GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #5

Assigned: 3-Feb-01
Due Date: Week of 12-Feb-01

Reading: Chapter 4 on *Sampling.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 5.1*:

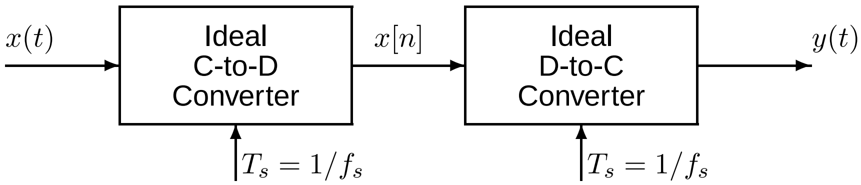


Figure 1: Ideal sampling and reconstruction system.

Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [20 + 20\cos(600\pi t)]\cos(2000\pi t - \pi/2)$$

(a) Use Euler's formulas for the sine and cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the minimum sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 4000$. Your plot should include labels on the frequency (on the $\hat{\omega}$ scale), amplitude and phase of each spectrum component.

PROBLEM 5.2*:

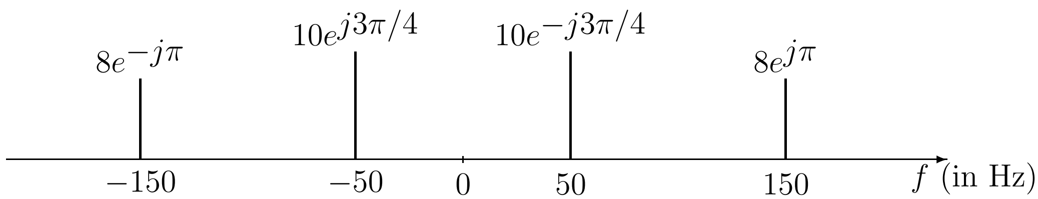
Again consider the ideal sampling and reconstruction system shown in Figure 1 of the previous problem.

(a) Suppose that the discrete-time signal $x[n]$ in Figure 1 is given by the formula

$$x[n] = 4\cos(0.2\pi n + \pi/8)$$

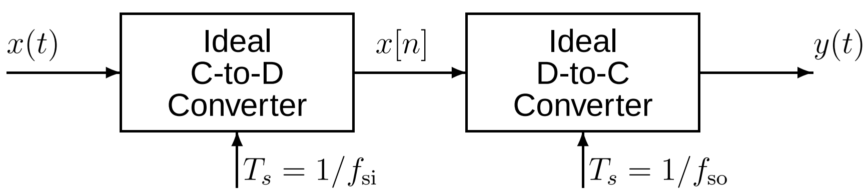
If the sampling rate of the C-to-D converter is $f_s = 8000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 8000 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_\mathrm{s}) = x_2(nT_\mathrm{s})$ if $T_\mathrm{s} = 1/8000$ secs.

(b) Now if the input $x(t)$ to the system in Figure 1 of Problem 5.1 has the two-sided spectrum representation shown below,what is the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input $x(t)$?



(c) Determine the spectrum for $x[n]$ when $f_\mathrm{s} = 150$ samples/sec. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.

PROBLEM 5.3*:



(a) Suppose that the input $x(t)$ is given by

$$x(t) = 10 + 10\cos(2\pi(2000)t - \pi) + 8\cos(2\pi(7000)t - 3\pi/4)$$

Determine the spectrum for $x[n]$ when $f_\mathrm{si} = 10000$ samples/sec. Make a plot for your answer, making sure to label the frequency, amplitude and phase of each spectral component.

(b) Using the discrete-time spectrum from part (a), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_\mathrm{so} = 10000$ Hz.

(c) Again using the discrete-time spectrum from part (b), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_\mathrm{so} = 20000$ Hz. In other words, the sampling rates of the two converters are different.

PROBLEM 5.4*:

A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.1$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} 32 & 0 \le n \le 4 \\ 32(.5)^{n-4} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus n.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its non-zero region.

PROBLEM 5.5*:

In the rotating disk and strobe demo described in Chapter 4 of *DSP First,* we observed that different flashing rates of the strobe light would make the spot on the disk stand still.

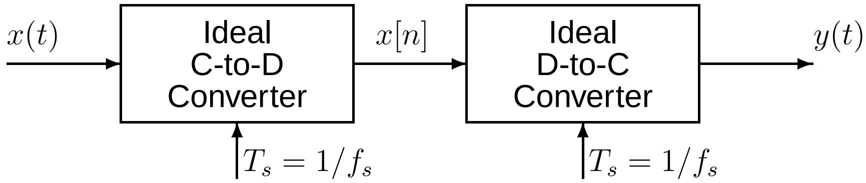
(a) Assume that the disk is rotating in the clockwise direction at a constant speed of 15 revolutions per second. Express the movement of the spot on the disk as a rotating complex phasor.

(b) If the strobe light can be flashed at a rate of n flashes *per second* where n is an integer greater than zero, determine all possible flashing rates such that the disk can be made to stand still.
NOTE: the only possible flashing rates are integers: 1 per second, 2 per second, 3 per second, etc.

(c) Now assume that the flashing rate is fixed so that the interval between flashes is 100 milliseconds. Explain how the spot will move and write a complex phasor that gives the position of the spot at each flash.

(d) Draw a spectrum plot of the discrete-time signal in part (c) to explain your answer.

PROBLEM 5.6:



Chirps are very useful signals for probing the behavior of sampling operations and illustrating the "folding" type of aliasing (see Fig. 4.4 in the book).

(a) If the input to the ideal C/D converter is $x(t) = 7\cos(1800\pi t + \pi/4)$, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the formula for the output signal.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(2000\pi t - 400\pi t^2) \qquad \text{for } 0 \leq t \leq 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the DSP-First function `plotspec()`.