6.1 $x(t) = 10\cos(800\pi t + \pi/4) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t)$

$f = 400Hz$ $f = 600Hz$ $f = 800Hz$

$\therefore$ by Nyquist rule $f_s \geq 2f_{MAX}$

$$f_s = 1600$$

6.2(a) NO

$$A\cos(\omega_0 n T_s + \phi) = A\cos[\omega_0(n+N)T_s + \phi]$$

$$= A\cos[\omega_0 n T_s + \underbrace{\omega_0 N T_s}_{2\pi} + \phi]$$

$\omega_0 N T_s = 2\pi$ $T_s = \dfrac{2\pi}{\omega_0 N}$

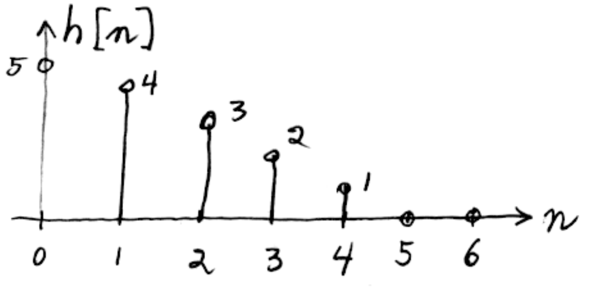
(b) $T_s = \dfrac{2\pi}{2000\pi(100)} = 10^{-5}$ sec.

6.3(a)

k	$5-k$
0	5
1	4
2	3
3	2
4	1

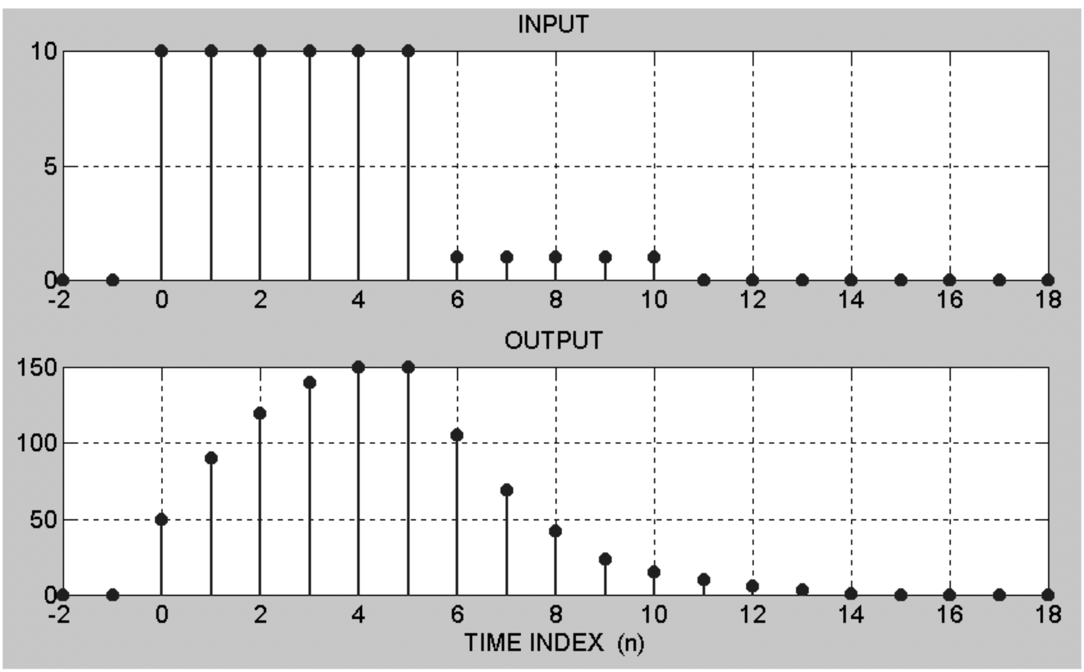
$[b_k] = [5\ \ 4\ \ 3\ \ 2\ \ 1]$

(b)

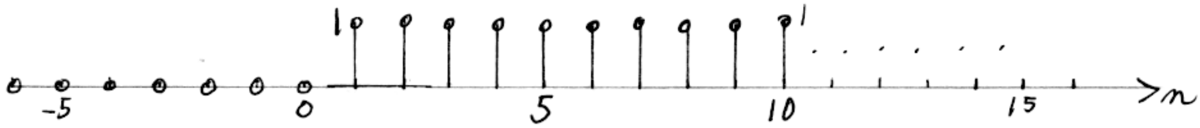


(c)

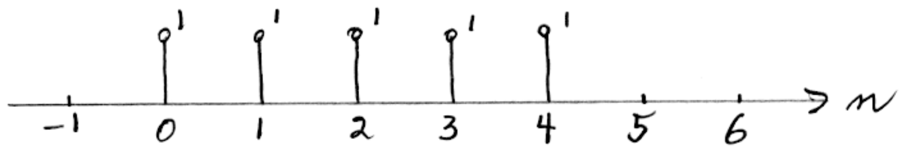
$n =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$h[n]$	5	4	3	2	1												
$x[n]$	10	10	10	10	10	10	1	1	1	1	1	0	0	0	. . . .		
	50	40	30	20	10												
		50	40	30	20	10											
			50	40	30	20	10										
				50	40	30	20	10									
					50	40	30	20	10								
						50	40	30	20	10							
							5	4	3	2	1						
								5	4	3	2	1					
									5	4	3	2	1				
										5	4	3	2	1			
											5	4	3	2	1		
	50	90	120	140	150	150	105	69	42	24	15	10	6	3	1	$= y[n]$	

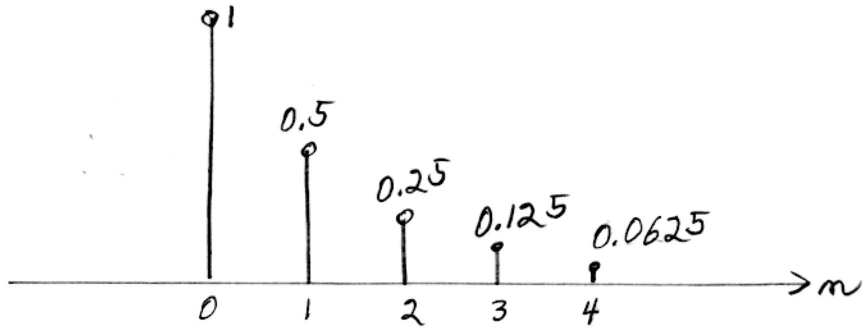


6.4(a)



(b) $u[n] - u[n-5]$





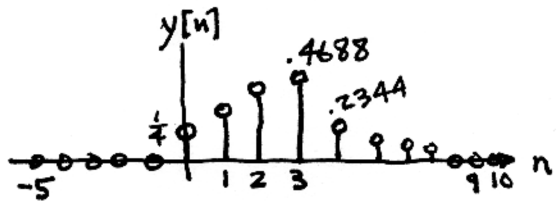
6.4(c) For a four point running averager, the impulse response is

$$h[n] = \frac{1}{4}\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\right)$$

Using convolution:

$n =$	0	1	2	3	4	5	6	7	8	9	10
$h[n] =$	1/4	1/4	1/4	1/4							
$x[n] =$	1	1/2	1/4	1/8	1/16						
	1/4	1/8	1/16	1/32	1/64						
		1/4	1/8	1/16	1/32	1/64					
			1/4	1/8	1/16	1/32	1/64				
				1/4	1/8	1/16	1/32	1/64			
$y[n] =$	1/4	.375	.4375	.4688	.2344	.1094	.0469	.0156			

for $n < -5$, $u[n] = 0$
for $n > 10$, $u[n] = 1$



$y[n] = 0$ for $n < 0$
and for $n > 7$

NOTE: $\text{length}\{y[n]\} = 8$
$= \text{len}\{x\} + \text{len}\{h\} - 1$
$= 5 + 4 - 1 = 8$

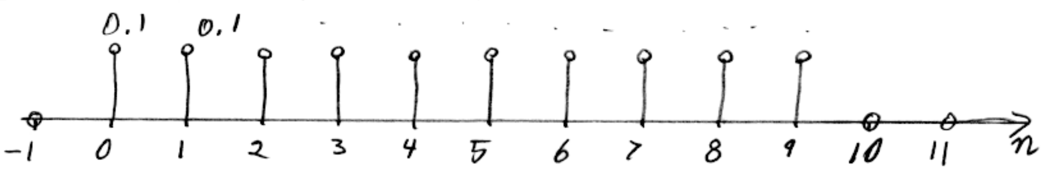
6.5 $y[n] = \sum_{k=5}^{10} b_k x[n-k]$

$x[n] = \sum_{m=5}^{20} c_m$

There is no output until $x[n-k] > 0$, or $[n-k] \geq 5$.
Since k must be ≥ 5, therefore $[n-5] \geq 5$, $n \geq 10$.
$\therefore N_3 = 10$.
No output for $x[n-k] = 0$ for $[n-k] > 20$
Since k must be ≤ 10, $[n-10] < 21$, $\therefore n \leq 30$
$\therefore N_4 = 30$

6.6 (a) $v[n] = x[n] * h_1[n] = \sum_{k=0}^{9} h_1[k]\, x[n-k]$

(b) $x[n] = \delta[n]$ $\quad v[n] = h_1[n]$



(c) $y[n] = v[n] - v[n-1]$ $\quad v[n] = \delta[n]$

$h[n] = \delta[n] - \delta[n-1]$

(d) $x[n] = \delta[n]$
$h[n] = h_1[n] * h_2[n] = 0.1\,\delta[n] - 0.1\,\delta[n-10]$

(e) $y[n] = 0.1\,x[n] - 0.1\,x[n-10]$

6.7 (a) $y[n] = x[n-2] + 2x[n] + x[n+2]$

This has the same form as an FIR filter with multiplies, adds and delays, so it is also LINEAR and TIME-INV.

It is NOT CAUSAL. Here is a counter-example:

Let $x[n] = \delta[n]$ ← this input starts at $n=0$.

Then $y[n] = \delta[n-2] + 2\delta[n] + \delta[n+2]$

This output starts at $n=-2$, so it starts before the input ⇒ NON-CAUSAL

6.7(b) $y[n] = n\,x[n]$

CAUSAL? Yes $y[n_0] = n_0 x[n_0]$ means that $y[n_0]$ depends only on the input at $n=n_0$.

LINEAR? Yes

$$\left.\begin{aligned} y_1[n] &= n x_1[n] \\ y_2[n] &= n x_2[n] \end{aligned}\right\}$$

If $x[n] = \alpha x_1[n] + \beta x_2[n]$ then

$$\begin{aligned} y[n] &= n(\alpha x_1[n] + \beta x_2[n]) \\ &= \alpha(n x_1[n]) + \beta(n x_2[n]) \\ &= \alpha y_1[n] + \beta y_2[n] \end{aligned}$$

TIME-INV? No

When the input is $x[n] = \delta[n]$, the output is $y[n] = n x[n] = n\delta[n] = 0$ for all n.

When the input is a "shifted impulse", e.g. $x_1[n] = \delta[n-1]$ then the output is $y_1[n] = n\,\delta[n-1] = 1\cdot\delta[n-1]$

But $y_1[n]$ is not the shifted version of $y[n]$ because $y_1[n] \neq y[n-1]$

6.7(c) $y[n] = (x[-n])^2$ ← "FLIP" & SQUARE

<u>LINEAR?</u> <u>NO</u>

$$\left.\begin{aligned} y_1[n] &= (x_1[-n])^2 \\ y_2[n] &= (x_2[-n])^2 \end{aligned}\right\}$$

If $x[n] = \alpha x_1[n] + \beta x_2[+n]$ then

$$\begin{aligned} y[n] = (x[-n])^2 &= (\alpha x_1[-n] + \beta x_2[-n])^2 \\ &= \alpha^2 (x_1[-n])^2 + 2\alpha\beta x_1[-n]\, x_2[-n] + \beta^2 (x_2[-n])^2 \\ &\neq \alpha (x_1[-n])^2 + \beta (x_2[-n])^2 \end{aligned}$$

<u>TIME-INV?</u> <u>NO</u>

Let $x[n] = \delta[n-1]$, then the output is $y[n] = (\delta[-n-1])^2$

Since the system flips the input and squares the output is $y[n] = \delta[n+1]$ ← SAME

Now shift the input: $x_2[n] = x[n-1] = \delta[n-2]$.

The output is $y_2[n] = (\delta[-n-2])^2 = \delta[n+2]$ ← An impulse at $n=-2$

But $y_2[n] \neq y[n-1]$ because $y[n-1] = \delta[n-1+1] = \delta[n]$.

<u>CAUSAL?</u> <u>NO</u>

If $x[n] = \delta[n-7]$, then $y[-7] = (x[-(-7)])^2 = (x[7])^2 = 1$

So the output at $n=-7$, needs a value from the <u>future</u>.