GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2000
Problem Set #11

Assigned: 11-Nov-00
Due Date: Week of 27-November-00

Quiz #3 will be on 20-November (Monday). It will cover material represented in Problem Sets #8, #9 and #10.

There will be a quiz review on Sunday (November 19) at 7:00 PM in the ECE Auditorium.

Reading: Finish reading Chapter 12 and begin reading Chapter 13.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

All **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 11.1*:

In each of the following cases, use known Fourier transform pairs together with Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = u(t+3)u(3-t) \quad \Longleftrightarrow \quad X(j\omega) =$

(b) $x(t) = \sin(4\pi t)\sin(50\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(c) $x(t) = \dfrac{\sin 4\pi t}{\pi t}\sin(50\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(d) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = \dfrac{\sin^2(200\omega)}{\omega^2}$

(e) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = \cos^2(\omega)$

PROBLEM 11.2*:

Let $x(t)$ be a triangular pulse defined by

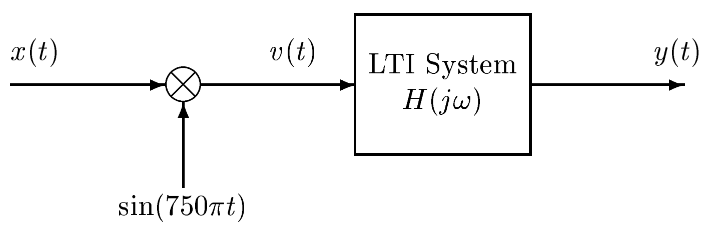
$$x(t) = \begin{cases} 1 - |t| & ; \quad |t| < 1 \\ 0 & ; \quad \text{else} \end{cases}$$

(a) By taking the derivative of $x(t)$, use the derivative property to find the Fourier transform of $x(t)$. Hint: Express the derivative as a sum of two pulses, one with an amplitude of one, and the other with an amplitude of minus one. From your table of Fourier transforms, and the delay property, you should be able to write down the transform without any integration.

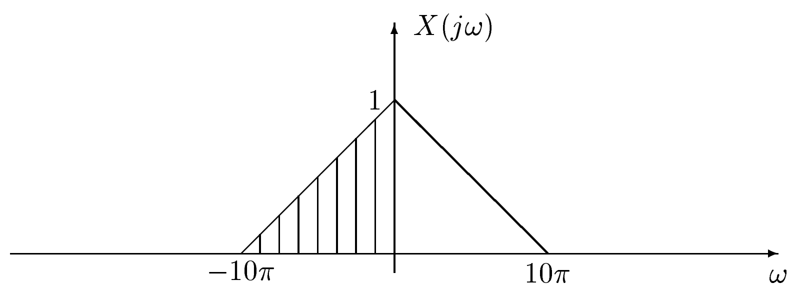
(b) Find the Fourier transform of $x(t)$ by differentiating $x(t)$ twice and using the derivative property. Compare you results.

PROBLEM 11.3*:

Consider the following amplitude modulation system:



Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below



and the linear system has the frequency response of an ideal bandpass filter:

$$H(j\omega) = \begin{cases} 1 & 750\pi < |\omega| < 760\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot the Fourier transform $H(j\omega)$ of the ideal BPF specified above.

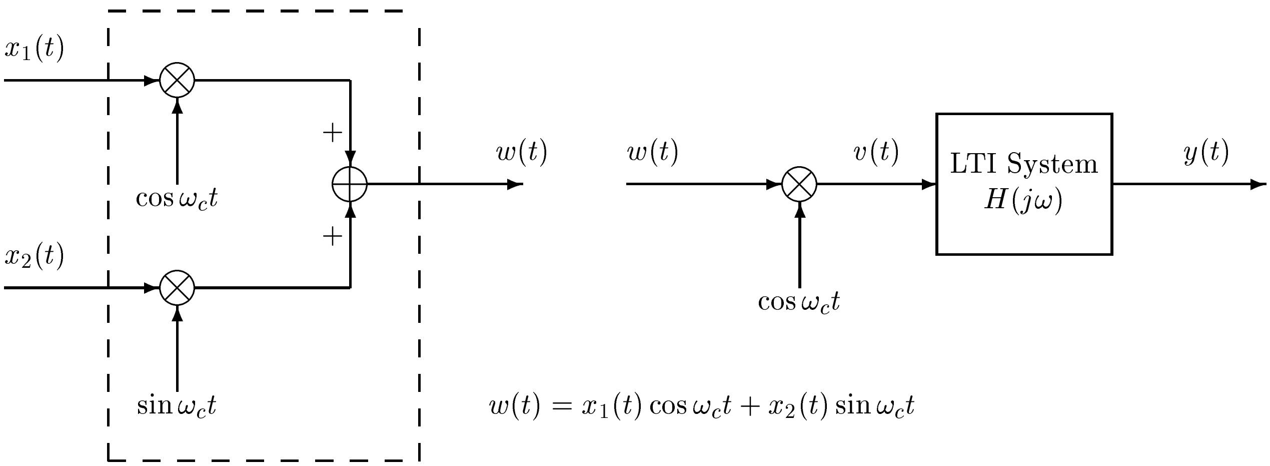
(b) Plot the Fourier transform $V(j\omega)$ of the signal $v(t)$ at the output of the multiplier.

(c) Plot the Fourier transform $Y(j\omega)$ of the output signal $y(t)$ from the filter.

Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plots of $V(j\omega)$ and $Y(j\omega)$.

PROBLEM 11.4*:

The system in the dashed box below is called a *quadrature modulation system.* It is a method of sending two bandlimited signals over the same channel.



Quadrature Modulator **Demodulator = Mixer, then Filter**

Assume that both input signals are bandlimited with highest frequency ω_m; i.e., $X_1(j\omega) = 0$ for $|\omega| \geq \omega_m$ and $X_2(j\omega) = 0$ for $|\omega| \geq \omega_m$, where $\omega_c \gg \omega_m$.

(a) Determine an expression for the Fourier transform $W(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Make a sketch of $W(j\omega)$. Assume simple shapes (but different) for the bandlimited Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$, and use them in making your sketch of $W(j\omega)$.

(b) From the expression found in part (a) and the sketch that you drew, you should see that $W(j\omega) = 0$ for $|\omega| \leq \omega_a$ and for $|\omega| \geq \omega_b$. Determine ω_a and ω_b.

(c) Given the trigonometric identities $2\sin\theta\cos\theta = \sin 2\theta$ and $2\cos^2\theta = (1+\cos 2\theta)$, show that in the "demodulator" figure on the right above, the output of the mixer is:

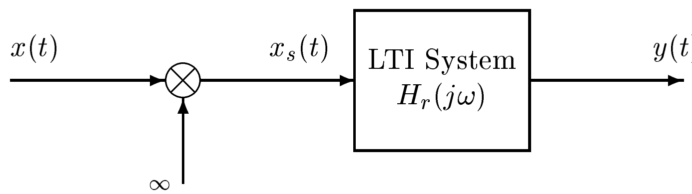
$$v(t) = \tfrac{1}{2}x_1(t)(1+\cos 2\omega_c t) + \tfrac{1}{2}x_2(t)\sin 2\omega_c t$$

(d) The signal $v(t)$ as determined in part (c) is the input to an LTI system. Determine the frequency response of that system so that its output is $y(t) = x_1(t)$. Give your answer as a carefully labeled plot of $H(j\omega)$.
You may use the result of part (c) even if you were unable to derive it.

(e) (e) Draw a block diagram of a demodulator system whose output will be $x_2(t)$ when its input is $w(t)$. This requires that you change the mixer.

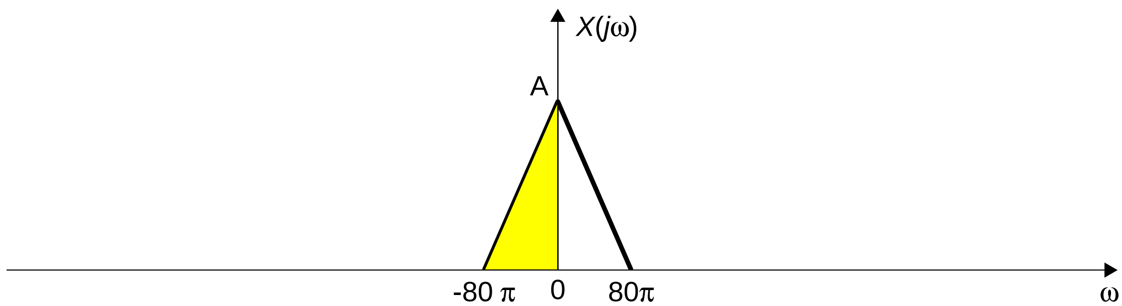
PROBLEM 11.5*:

The Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The "typical" bandlimited Fourier transform of the input is depicted below:



(a) For the input with Fourier transform depicted above, use the Sampling Theorem to choose the sampling rate $\omega_s = 2\pi/T_s$ so that $x_r(t) = x(t)$? Plot $X_s(j\omega)$ for the value of $\omega_s = 2\pi/T_s$ that is two times the Nyquist rate.

(b) If $\omega_s = 2\pi/T_s = 140\pi$ in the above system and $X(j\omega)$ is as depicted above, plot the Fourier transform $X_s(j\omega)$ and show that aliasing occurs. There will be an infinite number of shifted copies of $X(j\omega)$, so indicate what the pattern is versus ω.

(c) For the conditions of part (b), determine and sketch the Fourier transform of the output $X_r(j\omega)$ if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T_s & |\omega| \leq \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$