



**PROBLEM Spring-03-F.1:**(Circle exactly one answer<sup>5</sup> for each system,  $S_i$ )

$S_1$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_2$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_3$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_4$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_5$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_6$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_7$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9
$S_8$ :	#1	#2	#3	#4	#5	#6	#7	#8	#9

**PROBLEM Spring-03-F.2:**(Circle exactly one answer for each system,  $S_i$ )

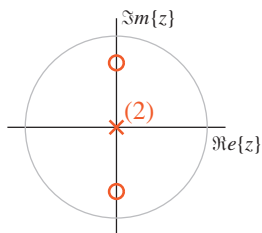
$S_1$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_2$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_3$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_4$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_5$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_6$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_7$ :	(A)	(B)	(C)	(D)	(E)	(F)	None
$S_8$ :	(A)	(B)	(C)	(D)	(E)	(F)	None

**PROBLEM Spring-03-F.3:**(Circle exactly one answer for each part)

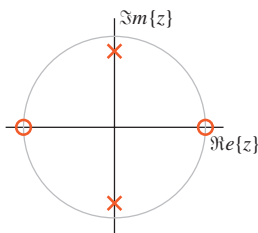
(a)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(d)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(e)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(h)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None

<sup>5</sup>If more than one answer is circled, the response will be considered wrong and will receive no credit.

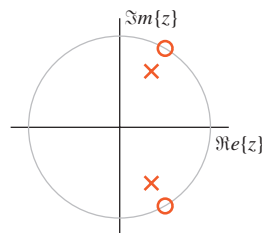
**PROBLEM Spring-03-F.1:**



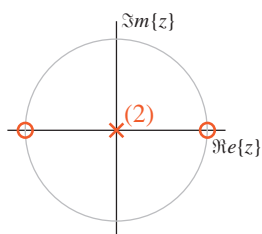
**Pole-Zero Plot #1**



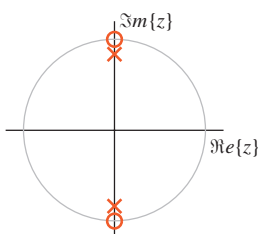
**Pole-Zero Plot #2**



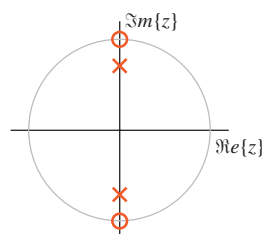
**Pole-Zero Plot #3**



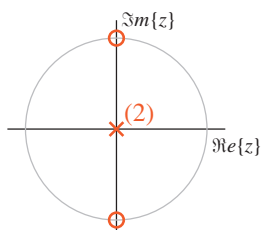
**Pole-Zero Plot #4**



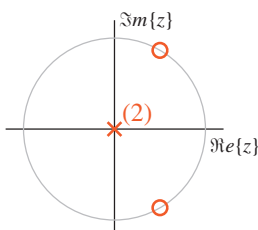
**Pole-Zero Plot #5**



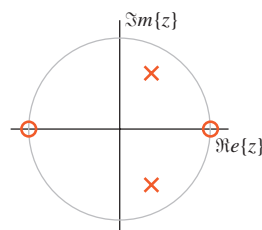
**Pole-Zero Plot #6**



**Pole-Zero Plot #7**



**Pole-Zero Plot #8**



**Pole-Zero Plot #9**

For each of systems below<sup>6</sup> determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

*Note:* the unit circle is shown for reference.

$$\mathcal{S}_1 : y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$$

$$\mathcal{S}_2 : H(z) = 5 - 5z^{-2}$$

$$\mathcal{S}_3 : y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$$

$$\mathcal{S}_4 : \text{filter}([10, -10, 10]/3, 1, \text{xx})$$

$$\mathcal{S}_5 : y[n] = 5x[n] + 5x[n-2]$$

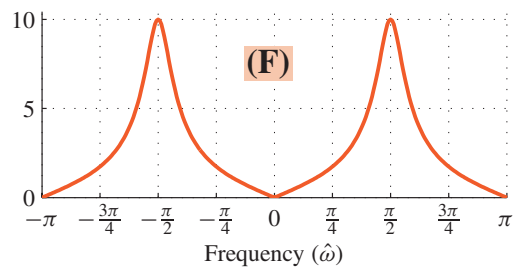
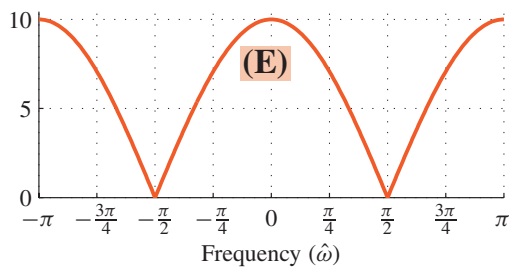
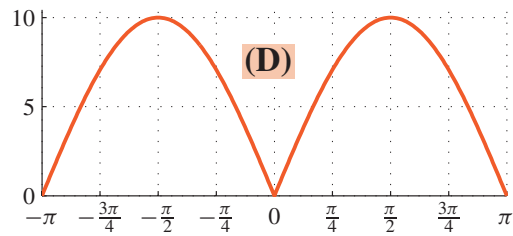
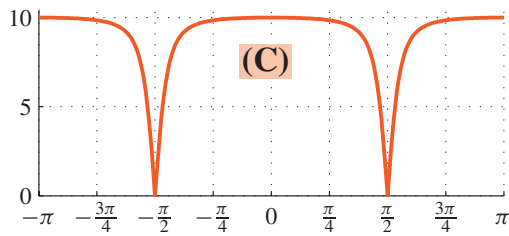
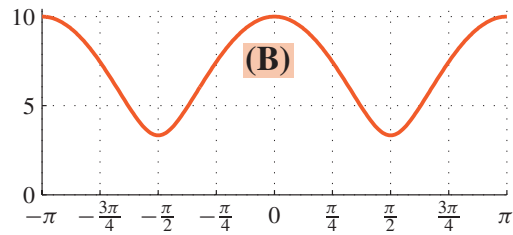
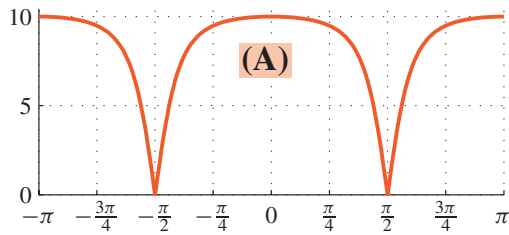
$$\mathcal{S}_6 : H(z) = \frac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$$

$$\mathcal{S}_7 : y[n] = -0.7y[n-2] + 8.5x[n] + 8.5x[n-2]$$

$$\mathcal{S}_8 : \text{filter}([1.5, 0, -1.5], [1, 0, 0.7], \text{xx})$$

<sup>6</sup>These same systems are also used in the next problem.

**PROBLEM Spring-03-F.2:**



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**

*Note:* the frequency axis is  $\hat{\omega}$ .

$$\mathcal{S}_1 : y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$$

$$\mathcal{S}_2 : H(z) = 5 - 5z^{-2}$$

$$\mathcal{S}_3 : y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$$

$$\mathcal{S}_4 : \text{filter}([10, -10, 10]/3, 1, \text{xx})$$

$$\mathcal{S}_5 : y[n] = 5x[n] + 5x[n-2]$$

$$\mathcal{S}_6 : H(z) = \frac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$$

$$\mathcal{S}_7 : y[n] = -0.7y[n-2] + 8.5x[n] + 8.5x[n-2]$$

$$\mathcal{S}_8 : \text{filter}([1.5, 0, -1.5], [1, 0, 0.7], \text{xx})$$

**PROBLEM Spring-03-F.3:**

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. *Write your answers on the answer sheet provided.* (The operator \* denotes convolution.)

$$(a) x(t) = \int_{-\infty}^t e^{-t+\tau} \delta(\tau - 4) d\tau$$

$$(b) x(t) = \cos(\pi t) \delta(t - 4)$$

$$(c) x(t) = \delta(t) - \delta(t - 8)$$

$$(d) x(t) = -e^{-t} u(t) + \delta(t)$$

$$(e) x(t) = u(t - 3) - u(t - 5)$$

$$(f) x(t) = \delta(t + 2) * e^{-t+1} u(t - 1) * \delta(t - 1)$$

$$(g) x(t) = u(t) - u(t - 8)$$

$$(h) x(t) = e^{-(t-4)} \int_{-\infty}^0 \delta(\tau - 4) d\tau$$

---

Each of the time signals above has a Fourier transform that might be in the list below.

$$[1] X(j\omega) = e^{-j4\omega} [\pi \delta(\omega - \pi) + \pi \delta(\omega + \pi)]$$

$$[2] X(j\omega) = e^{-j4\omega} [u(\omega) - u(\omega - 8)]$$

$$[3] X(j\omega) = \frac{j\omega}{1 + j\omega}$$

$$[4] X(j\omega) = \frac{1}{1 + j\omega}$$

$$[5] X(j\omega) = \frac{e^{-j4\omega}}{1 + j\omega}$$

$$[6] X(j\omega) = 2e^{-j4\omega} \frac{\sin(4\omega)}{\omega}$$

$$[7] X(j\omega) = j2e^{-j4\omega} \sin(4\omega)$$

$$[8] X(j\omega) = e^{-j4\omega}$$

$$[9] X(j\omega) = 2e^{-j4\omega} \frac{\sin(\omega)}{\omega}$$

$$[10] X(j\omega) = \frac{\sin(4\omega)}{\omega/2}$$

[None]  $X(j\omega)$  not in the list above.

### PROBLEM Spring-03-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

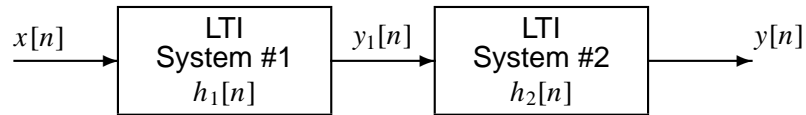


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{2z^{-2} - 2z^{-3}}{1 + 0.5z^{-1}}$$

but System #2 is unknown.

(a) Determine whether or not System #1 is stable. Give a reason to support your answer.

(b) When the input signal  $x[n]$  is a **unit-step** signal, determine the output of the first system,  $y_1[n]$ .

(c) When the input signal  $x[n]$  is a **unit-impulse** signal, the output  $y[n]$  of the overall cascaded system is:

$$y[n] = \delta[n - 3]$$

From this information, determine the system function  $H_2(z)$  for the second system. **Simplify the expression for  $H_2(z)$ .**

### PROBLEM Spring-03-F.5:

A few quick questions:

- (a) Consider the following MATLAB program:

```
nn = 0:16000;  
yy = 2*cos(1.5*pi*nn-pi/3);  
soundsc(yy,12000)
```

Neglecting the end effects in the convolution, the frequency *in hertz* for the output signal produced by the `soundsc( )` statement, i.e., the frequency that you hear.

Frequency =  Hz

- (b) Evaluate  $|H(e^{j\hat{\omega}})|^2$ , where  $H(e^{j\hat{\omega}}) = 6je^{-j(\hat{\omega}^2 - 7\hat{\omega})} \sin(\frac{1}{2}\hat{\omega})$ .

- (c) Determine the *minimum period* (in seconds) of the following signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin^2(k)}{k^2} e^{j\pi kt}$$

Period =  secs.

- (d) Solve the following relationship for  $A$  and  $\phi$

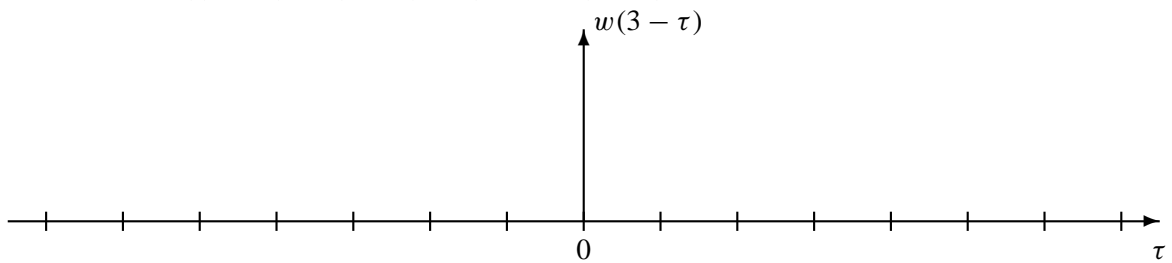
$$12 \cos(3t) + A \cos(3t + \phi) = 6 \cos(3t - 2\pi/3)$$

$A =$

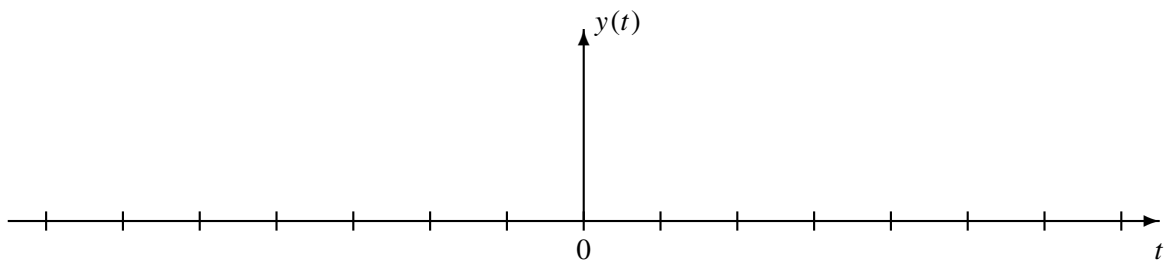
$\phi =$

**PROBLEM Spring-03-F.6:**

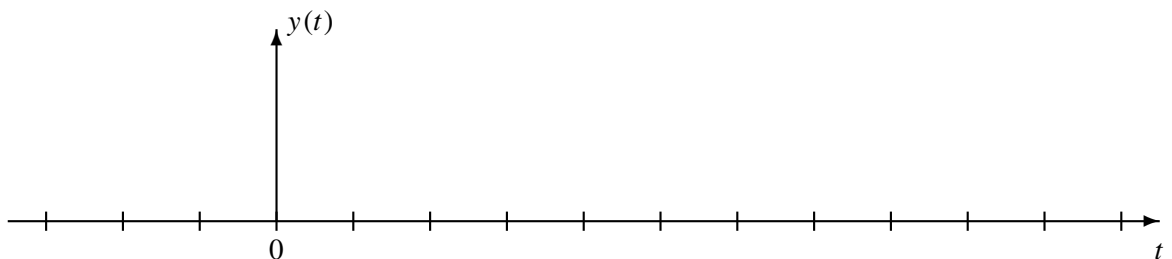
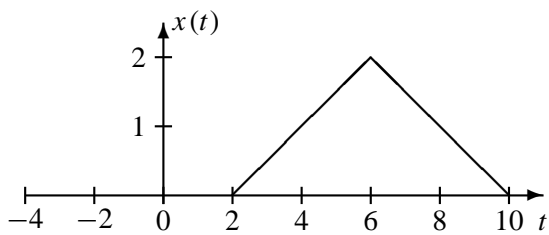
- (a) Assume that  $w(t) = u(t + 1) - u(t - 2)$ . Plot  $w(3 - \tau)$  as a function of  $\tau$ .



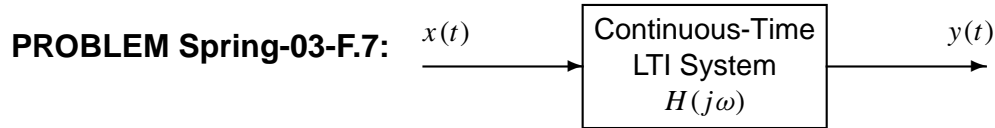
- (b) Perform the convolution  $y(t) = w(t) * u(t)$  where  $u(t)$  is the unit-step signal. Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.



- (c) If the input to an LTI system is  $x(t)$  below, determine the output signal  $y(t) = x(t) * h(t)$  when  $h(t) = 16\delta(t) + 16\delta(t - 4)$ . Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.





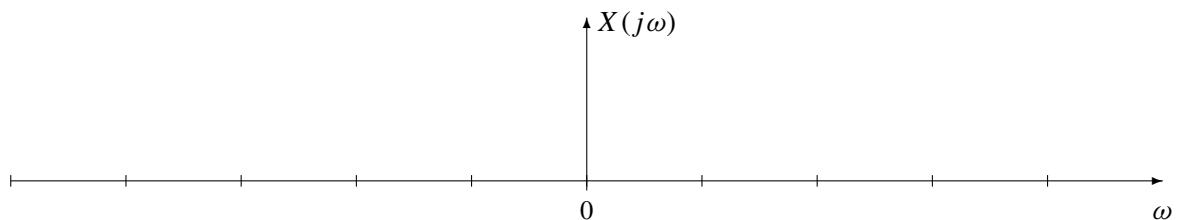


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^2 a_k e^{j7\pi kt}, \quad \text{where } a_k = \begin{cases} \frac{1}{\pi(1+|k|)} & k \neq 0 \\ \frac{1}{2} & k = 0 \end{cases}$$

- (a) Determine the Fourier transform of the periodic signal  $x(t)$ . Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



- (b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j5\omega}{\omega^2 - 49\pi^2 + j7\omega}$$

Determine the maximum magnitude of  $H(j\omega)$ , and the frequency where the maximum occurs.

$\max |H(j\omega)| =$

at  $\omega =$

- (c) For  $x(t)$  given above, the output signal can be written as

$$y(t) = B_0 + B_1 \cos(\omega_0 t + \psi_1) + B_2 \cos(2\omega_0 t + \psi_2)$$

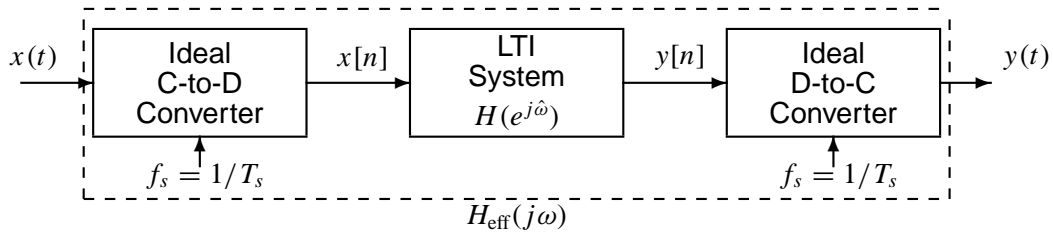
where  $B_k \geq 0$ . Determine the values of the parameters  $B_0$  and  $B_1$ .

$B_0 =$

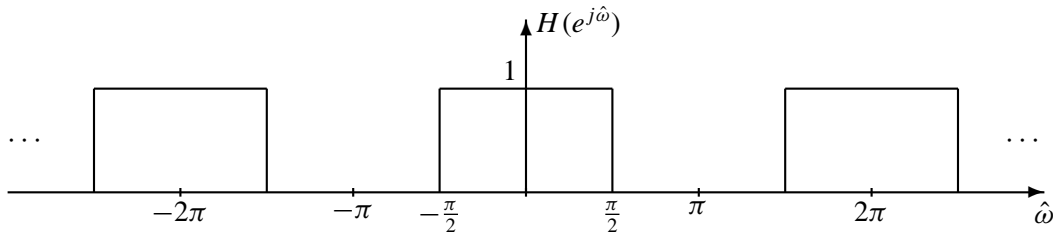
$B_1 =$

**PROBLEM Spring-03-F.8:**

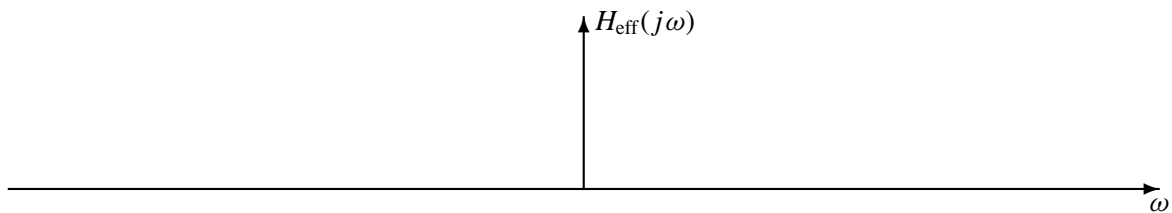
Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time system has frequency response  $H(e^{j\hat{\omega}})$  defined by the following plot:



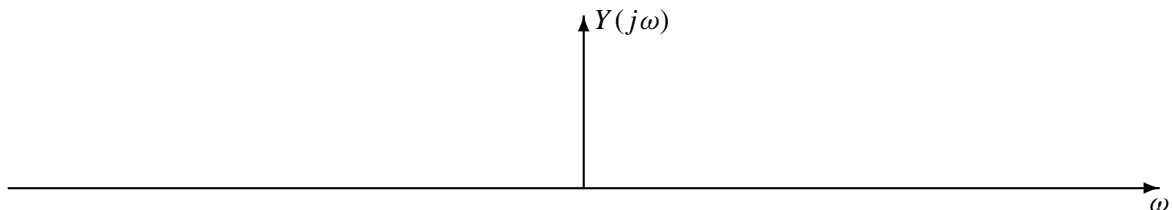
- (a) Now, if  $f_s = 200$  samples/sec, make a carefully labeled plot of  $H_{\text{eff}}(j\omega)$ , the effective frequency response of the overall system. State the frequency range where  $H_{\text{eff}}(j\omega)$  applies.



- (b) Assume that the input signal  $x(t)$  is a sum of cosines:

$$x(t) = 8 \cos(180\pi t + \pi/3) + 5 \cos(360\pi t - \pi/4)$$

For this input signal, determine the Fourier transform of the output signal  $y(t)$  when the sampling rate is  $f_s = 200$  samples/sec. Make a plot of  $Y(j\omega)$ .



**GEORGIA INSTITUTE OF TECHNOLOGY**  
**SCHOOL of ELECTRICAL & COMPUTER ENGINEERING**  
**FINAL EXAM**

DATE: Apr. 28, 2003

COURSE: ECE-2025

NAME: Answer Key  
LAST, FIRST

GT #: Version-3

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L01:Tues-9:30am (McLaughlin)		L02:Thur-9:30am (Barry)
	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
	L05:Tues-1:30pm (Li)		
L11:M-3pm (McClellan)	L07:Tues-3pm (Li)	L12:W-3pm (Hayes)	L08:Thur-3pm (Williams)
	L09:Tues-4:30pm (Zhou)	L14:W-4:30pm (Hayes)	
	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ( $8\frac{1}{2}'' \times 11''$ ) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning **CLEARLY** to receive partial credit.  
 Explanations are also required to receive full credit for any answer.
- You must write your answer **on the answer sheet** or in the space provided on the exam paper itself.  
 Only these answers will be graded. Circle your answers, or write them in the boxes provided.  
 If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

**PROBLEM Spring-03-F.1:**(Circle exactly one answer<sup>5</sup> for each system,  $S_i$ )

$S_1$ :	<input checked="" type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
$S_2$ :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input checked="" type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
$S_3$ :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input checked="" type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
$S_4$ :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input checked="" type="radio"/> #8	<input type="radio"/> #9
$S_5$ :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input checked="" type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
$S_6$ :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input checked="" type="radio"/> #9
$S_7$ :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input checked="" type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
$S_8$ :	<input type="radio"/> #1	<input checked="" type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9

**PROBLEM Spring-03-F.2:**(Circle exactly one answer for each system,  $S_i$ )

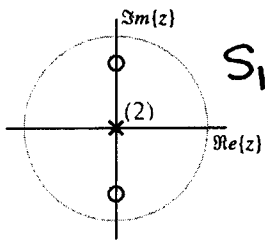
$S_1$ :	<input type="radio"/> (A)	<input checked="" type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
$S_2$ :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input checked="" type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
$S_3$ :	<input checked="" type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
$S_4$ :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input checked="" type="radio"/> None
$S_5$ :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input checked="" type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
$S_6$ :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input checked="" type="radio"/> None
$S_7$ :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input checked="" type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
$S_8$ :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input checked="" type="radio"/> (F)	<input type="radio"/> None

**PROBLEM Spring-03-F.3:**(Circle exactly one answer for each part)

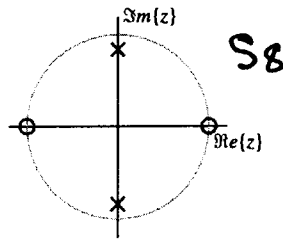
(a)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input checked="" type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(b)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input checked="" type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(c)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input checked="" type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(d)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input checked="" type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(e)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input checked="" type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(f)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input checked="" type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(g)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input checked="" type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> None
(h)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input checked="" type="radio"/> None

<sup>5</sup>If more than one answer is circled, the response will be considered wrong and will receive no credit.

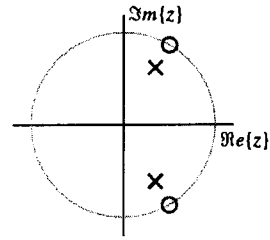
**PROBLEM Spring-03-F.1:**



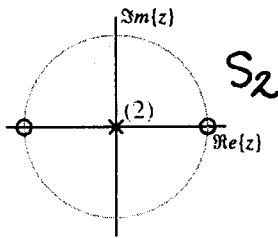
**Pole-Zero Plot #1**



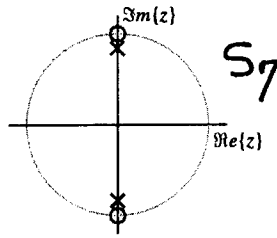
**Pole-Zero Plot #2**



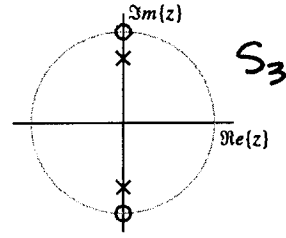
**Pole-Zero Plot #3**



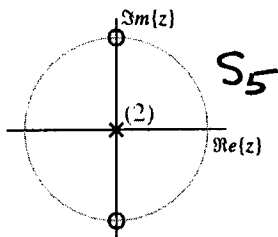
**Pole-Zero Plot #4**



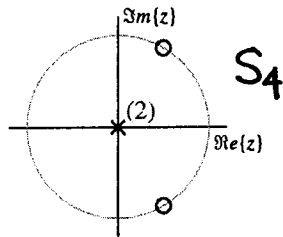
**Pole-Zero Plot #5**



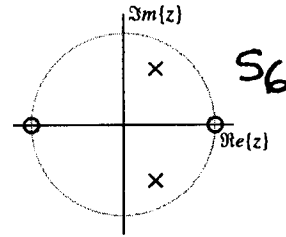
**Pole-Zero Plot #6**



**Pole-Zero Plot #7**



**Pole-Zero Plot #8**



**Pole-Zero Plot #9**

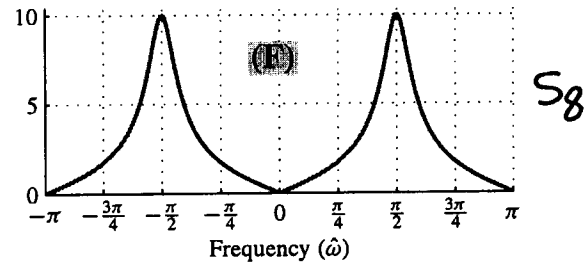
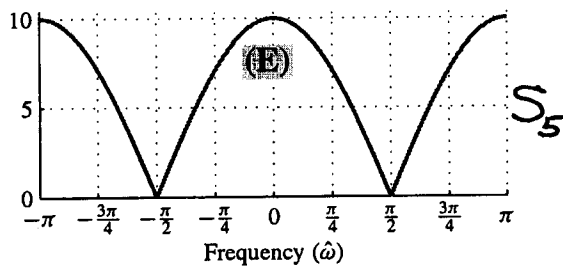
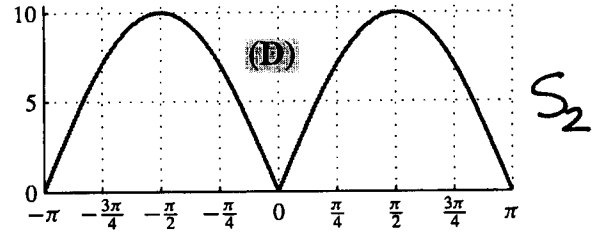
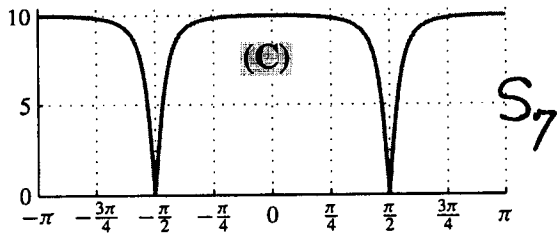
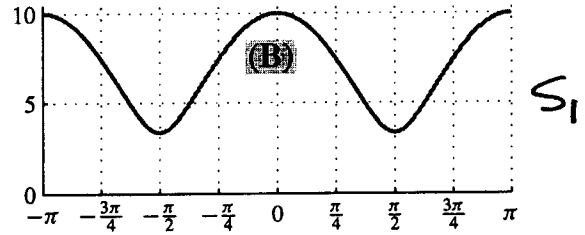
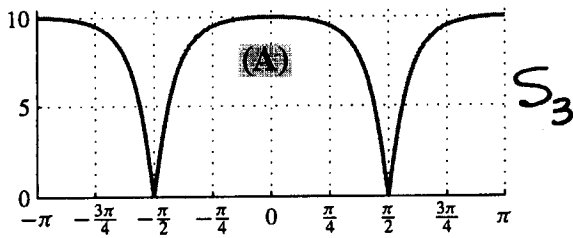
For each of systems below<sup>6</sup> determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

*Note:* the unit circle is shown for reference.

- $S_1: y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$   $\frac{20}{3}(1 + \frac{1}{2}z^{-2})$  zeros:  $\pm j\sqrt{.5} \approx \pm j0.7$
- $S_2: H(z) = 5 - 5z^{-2}$  zeros:  $\pm 1$
- $S_3: y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$   $\frac{7.5(1+z^{-2})}{1+0.5z^{-2}}$  zeros:  $\pm j$   
poles:  $\pm j\sqrt{.5}$
- $S_4: \text{filter}([10, -10, 10]/3, 1, \text{xx})$   $10(1-z^{-1}+z^{-2})$  zeros:  $e^{\pm j\pi/3}$
- $S_5: y[n] = 5x[n] + 5x[n-2]$   $5(1+z^{-2})$  zeros:  $\pm j$
- $S_6: H(z) = \frac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$  zeros:  $\pm 1$   
poles:  $.7e^{\pm j\pi/3}$
- $S_7: y[n] = -0.7y[n-2] + 8.5x[n] + 8.5x[n-2]$   $\frac{8.5(1+z^{-2})}{1+.7z^{-2}}$  zeros:  $\pm j$   
poles:  $\pm j\sqrt{.7}$
- $S_8: \text{filter}([1.5, 0, -1.5], [1, 0, 0.7], \text{xx})$   
 $\frac{1.5(1-z^{-2})}{1+.7z^{-2}}$  zeros:  $\pm 1$   
poles:  $\pm j\sqrt{.7}$

<sup>6</sup>These same systems are also used in the next problem.

**PROBLEM Spring-03-F.2:**



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**  
 Note: the frequency axis is  $\hat{\omega}$ .

$S_1: y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$   $\frac{20}{3}(1 + \frac{1}{2}z^{-2})$  zeros near  $\pm j$  but not on the unit circle  $\hat{\omega} = \pm \pi/2$

$S_2: H(z) = 5 - 5z^{-2}$  zeros at  $\hat{\omega} = 0, \pi$

$S_3: y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$   $\frac{7.5(1+z^{-2})}{1+0.5z^{-2}}$  zeros:  $\hat{\omega} = \pm \pi/3$  Notch @  $\pi/2$

$S_4: \text{filter}([10, -10, 10]/3, 1, \text{xx})$   $\frac{10}{3}(1 - z^{-1} + z^{-2})$  zeros:  $\hat{\omega} = \pm \pi/3$

$S_5: y[n] = 5x[n] + 5x[n-2]$   $5(1+z^{-2})$  zeros at  $\hat{\omega} = \pm \pi/2$

$S_6: H(z) = \frac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$  zeros:  $\hat{\omega} = 0, \pi$  poles near  $e^{\pm j\pi/3}$  BPF peak at  $\hat{\omega} = \pi/3$

$S_7: y[n] = -0.7y[n-2] + 8.5x[n] + 8.5x[n-2]$   $\frac{8.5(1+z^{-2})}{1+0.7z^{-2}}$  zeros:  $\hat{\omega} = \pm \pi/2$  Notch narrower

$S_8: \text{filter}([1.5, 0, -1.5], [1, 0, 0.7], \text{xx})$

$\frac{1.5(1-z^{-2})}{1+0.7z^{-2}}$  zeros:  $\hat{\omega} = 0, \pi$  pole near  $e^{\pm j\pi/2}$  BPF peak at  $\hat{\omega} = \pi/2$

**PROBLEM Spring-03-F.3:**

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. Write your answers on the answer sheet provided. (The operator \* denotes convolution.)

- (a)  $x(t) = \int_{-\infty}^t e^{-t+\tau} \delta(\tau - 4) d\tau = \int_{-\infty}^t e^{-t+4} \delta(\tau - 4) d\tau = e^{-(t-4)} u(t-4) \rightarrow \frac{e^{-j4\omega}}{1+j\omega}$
- (b)  $x(t) = \cos(\pi t) \delta(t - 4) = \cos(4\pi) \delta(t-4) = \delta(t-4) \rightarrow X(j\omega) = e^{-j4\omega}$
- (c)  $x(t) = \delta(t) - \delta(t - 8) \rightarrow 1 - e^{-j8\omega} = e^{-j4\omega} (e^{j4\omega} - e^{-j4\omega}) = e^{-j4\omega} \cdot 2j \sin(4\omega)$
- (d)  $x(t) = -e^{-t} u(t) + \delta(t) \rightarrow \frac{-1}{1+j\omega} + 1 = \frac{j\omega}{1+j\omega}$
- (e)  $x(t) = u(t - 3) - u(t - 5)$  rect  $\rightarrow$  sinc  $e^{-j4\omega} \frac{\sin \omega}{\omega/2}$
- (f)  $x(t) = \delta(t + 2) * e^{-t+1} u(t - 1) * \delta(t - 1) = e^{-t} u(t) \rightarrow \frac{1}{1+j\omega}$
- (g)  $x(t) = u(t) - u(t - 8) \rightarrow e^{-j4\omega} \frac{\sin(4\omega)}{\omega/2}$
- (h)  $x(t) = e^{-(t-4)} \int_{-\infty}^0 \delta(\tau - 4) d\tau = 0 \rightarrow 0$

Each of the time signals above has a Fourier transform that might be in the list below.

- [1]  $X(j\omega) = e^{-j4\omega} [\pi \delta(\omega - \pi) + \pi \delta(\omega + \pi)]$
- [2]  $X(j\omega) = e^{-j4\omega} [u(\omega) - u(\omega - 8)]$
- [3]  $X(j\omega) = \frac{j\omega}{1+j\omega}$  (d)
- [4]  $X(j\omega) = \frac{1}{1+j\omega}$  (f)
- [5]  $X(j\omega) = \frac{e^{-j4\omega}}{1+j\omega}$  (a)
- [6]  $X(j\omega) = 2e^{-j4\omega} \frac{\sin(4\omega)}{\omega}$  (g)
- [7]  $X(j\omega) = j2e^{-j4\omega} \sin(4\omega)$  (c)
- [8]  $X(j\omega) = e^{-j4\omega}$  (b)
- [9]  $X(j\omega) = 2e^{-j4\omega} \frac{\sin(\omega)}{\omega}$  (e)
- [10]  $X(j\omega) = \frac{\sin(4\omega)}{\omega/2}$

(f) is not in the list.

[None]  $X(j\omega)$  not in the list above.

### PROBLEM Spring-03-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

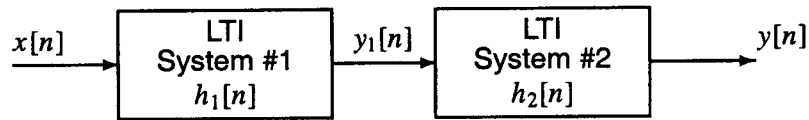


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{2z^{-2} - 2z^{-3}}{1 + 0.5z^{-1}}$$

but System #2 is unknown.

- (a) Determine whether or not System #1 is stable. Give a reason to support your answer.

Pole at  $z = -0.5$  is inside unit circle  
 $\Rightarrow$  system is **STABLE**

- (b) When the input signal  $x[n]$  is a *unit-step* signal, determine the output of the first system,  $y_1[n]$ .

$$x[n] = u[n] \Rightarrow X(z) = \frac{1}{1-z^{-1}}$$

$$Y_1(z) = H_1(z)X(z) = \frac{2z^{-2}(1-z^{-1})}{1+0.5z^{-1}} \cdot \frac{1}{1-z^{-1}} = \frac{2z^{-2}}{1+0.5z^{-1}}$$

$$y_1[n] = 2(-0.5)^{n-2}u[n-2]$$

- (c) When the input signal  $x[n]$  is a *unit-impulse* signal, the output  $y[n]$  of the overall cascaded system is:

$$y[n] = \delta[n-3]$$

From this information, determine the system function  $H_2(z)$  for the second system. *Simplify the expression for  $H_2(z)$ .*

$$x[n] = \delta[n] \Rightarrow X(z) = 1 \quad Y(z) = z^{-3}$$

$$Y(z) = H_1(z)H_2(z)X(z)$$

$$z^{-3} = H_1(z)H_2(z) \cdot 1$$

$$H_2(z) = \frac{z^{-3}}{H_1(z)} = \frac{z^{-3}}{\frac{2z^{-2}(1-z^{-1})}{1+0.5z^{-1}}} = \frac{1}{2} \frac{z^{-1}(1+0.5z^{-1})}{1-z^{-1}}$$

← Solve for  $H_2(z)$



### PROBLEM Spring-03-F.5:

A few quick questions:

(a) Consider the following MATLAB program:

```
nn = 0:16000;
yy = 2*cos(1.5*pi*nn-pi/3);
soundsc(yy, 12000)
```

$y[n] = 2 \cos(1.5\pi n - \pi/3)$   
 $\hat{\omega} = 1.5\pi$

Neglecting the end effects in the convolution, the frequency in hertz for the output signal produced by the soundsc ( ) statement, i.e., the frequency that you hear.

Frequency = 3000 Hz

$\hat{\omega} = 1.5\pi$  aliases to  $1.5\pi - 2\pi = -0.5\pi$

$y[n] = 2 \cos(-0.5\pi n - \pi/3)$

$\omega = \hat{\omega} f_s = (0.5\pi)(12000) = 6000\pi$

(b) Evaluate  $|H(e^{j\hat{\omega}})|^2$ , where  $H(e^{j\hat{\omega}}) = 6je^{-j(\hat{\omega}^2 - 7\hat{\omega})} \sin(\frac{1}{2}\hat{\omega})$ .

$|j| = 1$     $|e^{-j\theta}| = 1$

$|H(e^{j\hat{\omega}})|^2 = |6 \sin(\frac{1}{2}\hat{\omega})|^2 = 36 \sin^2(\frac{\hat{\omega}}{2})$

(c) Determine the *minimum period* (in seconds) of the following signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin^2(k)}{k^2} e^{j\pi kt}$$

Period = 2 secs.

$\omega_0 = \pi$

$\omega_0 = 2\pi/T \Rightarrow T = 2 \text{ secs}$

(d) Solve the following relationship for A and  $\phi$

$$12 \cos(3t) + A \cos(3t + \phi) = 6 \cos(3t - 2\pi/3)$$

convert to phasors

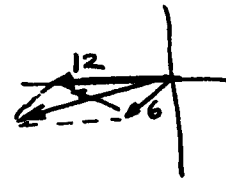
A = 15.87

$\phi =$  -2.808 rads.

$12 + Ae^{j\phi} = 6e^{-j2\pi/3}$

$Ae^{j\phi} = -12 + 6e^{-j2\pi/3}$

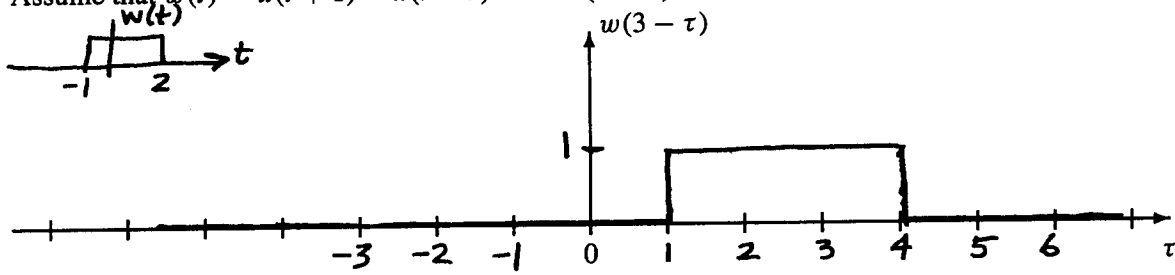
$= 15.87e^{-j2.808}$



$-2.808 = -0.894\pi \rightarrow -161^\circ$

**PROBLEM Spring-03-F.6:**

(a) Assume that  $w(t) = u(t + 1) - u(t - 2)$ . Plot  $w(3 - \tau)$  as a function of  $\tau$ .



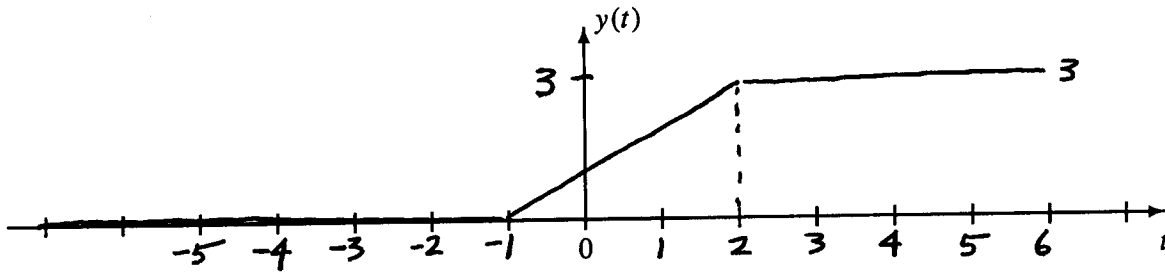
(b) Perform the convolution  $y(t) = w(t) * u(t)$  where  $u(t)$  is the unit-step signal. Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.

$$y(t) = u(t+1) * u(t) - u(t-2) * u(t)$$

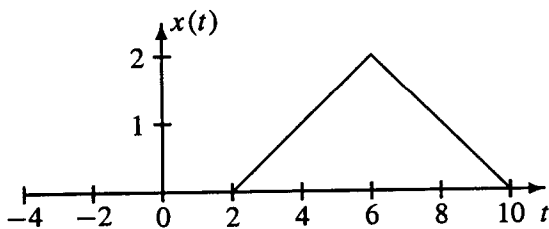
$$= (t+1)u(t+1) - (t-2)u(t-2)$$

starts at  $t = -1$   
Levels off at  $t = 2$

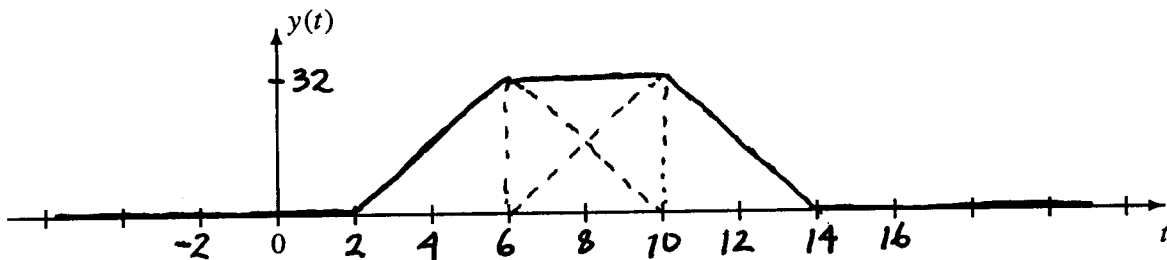
As  $t \rightarrow \infty$   $y(t) = 3$

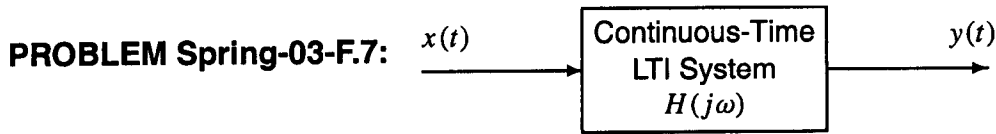


(c) If the input to an LTI system is  $x(t)$  below, determine the output signal  $y(t) = x(t) * h(t)$  when  $h(t) = 16\delta(t) + 16\delta(t - 4)$ . Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.



$$y(t) = 16x(t) + 16x(t-4)$$





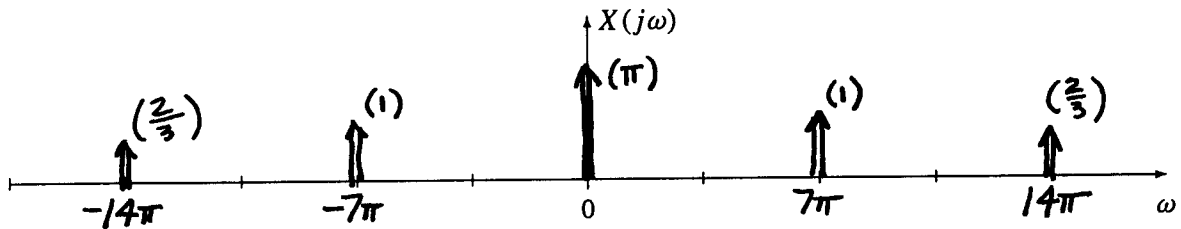
The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^2 a_k e^{j7\pi kt}, \quad \text{where } a_k = \begin{cases} \frac{1}{\pi(1+|k|)} & k \neq 0 \\ \frac{1}{2} & k = 0 \end{cases}$$

$k=1: \frac{1}{2\pi} = a_1$   
 $k=2: \frac{1}{3\pi}$   
 $k=0$

- (a) Determine the Fourier transform of the periodic signal  $x(t)$ . Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$$X(j\omega) = \sum_{k=-2}^2 2\pi a_k \delta(\omega - 7\pi k)$$



- (b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j5\omega}{\omega^2 - 49\pi^2 + j7\omega}$$

Determine the maximum magnitude of  $H(j\omega)$ , and the frequency where the maximum occurs.

$$\max |H(j\omega)| = \frac{5}{7}$$

$$\text{at } \omega = 7\pi \text{ rad/s}$$

Peak at  $\omega = 7\pi$  because  
 $\omega^2 - 49\pi^2 = 0$  at  $\omega = 7\pi$

$$H(j7\pi) = \frac{j5(7\pi)}{j7(7\pi)} = \frac{5}{7} = 0.7143$$

- (c) For  $x(t)$  given above, the output signal can be written as

$$y(t) = B_0 + B_1 \cos(\omega_0 t + \psi_1) + B_2 \cos(2\omega_0 t + \psi_2)$$

where  $B_k \geq 0$ . Determine the values of the parameters  $B_0$  and  $B_1$ .

$$B_0 = 0$$

$$B_1 = 0.2274$$

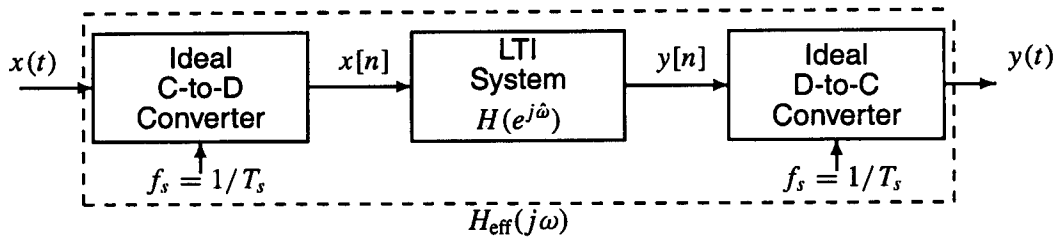
$$H(j0) = 0 \Rightarrow B_0 = 0$$

$$b_1 = a_1 H(j7\pi) = \frac{1}{2\pi} \left(\frac{5}{7}\right) = \frac{5}{14\pi}$$

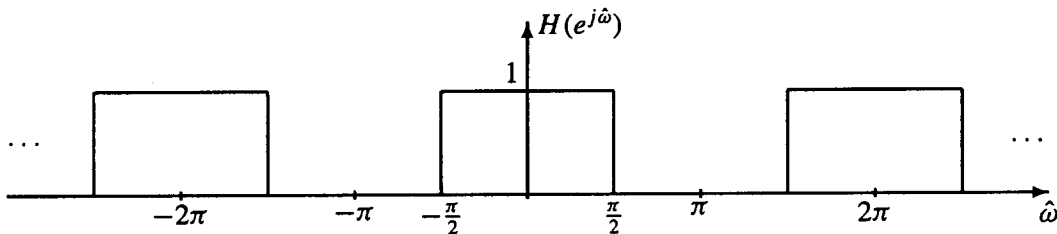
$$B_1 = 2|b_1| = \frac{5}{7\pi} = 0.2274$$

**PROBLEM Spring-03-F.8:**

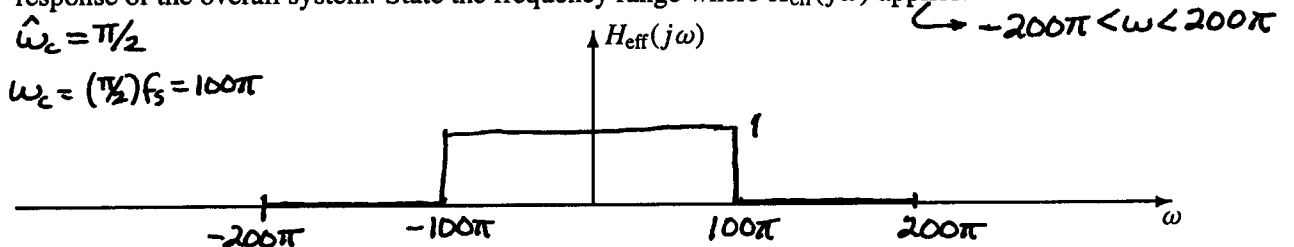
Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time system has frequency response  $H(e^{j\hat{\omega}})$  defined by the following plot:



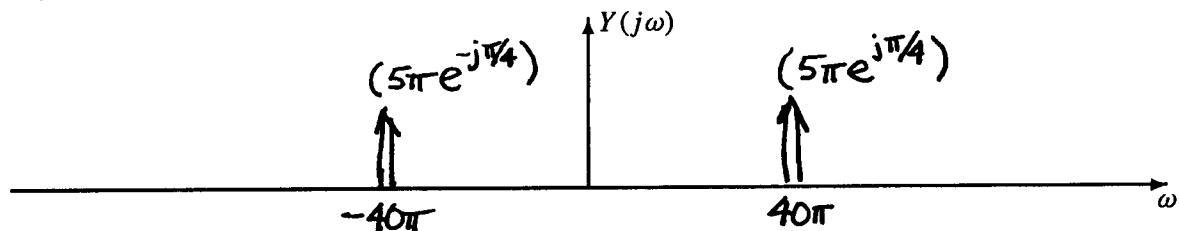
- (a) Now, if  $f_s = 200$  samples/sec, make a carefully labeled plot of  $H_{eff}(j\omega)$ , the effective frequency response of the overall system. State the frequency range where  $H_{eff}(j\omega)$  applies.



- (b) Assume that the input signal  $x(t)$  is a sum of cosines:

$$x(t) = 8 \cos(180\pi t + \pi/3) + 5 \cos(360\pi t - \pi/4)$$

For this input signal, determine the Fourier transform of the output signal  $y(t)$  when the sampling rate is  $f_s = 200$  samples/sec. Make a plot of  $Y(j\omega)$ .



$$\omega = 180\pi \rightarrow \hat{\omega} = \frac{\omega}{f_s} = \frac{180\pi}{200} = .9\pi \leftarrow \text{in STOP band}$$

$$\omega = 360\pi \rightarrow \hat{\omega} = 1.8\pi \text{ aliases to } 1.8\pi - 2\pi = -0.2\pi$$

$$y[n] = 5 \cos(-0.2\pi n - \pi/4)$$

$$y(t) = 5 \cos(-40\pi t - \pi/4)$$