13.1

a) $x_a[n] = 2(0.8)^n u[n]$, find $X_a(z)$

$$b_0 a_1^n u[n] \longrightarrow \frac{b_0}{1-a_1 z^{-1}}$$ from table & lecture

$b_0 = 2$
$a_1 = 0.8$

$$\boxed{X_a(z) = \frac{2}{1-0.8z^{-1}}}$$

b) $x_b[n] = 4(\tfrac{1}{2})^n u[n] - 2(\tfrac{1}{4})^{n-1} u[n-1]$, find $X_b(z)$

use linearity, transform from a),
and $x[n-1] \to z^{-1} X(z)$

$$4(\tfrac{1}{2})^n u[n] \longrightarrow \frac{4}{1-0.5z^{-1}}$$

$$\underbrace{2(\tfrac{1}{4})^n u[n]}_{=x[n]} \rightarrow \underbrace{\frac{2}{1-0.25z^{-1}}}_{=X(z)}, \quad x[n-1] \longrightarrow z^{-1}X(z) = \frac{2z^{-1}}{1-0.25z^{-1}}$$

As polynomials in z^{-1}

$$\boxed{X_b(z) = \frac{4}{1-0.5z^{-1}} - \frac{2z^{-1}}{1-0.25z^{-1}}} = \boxed{\frac{4-3z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-1}}}$$

c) $x_b[n] = \delta[n] + u[n-1]$, find $X_b(z)$

$$\delta[n] \longrightarrow 1, \quad u[n] \longrightarrow \frac{1}{1-z^{-1}}, \quad u[n-1] \rightarrow \frac{z^{-1}}{1-z^{-1}}$$

$$X_b(z) = 1 + \frac{z^{-1}}{1-z^{-1}} = \boxed{\frac{1}{1-z^{-1}}}$$

(note, $X_b(z)$ corresponds to $x_b[n] = u[n]$,
but $\delta[n] + u[n-1] = u[n]$)

13.2

a) $H_a(z) = \dfrac{1 + z^{-2}}{1 - 0.8z^{-1}}$, find $h[n]$

$$\frac{1+z^{-2}}{1-0.8z^{-1}} = \frac{1}{1-0.8z^{-1}} + (z^{-2})\frac{1}{1-0.8z^{-1}}, \quad \frac{1}{1-0.8z^{-1}} \longrightarrow (0.8)^n u[n]$$

$$h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2]$$

b) $H_b(z) = \dfrac{0.5}{1-0.8e^{j0.25\pi}z^{-1}} + \dfrac{0.5}{1-0.8e^{-j0.25\pi}z^{-1}}$, find $h[n]$

recall $b_0 a_1^n u[n] \rightarrow \dfrac{b_0}{1-a_1 z^{-1}}$, ($a_1$ can be complex)
and use linearity

$$h_b[n] = (0.5)(0.8)^n (e^{j0.25\pi})^n u[n] + (0.5)(0.8)^n (e^{-j0.25\pi})^n u[n]$$

$$= \left(\tfrac{1}{2}\right)(0.8)^n u[n] \left[e^{j0.25\pi n} + e^{-j0.25\pi n}\right]$$

$$= (0.8)^n u[n] \cos(0.25\pi n)$$

13.2(c) $H(z)=\dfrac{1+0.64z^{-1}}{1+0.64z^{-2}}$, find h[n].

$$\frac{1+0.64z^{-1}}{1+0.64z^{-2}}=\frac{1+0.64z^{-1}}{\left(1+j0.8z^{-1}\right)\left(1-j0.8z^{-1}\right)}=\frac{A}{1+j0.8z^{-1}}+\frac{B}{1-j0.8z^{-1}}$$

$$A=\left.\frac{1+0.64z^{-1}}{1-j0.8z^{-1}}\right|_{z=-j0.8}=\frac{1+\frac{0.64}{-j0.8}}{1-\frac{j0.8}{-j0.8}}=0.5+j0.4\approx 0.6403e^{j0.6747}$$

$$B=\left.\frac{1+0.64z^{-1}}{1+j0.8z^{-1}}\right|_{z=j0.8}=\frac{1+\frac{0.64}{j0.8}}{1+\frac{j0.8}{j0.8}}=0.5-j0.4\approx 0.6403e^{-j0.6747}$$

$$h[n]=(0.5+j0.4)(-j0.8)^n u[n]+(0.5-j0.4)(j0.8)^n u[n]$$

$$=(0.5+j0.4)\left(0.8e^{-j\frac{\pi}{2}}\right)^n u[n]+(0.5-j0.4)\left(0.8e^{j\frac{\pi}{2}}\right)^n u[n]$$

$$=(0.8)^n u[n]\left(0.5\left(\left(e^{-j\frac{\pi}{2}}\right)^n+\left(e^{j\frac{\pi}{2}}\right)^n\right)+j0.4\left(\left(e^{-j\frac{\pi}{2}}\right)^n-\left(e^{j\frac{\pi}{2}}\right)^n\right)\right)$$

$$=(0.8)^n u[n]\left(\cos\left(\frac{\pi}{2}n\right)+j0.4\left(-j2\sin\left(\frac{\pi}{2}n\right)\right)\right)$$

$$=(0.8)^n u[n]\left(\cos\left(\frac{\pi}{2}n\right)+0.8\sin\left(\frac{\pi}{2}n\right)\right)$$

$$=(0.8)^n u[n]\left(\cos\left(\frac{\pi}{2}n\right)+0.8\cos\left(\frac{\pi}{2}n-\frac{\pi}{2}\right)\right)$$

$$=(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}+0.8e^{j\frac{\pi}{2}n}e^{-j\frac{\pi}{2}}\right\}\right)$$

$$=(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}\left(1+0.8e^{-j\frac{\pi}{2}}\right)\right\}\right)$$

$$=(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}(1-j0.8)\right\}\right)$$

$$\approx(0.8)^n u[n]\left(\Re\left\{e^{j\frac{\pi}{2}n}\left(1.2806e^{-j0.6747}\right)\right\}\right)$$

$$=\left(1.2806\cos\left(\frac{\pi}{2}n-0.6747\right)\right)(0.8)^n u[n]$$

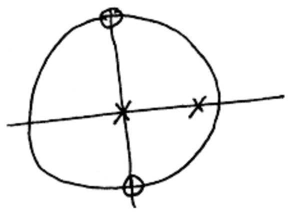
13.3 find poles and zeros

S_1: $Y(z) = 0.8\, z^{-1} Y(z) + X(z) + z^{-2} X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 - 0.8 z^{-1}}, \text{ multiply by } \frac{z^2}{z^2} \Rightarrow \frac{z^2 + 1}{(z)(z - 0.8)}$$

poles@ $(z)(z - 0.8) = 0 \Rightarrow z = 0,\ 0.8$

zeros@ $z^2 + 1 = 0$, $z = \pm j$

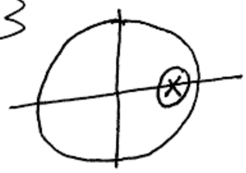


S_2: $H(z) = \dfrac{1.25 - z^{-1}}{1 - 0.8 z^{-1}} = \dfrac{1.25 z - 1}{z - 0.8}$

zero @ $z = 0.8$

pole @ $z = 0.8$

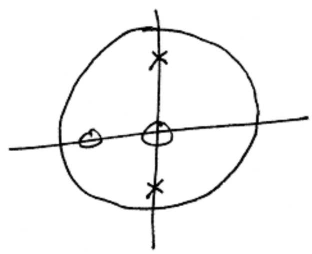
$\left\{ \text{note } H(z) = (1.25)\left(\dfrac{1.25 - z^{-1}}{1.25 - z^{-1}}\right) = 1.25 \right\}$



S_3: $H(z) = \dfrac{1 + 0.64 z^{-1}}{1 + 0.64 z^{-2}} = \dfrac{z^2 + 0.64 z}{z^2 + 0.64} = \dfrac{(z)(z + 0.64)}{(z + 0.8j)(z - 0.8j)}$

zeros @ $z = 0,\ -0.64$

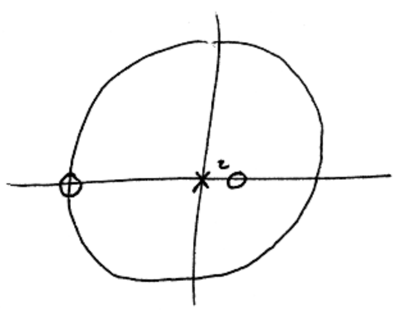
poles @ $z = \pm 0.8j$



S_4: $H(z) = \dfrac{1 + \frac{3}{4} z^{-1} - \frac{1}{4} z^{-2}}{1} = \dfrac{z^2 + \frac{3}{4} z - \frac{1}{4}}{z^2} = \dfrac{(z + 1)(z - \frac{1}{4})}{(z)(z)}$

zeros@ $z = -1$, $z = \frac{1}{4}$

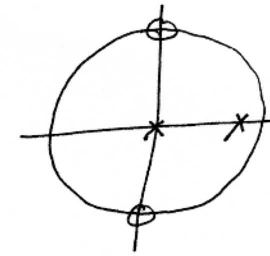
poles @ $z = 0, 0$



13.4) Causal LTI system

$$H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}}$$

a) $\left(\frac{1+z^{-2}}{1-0.8z^{-1}}\right)\left(\frac{z^2}{z^2}\right) = \frac{z^2+1}{z(z-0.8)}$ zeros @ $z = \pm j$ poles @ $z = 0, 0.8$



b) find $h[n]$. $H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}} = \frac{1}{1-0.8z^{-1}} + (z^{-2})\left(\frac{1}{1-0.8z^{-1}}\right)$

$$h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2]$$

c) stable? yes, poles inside unit circle

or $\sum_{n=-\infty}^{\infty} |h[n]| = \left|\frac{1}{1-0.8}\right| + \left|\frac{1}{1-0.8}\right| < \infty$

d) $H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{1+e^{-j2\omega}}{1-0.8e^{-j\omega}}$

e) $y[n] = H(e^{j\omega})x[n] = H(e^{j\pi/2})x[n]$

$$H(e^{j\pi/2}) = \frac{1+e^{-j2\pi/2}}{1-0.8e^{-j\pi/2}} = \frac{1-1}{1+0.8j} = 0$$

$y[n] = 0$

$\left\{ \hat{\omega} = \pi/2 \text{ is a zero of } H(z) \text{ from part a} \right\}$

13.5) $y[n] = 0.8y[n-1] + x[n] + x[n-2]$ 6

a) $x[n] = u[n]$

find $X(z) = \frac{1}{1-z^{-1}}$

find $H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}}$

find $Y(z) = X(z)H(z) = \frac{1+z^{-2}}{(1-z^{-1})(1-0.8z^{-1})} = \frac{A}{1-z^{-1}} + \frac{B+Cz^{-1}}{1-0.8z^{-1}}$

$$= \frac{10}{1-z^{-1}} + \frac{-9-z^{-1}}{1-0.8z^{-1}} = \frac{10}{1-z^{-1}} + \frac{-9}{1-0.8z^{-1}} + (z^{-1})\frac{-1}{1-0.8z^{-1}}$$

$y[n] = 10u[n] - 9(0.8)^n u[n] - (0.8)^{n-1}u[n-1]$

b) use freq domain & linearity

$H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}} \Rightarrow H(e^{j\omega}) = \frac{1+e^{-2j\omega}}{1-0.8e^{-j\omega}}$

$x[n] = 2\cos(0.5\pi n - \pi/2) + \cos(0.25\pi n - \pi)$

need $H(e^{j\pi/2})$, $H(e^{j\pi/4})$

$H(e^{j\pi/2}) = \frac{1+e^{-2j\pi/2}}{1-0.8e^{-j\pi/2}} = \frac{1+e^{-j\pi}}{1-0.8e^{-j\pi/2}} = 0$

$H(e^{j\pi/4}) = \frac{1+e^{-j\pi/2}}{1-0.8e^{-j\pi/4}} = 1.983 \measuredangle -1.7014 = 1.983e^{-j1.7014}$

$y[n] = 1.983\cos(0.25\pi n + 1.4402)$

13.5 c) time domain

$$x[n] = 10\,\delta[n-5]$$

$$H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}} = \frac{1}{1-0.8z^{-1}} + (z^{-2})\left(\frac{1}{1-0.8z^{-1}}\right)$$

$$h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2]$$

$$y[n] = x[n] * h[n] =$$

$$10(0.8)^{n-5} u[n-5] + 10(0.8)^{n-7} u[n-7]$$

d) use MATLAB to evaluate

$$y[n] = x[n] * h[n]$$

where $x[n]$ is the 10,000 numbers

$h[n]$ is the impulse response, as in part c

(note that while $h[n]$ is infinite in length, $h[n]$ for say $n > 200$ is so small that the computer will consider it to be zero)