GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #10

Assigned: 23 March
Due Date: Week of 2-April-01

Quiz #3 will be on 6-April (Friday). One page of notes will be allowed.

Reading: In *DSP First*, Chapter 11 (pp. 1100-1118) and Chapter 12 (pp. 1200-1214).

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 10.1*:

The impulse response of a continuous-time linear time-invariant system is

$$h(t) = \delta(t) - 0.1e^{-0.1t}u(t).$$

(a) Find the frequency response $H(j\omega)$ of the system. Express your answer as a single rational function with powers of $(j\omega)$ in the numerator and denominator.

(b) Plot the magnitude squared, $|H(j\omega)|^2 = H(j\omega)H^*(j\omega)$, versus ω. Also plot the phase $\angle H(j\omega)$ as a function of ω.

(c) At what frequency ω does the magnitude squared of the frequency response have it largest value? At what frequency is the magnitude squared of the frequency response equal to one half of its peak value? (This is referred to as the 3dB point of the filter since the frequency response magnitude measured in decibels, $10\log|H(j\omega)|^2$, is 3.01dB smaller at this frequency compared to its peak value when measured in decibels.

(d) Suppose that the input to this system is

$$x(t) = 10 + 20\cos(0.1t) + \delta(t - 0.2).$$

Use superposition to find the output $y(t)$. *Hint: To find the response of each term, use the easiest method, i.e., impulse response or frequency response.*

PROBLEM 10.2*:

A continuous-time LTI system is defined by the following input/output relation:

$$y(t) = x(t+1) + 2x(t) + x(t-2). \tag{1}$$

(a) Find the impulse response $h(t)$ of the system; i.e., determine the output when the input is an impulse.

(b) Substitute your answer for $h(t)$ into the the integral formula

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

to find the frequency response.

(c) Apply the system definition given in Eq. (1) directly to the input $x(t) = e^{j\omega t}$ for $-\infty < t < \infty$ and show that $y(t) = H(j\omega)e^{j\omega t}$, where $H(j\omega)$ is as determined in part (b).

PROBLEM 10.3*:

The delay property of Fourier transform states that if $X(j\omega)$ is the Fourier transform of $x(t)$, the the Fourier transform of $x(t-t_d)$ is $e^{-j\omega t_d}X(j\omega)$, i.e.,

$$x(t-t_d) \Longleftrightarrow e^{-j\omega t_d}X(j\omega).$$

Use this property to find the Fourier transforms of the following signals:

(a) $x(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

(b) $x(t) = \dfrac{\sin(100\pi(t-2))}{\pi(t-2)}$

(c) $x(t) = e^{-t}u(t) - e^{-t}u(t-4) = e^{-t}u(t) - e^{-4}e^{-(t-4)}u(t-4)$

PROBLEM 10.4*:

For each of the following cases, use the table of known Fourier transform pairs to complete the following Fourier transform pair relationships.

(a) Find $x(t)$ if $X(j\omega) = \dfrac{j\omega}{0.1+j\omega}e^{-0.2j\omega}$.

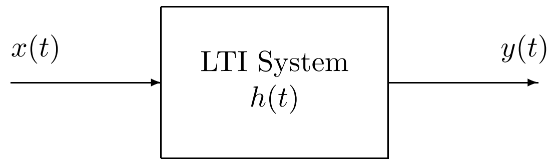
(b) Find $x(t)$ if $X(j\omega) = 2 + 2\cos(\omega)$.

(c) Find $x(t)$ if $X(j\omega) = \dfrac{1}{1+j\omega} - \dfrac{1}{2+j\omega}$.

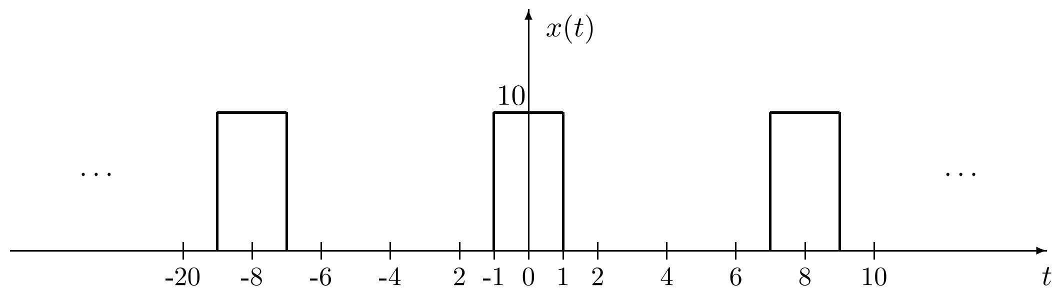
(d) Find $x(t)$ if $X(j\omega) = j\delta(\omega - 100\pi) - j\delta(\omega + 100\pi)$.

PROBLEM 10.5*:

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



The input can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \frac{10\sin(\pi k/4)}{\pi k}.$$

(a) Determine ω_0 in the Fourier series representation of $x(t)$. Also, write down the integral that must be evaluated to obtain the Fourier coefficients a_k.

(b) Plot the spectrum of the signal $x(t)$; i.e., make a plot showing the a_k's plotted at the frequencies $k\omega_0$ for $-4\omega_0 \le \omega \le 4\omega_0$.

(c) If the frequency response of the system is the ideal *highpass* filter

$$H(j\omega) = \begin{cases} 0 & |\omega| < \pi/8 \\ 1 & |\omega| > \pi/8 \end{cases}$$

plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: First determine what frequency is removed by the filter, and then determine what effect this will have on the waveform.*

(d) If the frequency response of the system is an ideal lowpass filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

where ω_c is the *cutoff frequency*, for what values of ω_c will the output of the system have the form

$$y(t) = A + B\cos(\omega_0 t + \phi)$$

where A and B are nonzero?

(e) If the frequency response of the LTI system is $H(j\omega) = 1 - e^{-j2\omega}$, plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: In this case it will be easiest to determine the impulse response $h(t)$ corresponding to $H(j\omega)$ and from $h(t)$ you can easily find an equation that relates $y(t)$ to $x(t)$. This will allow you to plot $y(t)$.*