GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #12

Assigned: 8-April
Due Date: Week of 16-April-2001

The Final Exam will be on 4-May at 2:50 pm (Friday, period 15). It will cover will cover the entire course, with slight emphasis on material since Quiz #3.

Reading: Finish reading Chapter 13. This is important since several of the homework problems on this set are discussed directly in Chapter 13. See in Section 13.2.1 in particular.

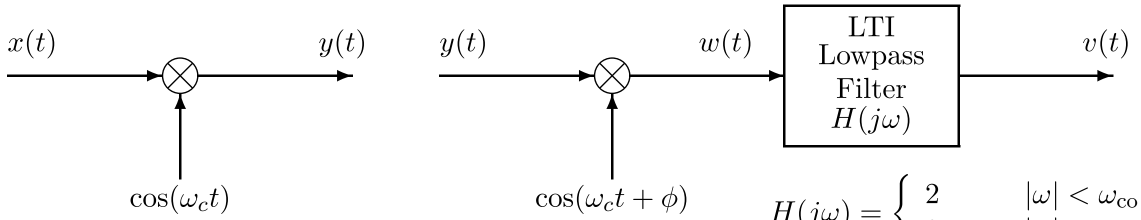
$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

All **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

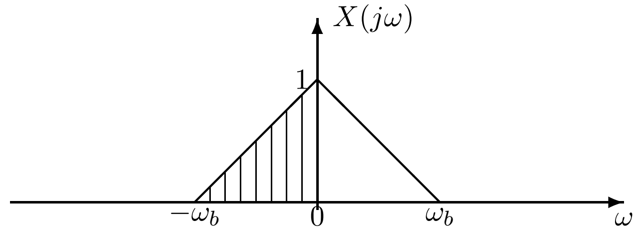
PROBLEM 12.1*:

You will use the results of this problem in Lab #12 so work it first.[1]



$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{\text{co}} \\ 0 & |\omega| > \omega_{\text{co}} \end{cases}$$

We have shown that if $x(t)$ has a bandlimited Fourier transform such that $X(j\omega) = 0$ for $|\omega| \geq \omega_b$ and $\omega_c > \omega_b$ and $\phi = 0$ and $\omega_b < \omega_{\text{co}} < 2\omega_c - \omega_b$, then the AMDSB/SC signal $y(t) = x(t)\cos\omega_c t$ can be demodulated by the above system. That is, for precise adjustment of the demodulator frequency and phase, $v(t) = x(t)$. In the following parts, assume that the input signal $x(t)$ has a bandlimited Fourier transform represented by the following plot:

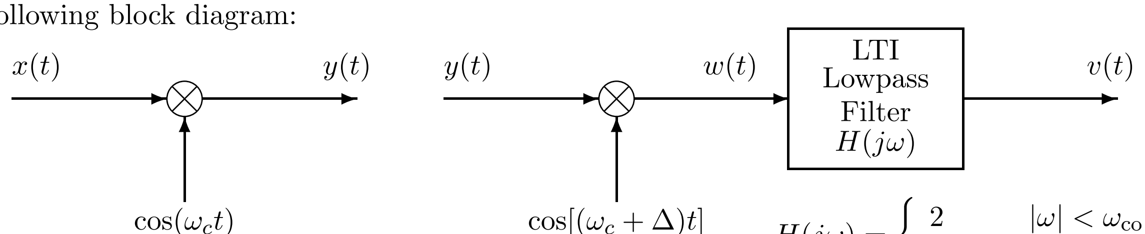


(a) Now suppose that $\phi \neq 0$. Use a well known trig identity to show that

$$w(t) = y(t)\cos(\omega_c t + \phi) = x(t)\cos(\omega_c t)\cos(\omega_c t + \phi) = \tfrac{1}{2}x(t)\cos\phi + \tfrac{1}{2}x(t)\cos(2\omega_c t + \phi).$$

From this equation obtain an equation for $W(j\omega)$ in terms of $X(j\omega)$ and use this equation to make a plot of $W(j\omega)$ for the given $X(j\omega)$. From this plot, determine a plot of $V(j\omega)$ and from that plot, obtain an equation for $v(t)$ in terms of $x(t)$ and ϕ.

(b) Now using the same assumptions on $X(j\omega)$ and ω_c and $\phi = 0$, consider the case when the demodulator carrier frequency is mismatched by a small amount $\Delta \ll \omega_c$ as depicted in the following block diagram:



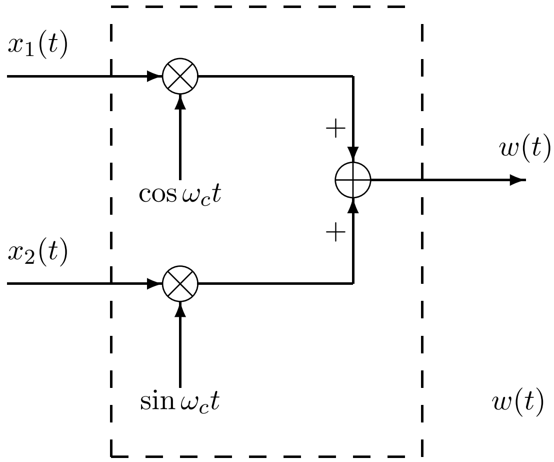
$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{\text{co}} \\ 0 & |\omega| > \omega_{\text{co}} \end{cases}$$

Again make a plot of $W(j\omega)$ for the given $X(j\omega)$. Use this plot to help you determine an equation for $v(t)$ in terms of $x(t)$ and Δ. In this case, you will need to assume that the cutoff frequency of the ideal lowpass filter satisfies $(\omega_b + \Delta) < \omega_{\text{co}} < (2\omega_c + \Delta - \omega_b)$.

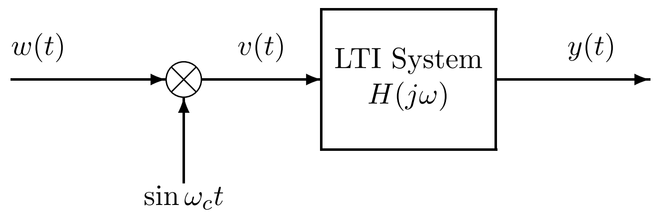
[1] See Section 13.2.1 of the Notes for a discussion of AMDSB modulation and demodulation and the issues addressed by this problem.

PROBLEM 12.2*:

The system in the dashed box below is called a *quadrature modulation system.* It is a method of sending two bandlimited signals in the same band of frequency.



Quadrature Modulator



$$w(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t$$

Demodulator = Mixer, then Filter

Assume that both input signals are bandlimited with highest frequency ω_m; i.e., $X_1(j\omega) = 0$ for $|\omega| \geq \omega_m$ and $X_2(j\omega) = 0$ for $|\omega| \geq \omega_m$, where $\omega_c \gg \omega_m$.

(a) Determine an expression for the Fourier transform $W(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Make a sketch of $W(j\omega)$. Assume simple ("typical") shapes (each different) for the bandlimited Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$, and use them in making your sketch of $W(j\omega)$.

(b) From the expression found in part (a) and the sketch that you drew, you should see that $W(j\omega) = 0$ for $|\omega| \leq \omega_a$ and for $|\omega| \geq \omega_b$. Determine ω_a and ω_b.

(c) Given the trigonometric identities $2\sin\theta\cos\theta = \sin 2\theta$, $2\sin^2\theta = (1 - \cos 2\theta)$, and $2\cos^2\theta = (1 + \cos 2\theta)$, show that in the "demodulator" figure on the right above, the output of the signal multiplier (sometimes called a *mixer*) is:

$$v(t) = \tfrac{1}{2}x_2(t)(1 - \cos 2\omega_c t) + \tfrac{1}{2}x_1(t)\sin 2\omega_c t$$

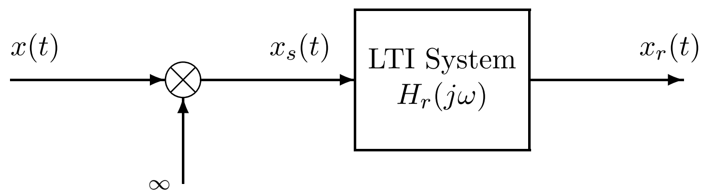
Use your assumed "typical bandlimited Fourier transforms" for $X_1(j\omega)$ and $X_2(j\omega)$ with the above expression for $v(t)$ to construct a "typical" plot of $V(j\omega)$.

(d) The signal $v(t)$ as determined in part (c) is the input to an LTI system. Determine the frequency response of that system so that its output is $y(t) = x_2(t)$. Give your answer as a carefully labeled plot of $H(j\omega)$.
You should use the plot that you constructed in part (c) as an aid in determining $H(j\omega)$.

(e) Draw a block diagram of a demodulator system whose output will be $x_1(t)$ when its input is $w(t)$. This requires that you change the signal multiplier in the demodulator.

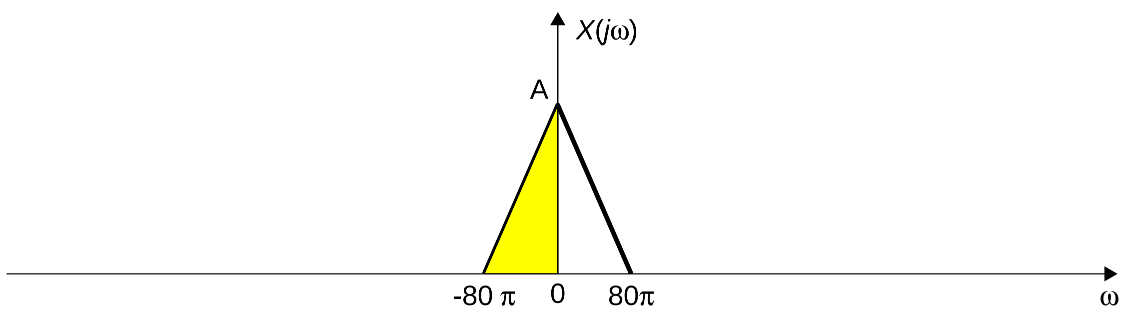
PROBLEM 12.3*:

The derivation of the Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The "typical" bandlimited Fourier transform of the input is depicted below:



(a) For the input with Fourier transform depicted above, use the Sampling Theorem to choose the sampling rate $\omega_s = 2\pi/T_s$ so that $x_r(t) = x(t)$. Plot $X_s(j\omega)$ for the value of $\omega_s = 2\pi/T_s$ that is equal to the *Nyquist* rate.[2]

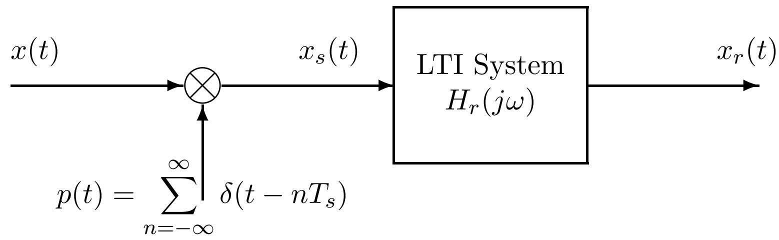
(b) If $\omega_s = 2\pi/T_s = 100\pi$ in the above system and $X(j\omega)$ is as depicted above, plot the Fourier transform $X_s(j\omega)$ and show that aliasing occurs. There will be an infinite number of shifted copies of $X(j\omega)$, so indicate what the pattern is versus ω.

(c) For the conditions of part (b), determine and sketch the Fourier transform of the output $X_r(j\omega)$ if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T_s & |\omega| \leq \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

[2] Remember that the Nyquist rate is the *lowest* possible sampling rate that does not cause aliasing.

PROBLEM 12.4*:



The input signal for the above sampling/reconstruction system is

$$x(t) = 2\cos(100\pi t - \pi/4) + \cos(300\pi t + \pi/3) \qquad -\infty < t < \infty$$

and the frequency response of the lowpass reconstruction filter is

$$H_r(j\omega) = \begin{cases} T_s & |\omega| < \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

where T_s is the sampling period.

(a) Determine the Fourier transform $X(j\omega)$ and plot the Fourier transform $X_s(j\omega)$ for $-2\pi/T_s < \omega < 2\pi/T_s$ for the case where $2\pi/T_s = 1000\pi$. Carefully label your sketch to receive full credit. What is the output $x_r(t)$ in this case?

(b) Now assume that $\omega_s = 2\pi/T_s = 500\pi$. Determine an equation for the output $x_r(t)$.

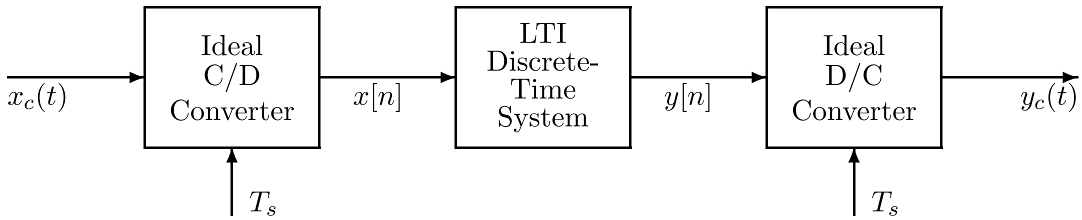
(c) Is it possible to choose the sampling rate so that

$$x_r(t) = A + 2\cos(100\pi t - \pi/4)$$

where A is a constant? If so, what is the value of T_s and what is the numerical value of A?

PROBLEM 12.5*:

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

In all parts of this problem, assume that $X_c(j\omega) = 0$ for $|\omega| \geq 1000\pi$.

(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T_s$ such that $y_c(t) = x_c(t)$?

(b) Suppose that the LTI D-T system has system function $H(z) = z^{-10}$ and assume that the sampling rate satisfies the condition of (a). Determine the overall effective frequency response $H_{\text{eff}}(j\omega)$ and from it determine a general relationship between $y_c(t)$ and $x_c(t)$.

(c) The input/output relation for the discrete-time system is

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

For the value of T_s chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{eff}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{eff}(j\omega)$. Plot the magnitude and phase of $H_{eff}(j\omega)$. Use MATLAB to do this or sketch it by hand.