

**EE-2025**

**Fall-2001**

**LECTURE #3**

**Phasor Addition Theorem**

**27-Aug-01**

## Web-CT Info

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- Check the Bulletin Board for msgs
  - **MAKE YOUR OWN POSTINGS**
- Lectures are being posted
  - PDF format (4 per page)
- Get PDF file of Lab#1 from WebCT
- On-Line HW #0 closes tonight @ midnite
- Get PDF file of HW#1 from WebCT
  - **HW #1 due next week (in Recitation)**

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## Homework Formatting

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- Include a Cover page with
  - Name
  - Lab section, ie, L05, L20, etc.
  - Recitation Prof's name
  - **Download example from Web-CT**
- Write on **ONE** side only
  - Use Engineer's paper or plain paper
- STAPLE

## Lab Info

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- NT passwd = **SSN or old password**
- Lab **FAQs** are also being posted
- **Be prepared for on-Line Pre-Lab Questions**
- Lab #1 Verifications: Turn in at end of Lab
- MATLAB Help: Mon,T,Wed 6PM VL-456 ??

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## Lab Info: next week

- Lab #1 Report
  - Due week of 4-Sept
    - Monday Labs, turn in on Mon 10-Sept @ Lab
  - Turn in at **BEGINNING** of your lab time
  - Write-up section 3 (short)
- Learn **your** Lab TA's format requirements
- Week of Sept 3rd (Labor Day)
  - Monday Labs will be shifted to 10-Sept

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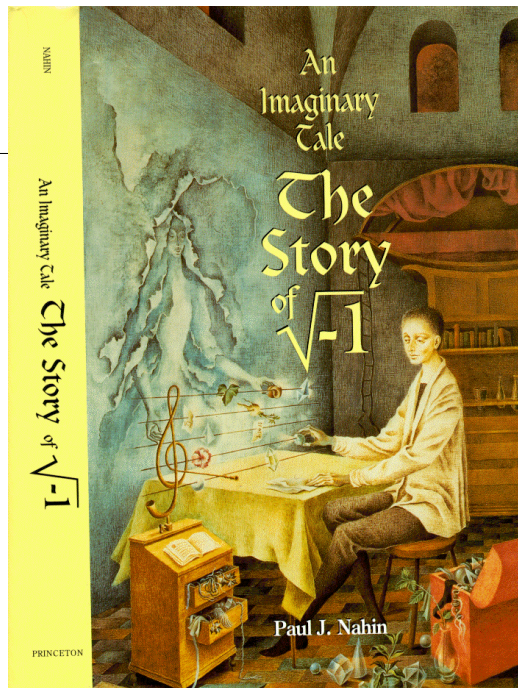
## PRINTING QUOTA

- ECE Labs have printers, but...
- Limit your printing to essentials
  - Your account has a quota
- 10 pages/week
  - 2000 students
  - 3 courses/student
  - 15 weeks/semester
  - 900,000 pages !

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**LECTURE**

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## READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 31-43
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

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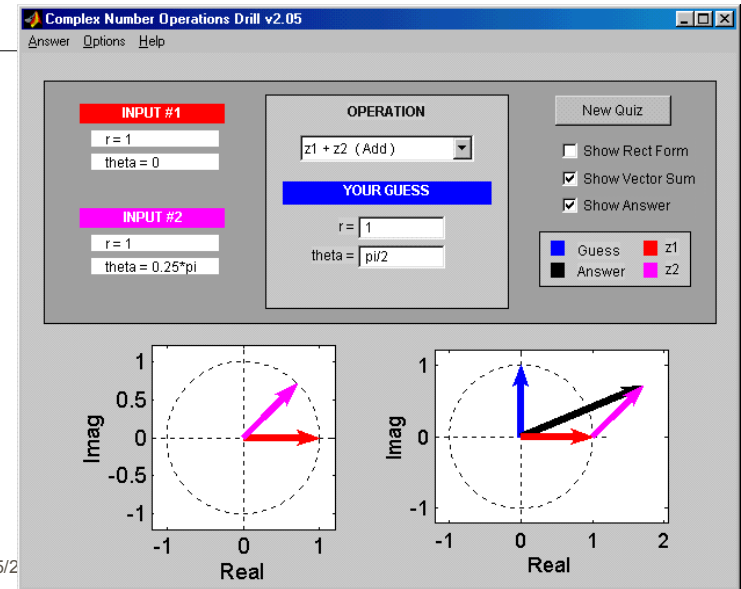
# LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\phi})e^{j\omega t}$$

- Develop the ABSTRACTION:
  - Adding Sinusoids = Complex Addition
  - **PHASOR ADDITION THEOREM**

# Z DRILL (Complex Arith)



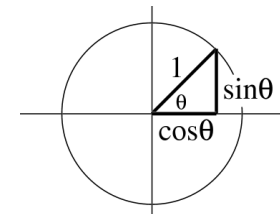
# AVOID Trigonometry

- Algebra, even complex, is **EASIER !!!**
- Can you recall  $\cos(\theta_1 + \theta_2)$  ?
- Use: real part of  $e^{j(\theta_1 + \theta_2)} = \cos(\theta_1 + \theta_2)$

$$\begin{aligned} e^{j(\theta_1 + \theta_2)} &= e^{j\theta_1} e^{j\theta_2} \\ &= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2) \\ &= \boxed{(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)} + j(\dots) \end{aligned}$$

# Euler's FORMULA

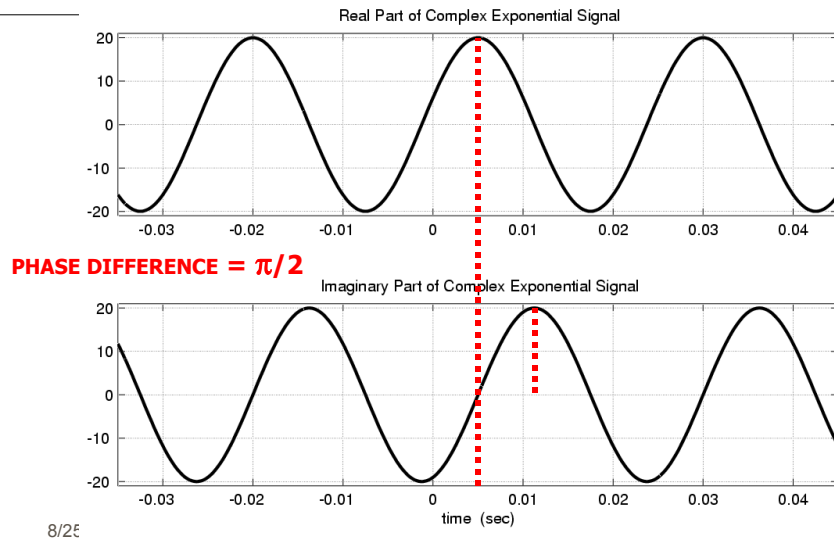
- **Complex Exponential**
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

# Real & Imaginary Part Plots



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# COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

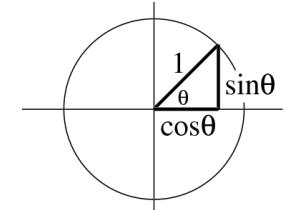
Interpret this as a **Rotating Vector**

■  $\theta = \omega t$

■ Angle changes vs. time

■ ex:  $\omega = 20\pi$  rad/s

■ Rotates  $0.2\pi$  in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

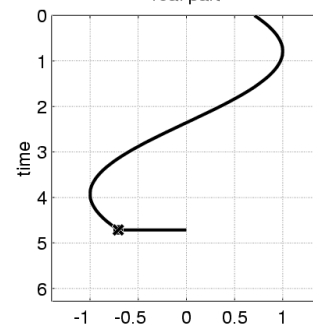
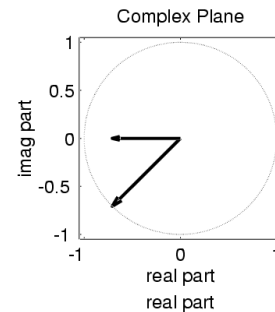
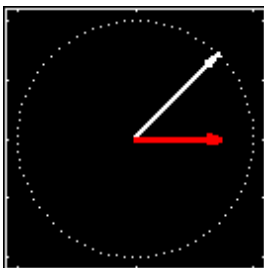
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# Rotating Phasor

See Demo on CD-ROM  
Chapter 2



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# Cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A \cos(\omega t + \phi)$$

So,

$$\begin{aligned} A \cos(\omega t + \phi) &= \Re\{A e^{j(\omega t + \phi)}\} \\ &= \Re\{A e^{j\phi} e^{j\omega t}\} \end{aligned}$$

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## COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A \cos(\omega t + \varphi) = \Re \{ A e^{j\varphi} e^{j\omega t} \}$$

- Sinusoid = REAL PART of  $(Ae^{j\varphi})e^{j\omega t}$

$$x(t) = \Re \{ X e^{j\omega t} \} = \Re \{ z(t) \}$$

- Complex AMPLITUDE = X**

$$z(t) = X e^{j\omega t} \quad X = A e^{j\varphi}$$

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## POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3} \cos(77\pi t + 0.5\pi)$$

- Use EULER'S FORMULA:

$$\begin{aligned} x(t) &= \Re \{ \sqrt{3} e^{j(77\pi t + 0.5\pi)} \} \\ &= \Re \{ \sqrt{3} e^{j0.5\pi} e^{j77\pi t} \} \end{aligned}$$

$$X = \sqrt{3} e^{j0.5\pi}$$

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## WANT to ADD SINUSOIDS

- ALL SINUSOIDS HAVE **SAME** FREQUENCY
- HOW to GET **{Amp,Phase}** of RESULT ?

$$x_1(t) = 1.7 \cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9 \cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532 \cos(2\pi(10)t + 141.79\pi/180)$$

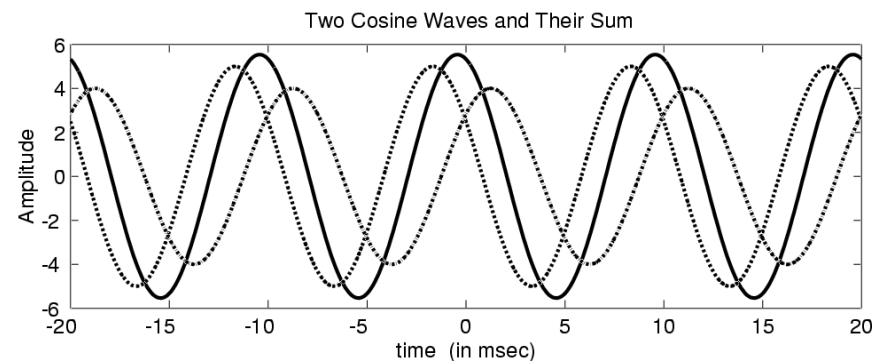
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## ADD SINUSOIDS

- Sum Sinusoid has **SAME** Frequency



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## PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^N A_k \cos(\omega_0 t + \phi_k)$$

$$= A \cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^N A_k e^{j\phi_k} = A e^{j\phi}$$

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## Phasor Addition Proof

$$\sum_{k=1}^N A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^N \Re \{ A_k e^{j(\omega_0 t + \phi_k)} \}$$

$$= \Re \left\{ \sum_{k=1}^N A_k e^{j\phi_k} e^{j\omega_0 t} \right\}$$

$$= \Re \left\{ \left( \sum_{k=1}^N A_k e^{j\phi_k} \right) e^{j\omega_0 t} \right\}$$

$$= \Re \{ (A e^{j\phi}) e^{j\omega_0 t} \} = A \cos(\omega_0 t + \phi)$$

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## POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$

$$x_2(t) = \sqrt{3} \cos(77\pi t + 0.5\pi)$$

- COMPLEX ADDITION:

$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$

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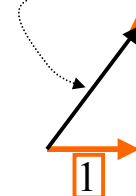
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## POP QUIZ (answer)

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$



$$j\sqrt{3} = \sqrt{3}e^{j0.5\pi}$$

- CONVERT back to cosine form:

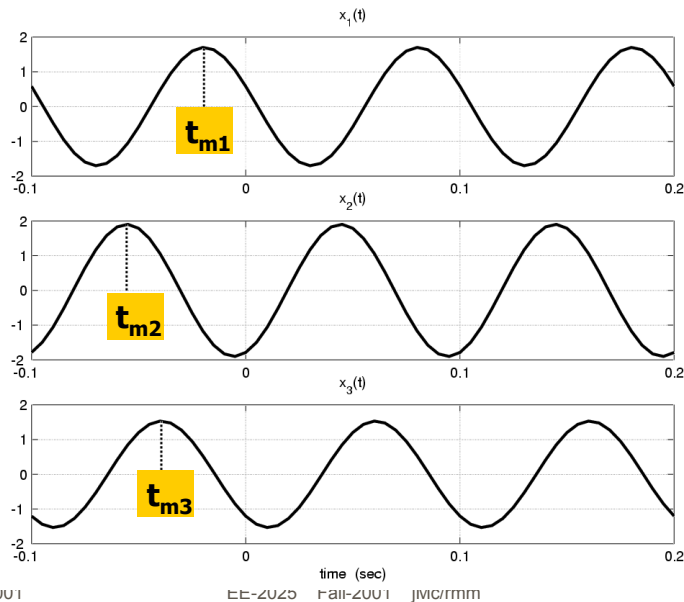
$$x_3(t) = 2 \cos(77\pi t + \frac{\pi}{3})$$

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# ADD SINUSOIDS EXAMPLE



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# Convert Time-Shift to Phase

- Measure **peak times**:
  - $t_{m1} = -0.0194$ ,  $t_{m2} = -0.0556$ ,  $t_{m3} = -0.0394$
- Convert to **phase** ( $T=0.1$ )
  - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$ ,
  - $\phi_2 = 200\pi/180$
- Amplitudes
  - $A_1 = 1.7$ ,  $A_2 = 1.9$ ,  $A_3 = 1.532$

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# Phasor Add: Numerical

- Convert Polar to Cartesian

- $X_1 = 0.5814 + j1.597$

- $X_2 = -1.785 - j0.6498$

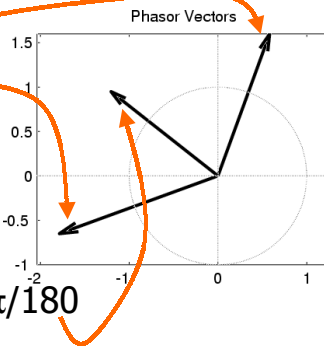
- sum =

- $X_3 = -1.204 + j0.9476$

- Convert back to Polar

- $X_3 = 1.532$  at angle  $141.79\pi/180$

- This is the sum



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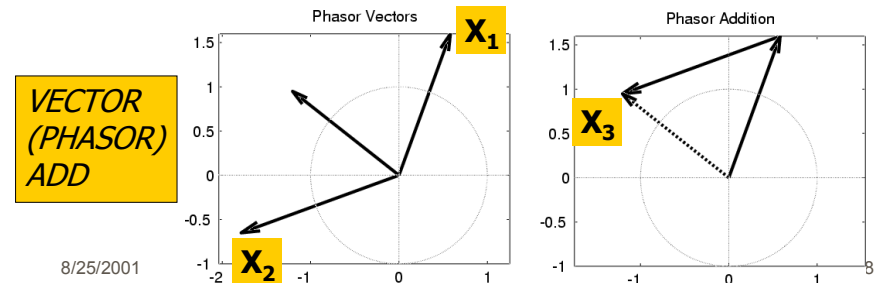
# ADD SINUSOIDS

$$x_1(t) = 1.7 \cos(2\pi(10)t + 70\pi/180)$$

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