ECE 2025   Spring 2001
Problem Set #11

Assigned: 30-March
Due Date: Week of 9-April-2001

Quiz #3 will be on 6-April (Friday). It will cover material represented in Problem Sets #8, #9 and #10.

There will be a quiz review on Sunday (5-April) at 8:00 PM in the ECE Auditorium.

Reading: Finish reading Chapter 12 and begin reading Chapter 13.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

All **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 11.1:

This problem appeared on Problem Set #11 of Fall, 2000.

In each of the following cases, use known Fourier transform pairs together with Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = u(t+3)u(3-t) \quad \Longleftrightarrow \quad X(j\omega) =$

(b) $x(t) = \sin(4\pi t)\sin(50\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(c) $x(t) = \dfrac{\sin 4\pi t}{\pi t}\sin(50\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

(d) $x(t) = \qquad \Longleftrightarrow \quad X(j\omega) = \left(\dfrac{\sin(200\omega)}{\omega}\right)^2$

(e) $x(t) = \qquad \Longleftrightarrow \quad X(j\omega) = \cos^2(\omega)$

PROBLEM 11.2:

This problem appeared on Problem Set #11 of Fall, 2000.

Let $x(t)$ be a triangular pulse defined by

$$x(t) = \begin{cases} 1 - |t| & ; \quad |t| < 1 \\ 0 & ; \quad \text{else} \end{cases}$$

(a) By taking the derivative of $x(t)$, use the derivative property to find the Fourier transform of $x(t)$. Hint: Express the derivative as a sum of two pulses, one with an amplitude of one, and the other with an amplitude of minus one. From your table of Fourier transforms, and the delay property, you should be able to write down the transform without any integration.

(b) Find the Fourier transform of $x(t)$ by differentiating $x(t)$ twice and using the derivative property. Compare your results.

PROBLEM 11.3*:

In the following, either the time-domain signal $x(t)$ or the Fourier transform is given. Use the tables of Fourier transforms and Fourier transform properties to determine the Fourier transform for each of the signals or the inverse Fourier transform for each of the given Fourier transforms. You may give your answer either as an equation or a carefully labelled plot, whichever is most convenient.

(a) $x(t) = \dfrac{d}{dt}\left(\dfrac{10\sin(200\pi t)}{\pi t}\right)$

(b) $x(t) = \dfrac{2\sin(400\pi t)}{\pi t}\cos(2000\pi t)$

(c) $x(t) = \displaystyle\sum_{n=-\infty}^{\infty} \delta(t - 10n)$ \qquad (Periodic impulse train)

(d) $X(j\omega) = \dfrac{e^{-j\omega 3}}{2 + j\omega}$

(e) $X(j\omega) = \dfrac{j\omega}{2 + j\omega}$

(f) $X(j\omega) = \dfrac{(j\omega)}{2 + j\omega}e^{-j\omega 3}$

(g) $X(j\omega) = \left(\dfrac{2\sin(\omega)}{\omega}\right)\displaystyle\sum_{k=-\infty}^{\infty}\left(\dfrac{2\pi}{10}\right)\delta(\omega - 2\pi k/10))$

   Work part (g) as a convolution problem in the time-domain.

PROBLEM 11.4*:

An LTI system has impulse response given by

$$h(t) = \frac{10 \sin[4\pi(t-1)]}{\pi(t-1)}.$$

(a) First make a detailed and accurately labeled sketch of $h(t)$. Mark the important amplitudes and time locations of peaks and zero crossings.

(b) Now determine the Fourier transform $H(j\omega)$ of this impulse response; i.e., $H(j\omega)$ is the frequency response of the system. Make detailed plots of $|H(j\omega)|$ and $\angle H(j\omega)$ versus ω. Label your plots carefully.

PROBLEM 11.5*:

The frequency response of an ideal lowpass LTI system is

$$H(j\omega) = \begin{cases} 10e^{-j\omega 0.0025} & |\omega| < 1000\pi \\ 0 & |\omega| > 1000\pi. \end{cases}$$

In each of the following cases, determine the Fourier transform of the input signal and then use frequency-domain methods to determine the corresponding output signal.

(a) Using frequency-domain methods, determine the output of the system when the input is

$$x(t) = \cos(200\pi t) + \frac{2\sin(2000\pi t)}{\pi t}.$$
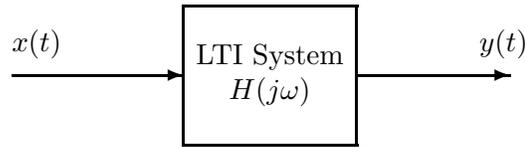
(b) Determine the output if the input is

$$x(t) = \cos(200\pi t) + \frac{2\sin(2000\pi t)}{\pi t} + \cos(3000\pi t).$$

(c) Determine the output if the input is

$$x(t) = \cos(200\pi t) + 2\delta(t).$$

(d) What is it about this system that accounts for the results you obtained above?

PROBLEM 11.6*:

$$x(t) \longrightarrow \boxed{\begin{array}{c}\text{LTI System} \\ H(j\omega)\end{array}} \longrightarrow y(t)$$

The impulse response of the above system is

$$h(t) = \frac{4\sin(\omega_{\text{co}}t)}{\pi t},$$

and the input to this system is a periodic signal (with period $T_0 = 1$) given by the following equation:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n) = \sum_{k=-\infty}^{\infty} e^{j2\pi kt}.$$

(a) Determine the Fourier transform $X(j\omega)$ of the input signal.[1] Plot $X(j\omega)$ over the range $-6\pi < \omega < 6\pi$.

(b) Determine the frequency response $H(j\omega)$ of the system and plot $|H(j\omega)|$ on the same graph as $X(j\omega)$ for the case $\omega_{\text{co}} = 5\pi$.

(c) Use your plot in (b) to determine $y(t)$, the output of the LTI system for the given input $x(t)$ when the cutoff frequency is $\omega_{\text{co}} = 5\pi$.

(d) How would you choose ω_{co} so that the output is a constant; i.e., $y(t) = C$ for all t. What is the constant C?

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[1]Recall   that   if $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$,   then   the   Fourier   transform   is
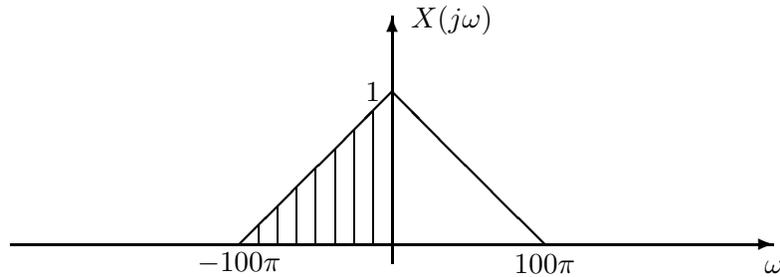
$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$, where $\omega_0 = 2\pi/T_0$.

PROBLEM 11.7*:

Consider the following amplitude modulation system:
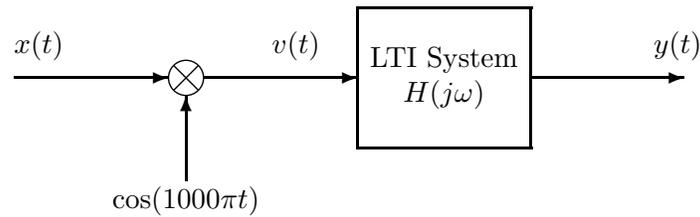
Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below

and the linear system has the frequency response of an ideal bandpass filter:

$$H(j\omega) = \begin{cases} 1 & 900\pi < |\omega| < 1000\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot the Fourier transform $H(j\omega)$ of the ideal BPF specified above. *Be sure to plot for both negative and positive frequencies.*

(b) Plot the Fourier transform $V(j\omega)$ of the signal $v(t)$ at the output of the multiplier.

(c) Plot the Fourier transform $Y(j\omega)$ of the output signal $y(t)$ from the filter. *Do not try to find $y(t)$.*

(d) The output signal is called a "single side-band" signal. Can you see why?

Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plots of $V(j\omega)$ and $Y(j\omega)$.