poles at $z=0$

$$H(z) = \frac{1}{z^3}(z-1)(z^2+1)$$



(c) $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

(d) $H(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$

8.4 $H(z) = (1+z^{-1})(1-e^{j2\pi/5}z^{-1})(1-e^{-j2\pi/5}z^{-1})$

(a) $H(z) = (1+z^{-1})(1-(e^{j\frac{2\pi}{5}}+e^{-j2\pi/5})z^{-1}+z^{-2})$

$$= (1+z^{-1})(1-2\cos\tfrac{2\pi}{5}z^{-1}+z^{-2})$$

$$= (1+z^{-1})(1-0.618z^{-1}+z^{-2}) = 1 - 0.618z^{-1} + z^{-2}$$
$$+ 1.0z^{-1} - 0.618z^{-2} + z^{-3}$$

$$= 1 + 0.382z^{-1} + 0.382z^{-2} + z^{-3}$$

$$Y(z) = H(z)\cdot X(z) = (1+0.382z^{-1}+0.382z^{-2}+z^{-3})(-z^{-2}-z^{-3}-z^{-4})$$

$$= -z^{-2} - 0.382z^{-3} - 0.382z^{-4} - z^{-5}$$
$$-1.0z^{-3} - 0.382z^{-4} - 0.382z^{-5} - z^{-6}$$
$$-1.0z^{-4} - 0.382z^{-5} - 0.382z^{-6} - z^{-7}$$
$$= -z^{-2} - 1.382z^{-3} - 1.764z^{-4} - 1.764z^{-5} - 1.382z^{-6} - z^{-7}$$

$$y[n] = -\delta[n-2] - 1.382\delta[n-3] - 1.764\delta[n-4]$$
$$-1.764\delta[n-5] - 1.382\delta[n-6] - \delta[n-7]$$

(b) Since $H(z) = (z+1)(z-e^{j2\pi/5})(z-e^{-j2\pi/5})/z^3$

it has zeros at

$$\left.\begin{aligned} z &= -1 = e^{j\pi} = e^{-j\pi} \\ z &= e^{j2\pi/5} \\ z &= e^{-j2\pi/5} \end{aligned}\right\}$$

these are all located <u>on</u> the unit circle

Zeros on the unit circle are zeros of the frequency response $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

$\Rightarrow$ The frequency response is zero at

$$\hat{\omega} = \pm 2\pi/5 \text{ and } \pm\pi$$

When the input is $x[n] = Ae^{j\varphi}e^{j\hat{\omega}n}$

the output will be $y[n] = H(e^{j\hat{\omega}})Ae^{j\varphi}e^{j\hat{\omega}n}$

Thus the output will be zero when $H(e^{j\hat{\omega}}) = 0$

So we get zero output for

$$\hat{\omega} = -\pi, -2\pi/5, 2\pi/5, \pi$$

8.5 $y[n] = x[n] + a\,x[n-1] + b\,x[n-2]$

$Y(z) = 1 + az^{-1} + bz^{-2}$

for $f = 60$ Hz, $\hat{\omega} = \frac{2\pi 60}{8000} = \frac{3\pi}{200}$

To accomplish nulling, a conjugate pair of zeros is needed at $z = e^{j\frac{3\pi}{200}}$ $z = e^{-j\frac{3\pi}{200}}$

$$H(z) = \left(z - e^{j\frac{3\pi}{200}}\right)\left(z - e^{-j\frac{3\pi}{200}}\right) = z^2 - \left(2\cos\frac{3\pi}{200}\right)z + 1$$

However, two poles are needed to get $H(z)$ into the required form:

$$H(z) = \frac{1}{z^2}\left(z^2 - \left(2\cos\frac{3\pi}{200}\right)z + 1\right) = 1 - \left(2\cos\frac{3\pi}{200}\right)z^{-1} + z^{-2}$$

Identifying corresponding terms in the original equation:

$$a = -2\cos\left(\frac{3\pi}{200}\right)$$

$$b = 1$$

8.6

(a) $$\int_{-\infty}^{t} e^{-(\tau-1)}\,d\tau = e\int_{-\infty}^{t} e^{-\tau}\,d\tau = -e\left[e^{-\tau}\Big|_{-\infty}^{t}\right. = -e\left[e^{-\tau} - e^{\infty}\right]$$
$$= \infty$$

(b) $$\int_{0}^{t} e^{-2(t-\tau)}e^{\tau}\,d\tau = \int_{0}^{t} e^{-2t}e^{2\tau}e^{\tau}\,d\tau = e^{-2t}\int_{0}^{t} e^{3\tau}\,d\tau$$

$$= \frac{e^{-2t}}{3}\left[e^{3\tau}\Big|_{0}^{t}\right. = \frac{e^{-2t}}{3}\left(e^{3t} - e^{0}\right) = \frac{1}{3}\left(e^{t} - e^{-2t}\right)$$

8.6(c)

$$\int_0^t \tau \cdot e^{(t-\tau)} \cdot d\tau = \int_0^t \tau \cdot e^t \cdot e^{-\tau} \cdot d\tau$$

$$= e^t \cdot \int_0^t \tau \cdot e^{-\tau} \cdot d\tau$$

$$= e^t \cdot \left\{ -\tau \cdot e^{-\tau}\Big|_0^t - \int_0^t \left(-e^{-\tau}\right) \cdot d\tau \right\}$$

$$= e^t \cdot \left\{ -t \cdot e^{-t} - \left(e^{-\tau}\Big|_0^t \right) \right\}$$

$$= e^t \cdot \left\{ -t \cdot e^{-t} - \left(e^{-t} - 1 \right) \right\}$$

$$= e^t \cdot \left\{ -(t+1) \cdot e^{-t} + 1 \right\}$$

$$= -t - 1 + e^t$$

For integration by parts, the following was used.

$$u = \tau,\ dv = e^{-\tau} \cdot d\tau$$
$$\therefore du = d\tau,\ v = -e^{-\tau}$$
$$\int u \cdot dv = uv - \int v \cdot du$$