GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 4-28-2003 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L01:Tues-9:30am (McLaughlin)		L02:Thur-9:30am (Barry)
	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
	L05:Tues-1:30pm (Li)		
L11:M-3pm (McClellan)	L07:Tues-3pm (Li)	L12:W-3pm (Hayes)	L08:Thur-3pm (Williams)
	L09:Tues-4:30pm (Zhou)	L14:W-4:30pm (Hayes)	
	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- JUSTIFY your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer ***on the answer sheet*** or in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	

PROBLEM spr-03-F.1:

(Circle exactly <u>one</u> answer[7] for each system, $\mathcal{S}_i$)

$\mathcal{S}_1$:	#1	#2	#3	#4	#5	#6	#7	#8	#9
$\mathcal{S}_2$:	#1	#2	#3	#4	#5	#6	#7	#8	#9
$\mathcal{S}_3$:	#1	#2	#3	#4	#5	#6	#7	#8	#9
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$\mathcal{S}_7$:	#1	#2	#3	#4	#5	#6	#7	#8	#9
$\mathcal{S}_8$:	#1	#2	#3	#4	#5	#6	#7	#8	#9

PROBLEM spr-03-F.2:

(Circle exactly <u>one</u> answer for each system, $\mathcal{S}_i$)

$\mathcal{S}_1$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_2$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_3$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_4$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_5$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_6$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_7$:	(A)	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_8$:	(A)	(B)	(C)	(D)	(E)	(F)	None

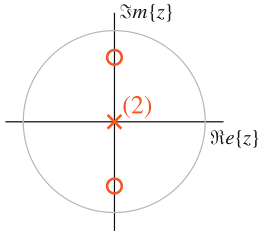
PROBLEM spr-03-F.3:

(Circle exactly <u>one</u> answer for each part)

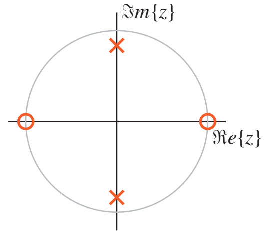
(a)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(d)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(e)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(h)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None

[7] If more than one answer is circled, the response will be considered wrong and will receive no credit.

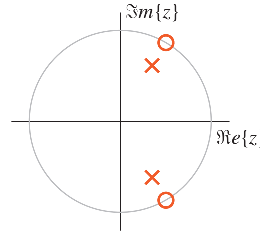
PROBLEM spr-03-F.1:



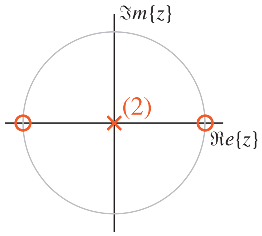
Pole–Zero Plot #1



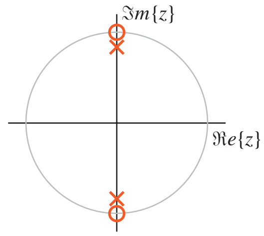
Pole–Zero Plot #2



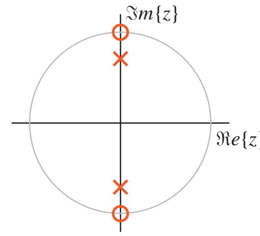
Pole–Zero Plot #3



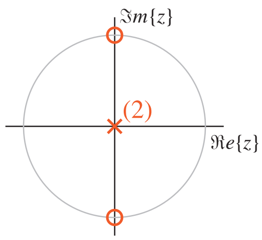
Pole–Zero Plot #4



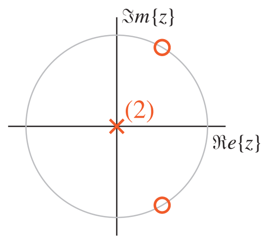
Pole–Zero Plot #5



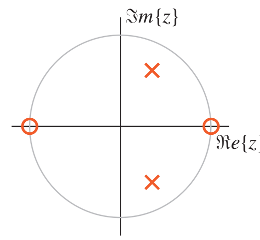
Pole–Zero Plot #6



Pole–Zero Plot #7



Pole–Zero Plot #8



Pole–Zero Plot #9

For each of systems below[8] determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. ***Mark your answers on the answer sheet provided.***
Note: the unit circle is shown for reference.

$\mathcal{S}_1$: `filter([1.5, 0, -1.5], [1, 0, 0.7], xx)`

$$\mathcal{S}_2: \quad y[n] = 0.7y[n-1] - 0.5y[n-2] + \tfrac{22}{3}x[n] - \tfrac{22}{3}x[n-1] + \tfrac{22}{3}x[n-2]$$

$$\mathcal{S}_3: \quad H(z) = \frac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$$

$$\mathcal{S}_4: \quad y[n] = 5x[n] + 5x[n-2]$$

$$\mathcal{S}_5: \quad H(z) = \frac{8.5 + 8.5z^{-2}}{1 + 0.7z^{-2}}$$

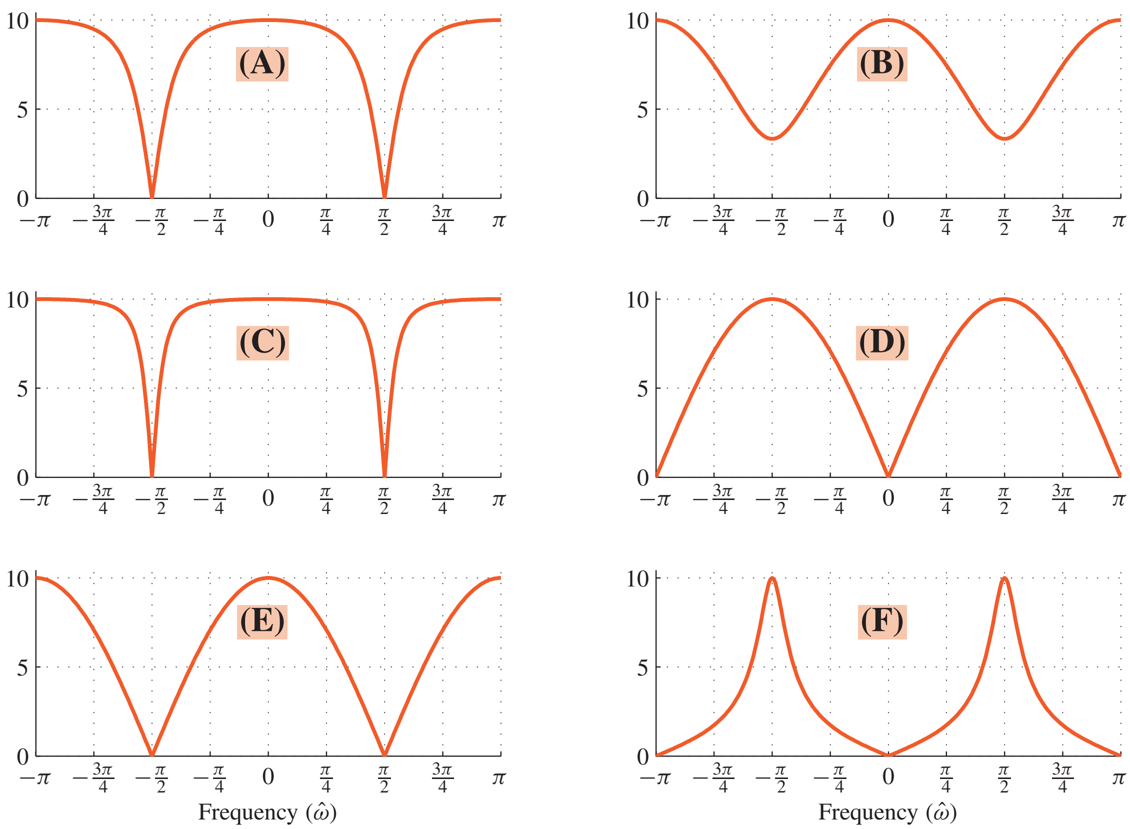
$$\mathcal{S}_6: \quad y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$$

$\mathcal{S}_7$: `filter([5, 0, -5], 1, xx)`

$$\mathcal{S}_8: \quad y[n] = \tfrac{20}{3}x[n] + \tfrac{10}{3}x[n-2]$$

[8] These same systems are also used in the next problem.

PROBLEM spr-03-F.2:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. ***Mark your answers on the answer sheet provided.***
Note: the frequency axis is $\hat{\omega}$.

$\mathcal{S}_1$: `filter([1.5, 0, -1.5], [1, 0, 0.7], xx)`

$\mathcal{S}_2$: $y[n] = 0.7y[n-1] - 0.5y[n-2] + \frac{22}{3}x[n] - \frac{22}{3}x[n-1] + \frac{22}{3}x[n-2]$

$\mathcal{S}_3$: $H(z) = \dfrac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$

$\mathcal{S}_4$: $y[n] = 5x[n] + 5x[n-2]$

$\mathcal{S}_5$: $H(z) = \dfrac{8.5 + 8.5z^{-2}}{1 + 0.7z^{-2}}$

$\mathcal{S}_6$: $y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$

$\mathcal{S}_7$: `filter([5, 0, -5], 1, xx)`

$\mathcal{S}_8$: $y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$

PROBLEM spr-03-F.3:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers on the answer sheet provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \delta(t+2) * e^{-t+1}u(t-1) * \delta(t-1)$

(b) $x(t) = u(t) - u(t-8)$

(c) $x(t) = e^{-(t-4)} \int\limits_{-\infty}^{0} \delta(\tau - 4) d\tau$

(d) $x(t) = u(t-3) - u(t-5)$

(e) $x(t) = -e^{-t}u(t) + \delta(t)$

(f) $x(t) = \delta(t) - \delta(t-8)$

(g) $x(t) = \cos(\pi t)\delta(t-4)$

(h) $x(t) = \int\limits_{-\infty}^{t} e^{-t+\tau}\delta(\tau - 4) d\tau$

Each of the time signals above has a Fourier transform that might be in the list below.

[1] $X(j\omega) = 2e^{-j4\omega} \dfrac{\sin(\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{\sin(4\omega)}{\omega/2}$

[3] $X(j\omega) = e^{-j4\omega}[\pi\delta(\omega - \pi) + \pi\delta(\omega + \pi)]$

[4] $X(j\omega) = e^{-j4\omega}[u(\omega) - u(\omega - 8)]$

[5] $X(j\omega) = \dfrac{j\omega}{1 + j\omega}$

[6] $X(j\omega) = \dfrac{1}{1 + j\omega}$

[7] $X(j\omega) = \dfrac{e^{-j4\omega}}{1 + j\omega}$

[8] $X(j\omega) = 2e^{-j4\omega} \dfrac{\sin(4\omega)}{\omega}$

[9] $X(j\omega) = j2e^{-j4\omega} \sin(4\omega)$

[10] $X(j\omega) = 0$

[None] $X(j\omega)$ not in the list above.

PROBLEM spr-03-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

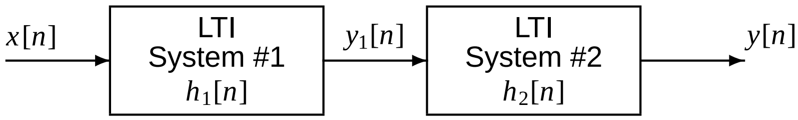


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{5z^{-3} - 5z^{-4}}{1 - 3z^{-1}}$$

but System #2 is unknown.

(a) Determine whether or not System #1 is stable. Give a reason to support your answer.

(b) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $y_1[n]$.

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n-5]$$

From this information, determine the system function $H_2(z)$ for the second system. ***Simplify the expression for*** $H_2(z)$**.**

PROBLEM spr-03-F.5:

A few quick questions:

(a) Consider the following MATLAB program:

```
nn = 0:16000;
yy = pi*cos(1.6*pi*nn);
soundsc(yy,48000)
```

Neglecting the end effects in the convolution, the frequency ***in hertz*** for the output signal produced by the `soundsc( )` statement, i.e., the frequency that you hear.

Frequency = ☐ Hz

(b) Evaluate $\left|H(e^{j\hat{\omega}})\right|^2$, where $H(e^{j\hat{\omega}}) = 3je^{-j(3\hat{\omega}^2+\hat{\omega})}\sin(2\hat{\omega})$.

(c) Determine the ***minimum period*** (in seconds) of the following signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin^2(k/4)}{k^2} e^{j\pi kt/4}$$

Period = ☐ secs.

(d) Solve the following relationship for A and ϕ

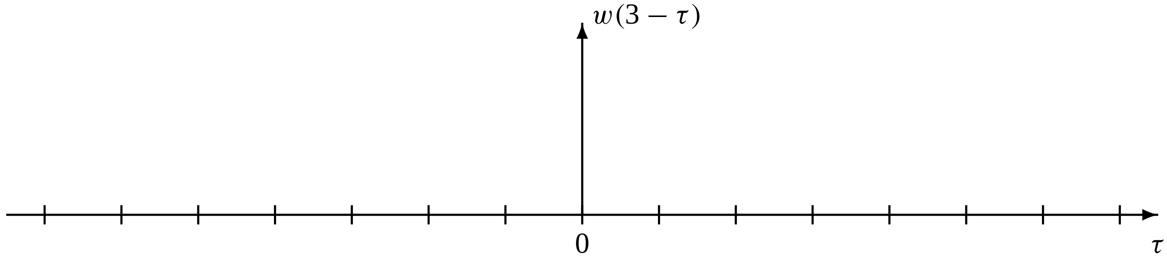
$$12\cos(3t) + A\cos(3t+\phi) = 9\cos(3t-\pi/2)$$

$A =$ ☐

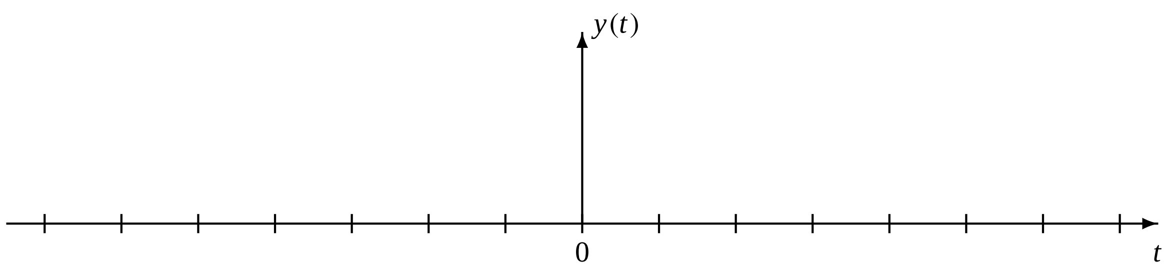
$\phi =$ ☐

PROBLEM spr-03-F.6:

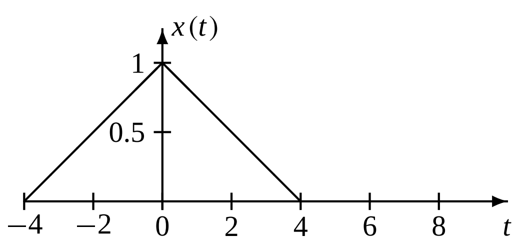
(a) Assume that $w(t) = u(t+2) - u(t-1)$. Plot $w(3-\tau)$ as a function of τ.

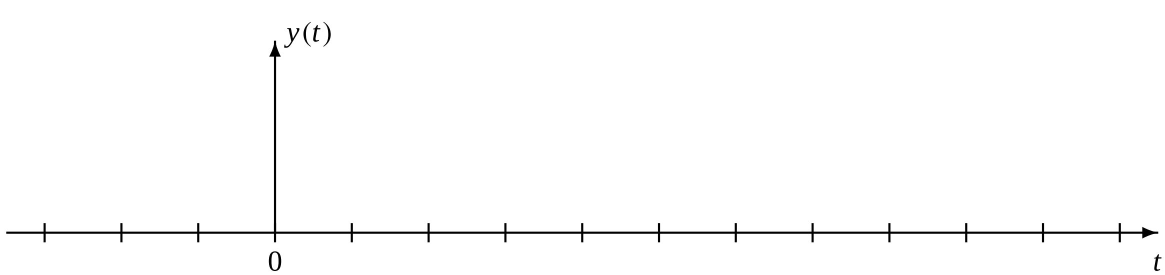


(b) Perform the convolution $y(t) = w(t) * u(t)$ where $u(t)$ is the unit-step signal. Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.

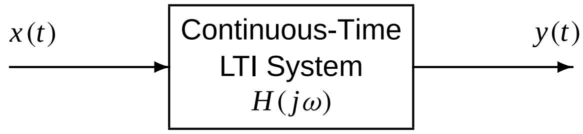


(c) If the input to an LTI system is $x(t)$ below, determine the output signal $y(t) = x(t) * h(t)$ when $h(t) = 10\delta(t) - 10\delta(t-4)$. Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.





PROBLEM spr-03-F.7:

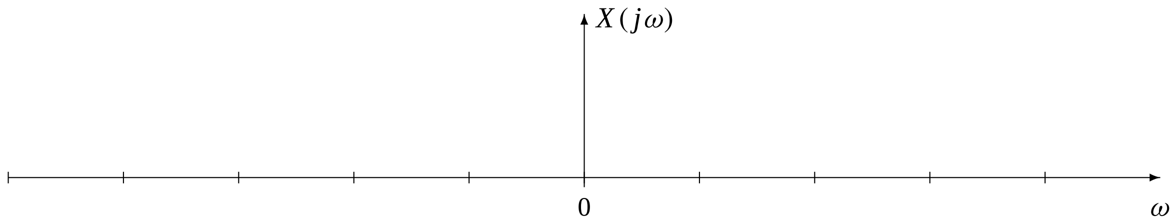


The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-2}^{2} a_k e^{j10\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1-2k^2} & k \neq 0 \\ 0.2 & k = 0 \end{cases}$$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$X(j\omega) =$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j32\omega}{\omega^2 - 100\pi^2 + j10\omega}$$

Determine the maximum magnitude of $H(j\omega)$, and the frequency where the maximum occurs.

$\max |H(j\omega)| =$

at $\omega =$

(c) For $x(t)$ given above, the output signal can be written as

$$y(t) = B_0 + B_1 \cos(\omega_0 t + \psi_1) + B_2 \cos(2\omega_0 t + \psi_2)$$

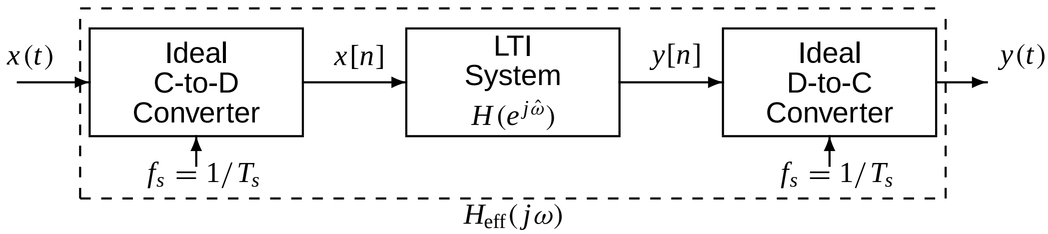
where $B_k \geq 0$. Determine the values of the parameters B_0 and B_1.

$B_0 =$

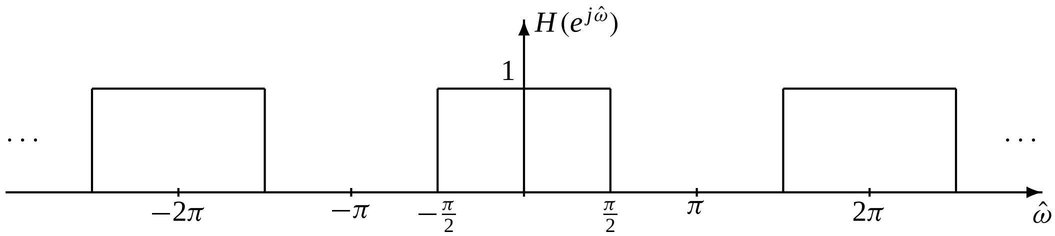
$B_1 =$

PROBLEM spr-03-F.8:

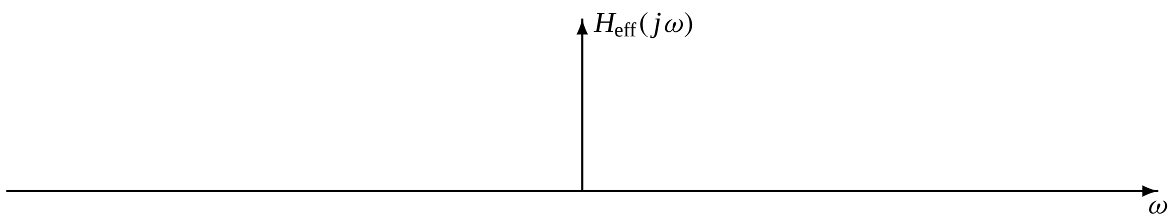
Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot:



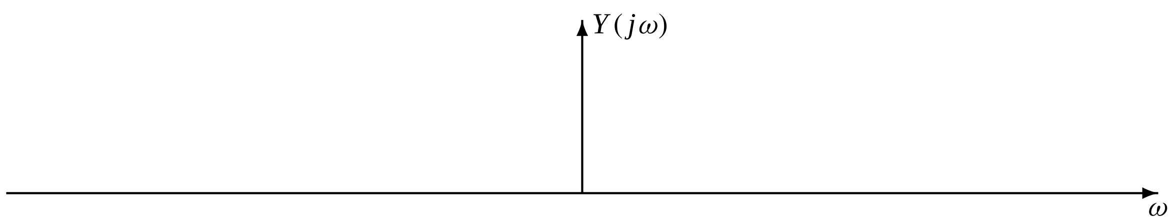
(a) Now, if f_s = **1000 samples/sec**, make a carefully labeled plot of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system. State the frequency range where $H_{\text{eff}}(j\omega)$ applies.



(b) Assume that the input signal $x(t)$ is a sum of cosines:

$$x(t) = 9\cos(850\pi t + \pi/3) + 3\cos(1700\pi t + \pi/4)$$

For this input signal, determine the Fourier transform of the output signal $y(t)$ when the sampling rate is f_s = **1000 samples/sec.** Make a plot of $Y(j\omega)$.



GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 4-28-2003 COURSE: ECE-2025

NAME: Answer Key (LAST, FIRST) GT #: Version-4

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8	20	

PROBLEM spr-03-F.1:

(Circle exactly one answer[7] for each system, $\mathcal{S}_i$)

$\mathcal{S}_1$:	#1	**(#2)**	#3	#4	#5	#6	#7	#8	#9
$\mathcal{S}_2$:	#1	#2	**(#3)**	#4	#5	#6	#7	#8	#9
$\mathcal{S}_3$:	#1	#2	#3	#4	#5	#6	#7	#8	**(#9)**
$\mathcal{S}_4$:	#1	#2	#3	#4	#5	#6	**(#7)**	#8	#9
$\mathcal{S}_5$:	#1	#2	#3	#4	**(#5)**	#6	#7	#8	#9
$\mathcal{S}_6$:	#1	#2	#3	#4	#5	**(#6)**	#7	#8	#9
$\mathcal{S}_7$:	#1	#2	#3	**(#4)**	#5	#6	#7	#8	#9
$\mathcal{S}_8$:	**(#1)**	#2	#3	#4	#5	#6	#7	#8	#9

PROBLEM spr-03-F.2:

(Circle exactly one answer for each system, $\mathcal{S}_i$)

$\mathcal{S}_1$:	(A)	(B)	(C)	(D)	(E)	**(F)**	None
$\mathcal{S}_2$:	(A)	(B)	(C)	(D)	(E)	(F)	**None**
$\mathcal{S}_3$:	(A)	(B)	(C)	(D)	(E)	(F)	**None**
$\mathcal{S}_4$:	(A)	(B)	(C)	(D)	**(E)**	(F)	None
$\mathcal{S}_5$:	(A)	(B)	**(C)**	(D)	(E)	(F)	None
$\mathcal{S}_6$:	**(A)**	(B)	(C)	(D)	(E)	(F)	None
$\mathcal{S}_7$:	(A)	(B)	(C)	**(D)**	(E)	(F)	None
$\mathcal{S}_8$:	(A)	**(B)**	(C)	(D)	(E)	(F)	None

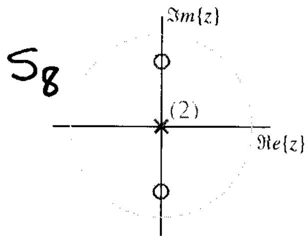
PROBLEM spr-03-F.3:

(Circle exactly one answer for each part)

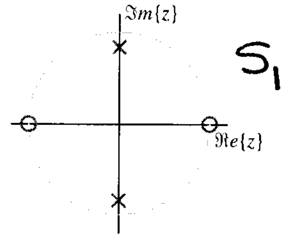
(a)	[1]	[2]	[3]	[4]	[5]	**[6]**	[7]	[8]	[9]	[10]	None
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	**[8]**	[9]	[10]	None
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	**[10]**	None
(d)	**[1]**	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	None
(e)	[1]	[2]	[3]	[4]	**[5]**	[6]	[7]	[8]	[9]	[10]	None
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	**[9]**	[10]	None
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	**None**
(h)	[1]	[2]	[3]	[4]	[5]	[6]	**[7]**	[8]	[9]	[10]	None

[7] If more than one answer is circled, the response will be considered wrong and will receive no credit.

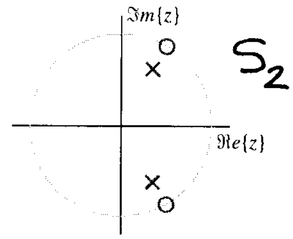
PROBLEM spr-03-F.1:



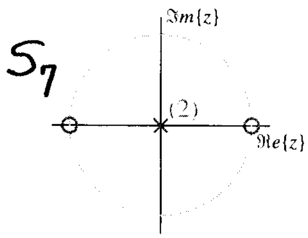
Pole–Zero Plot #1



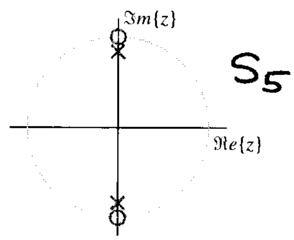
Pole–Zero Plot #2



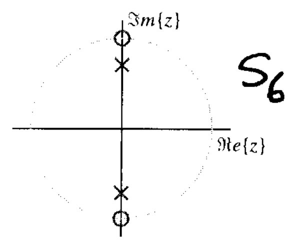
Pole–Zero Plot #3



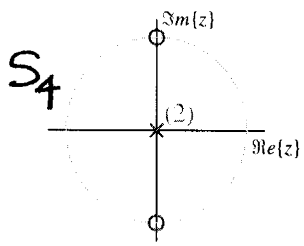
Pole–Zero Plot #4



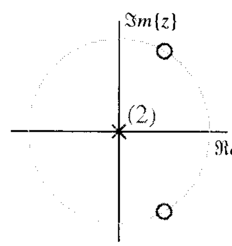
Pole–Zero Plot #5



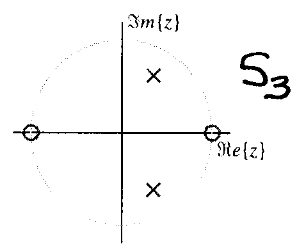
Pole–Zero Plot #6



Pole–Zero Plot #7



Pole–Zero Plot #8



Pole–Zero Plot #9

For each of systems below[8] determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. ***Mark your answers on the answer sheet provided.***
Note: the unit circle is shown for reference.

$\mathcal{S}_1$: `filter([1.5, 0, -1.5], [1, 0, 0.7], xx)` $\dfrac{1.5(1-z^{-2})}{1+.7z^{-2}}$ zeros: ± 1 poles: $\pm j\sqrt{0.7}$

$\mathcal{S}_2$: $y[n] = 0.7y[n-1] - 0.5y[n-2] + \frac{22}{3}x[n] - \frac{22}{3}x[n-1] + \frac{22}{3}x[n-2]$ $\dfrac{22/3(1-z^{-1}+z^{-2})}{1-0.7z^{-1}+0.5z^{-2}}$ zeros: $1e^{\pm j\pi/3}$

$\mathcal{S}_3$: $H(z) = \dfrac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$ zeros: ± 1 poles: $0.707e^{\pm j\pi/3}$

$\mathcal{S}_4$: $y[n] = 5x[n] + 5x[n-2]$ $\Rightarrow 5(1+z^{-2}) \Rightarrow$ FIR $\Rightarrow$ zeros: $\pm j$

$\mathcal{S}_5$: $H(z) = \dfrac{8.5 + 8.5z^{-2}}{1 + 0.7z^{-2}}$ $\Rightarrow$ zeros: $\pm j$ poles: $\pm j\sqrt{.7} \approx \pm j0.84$

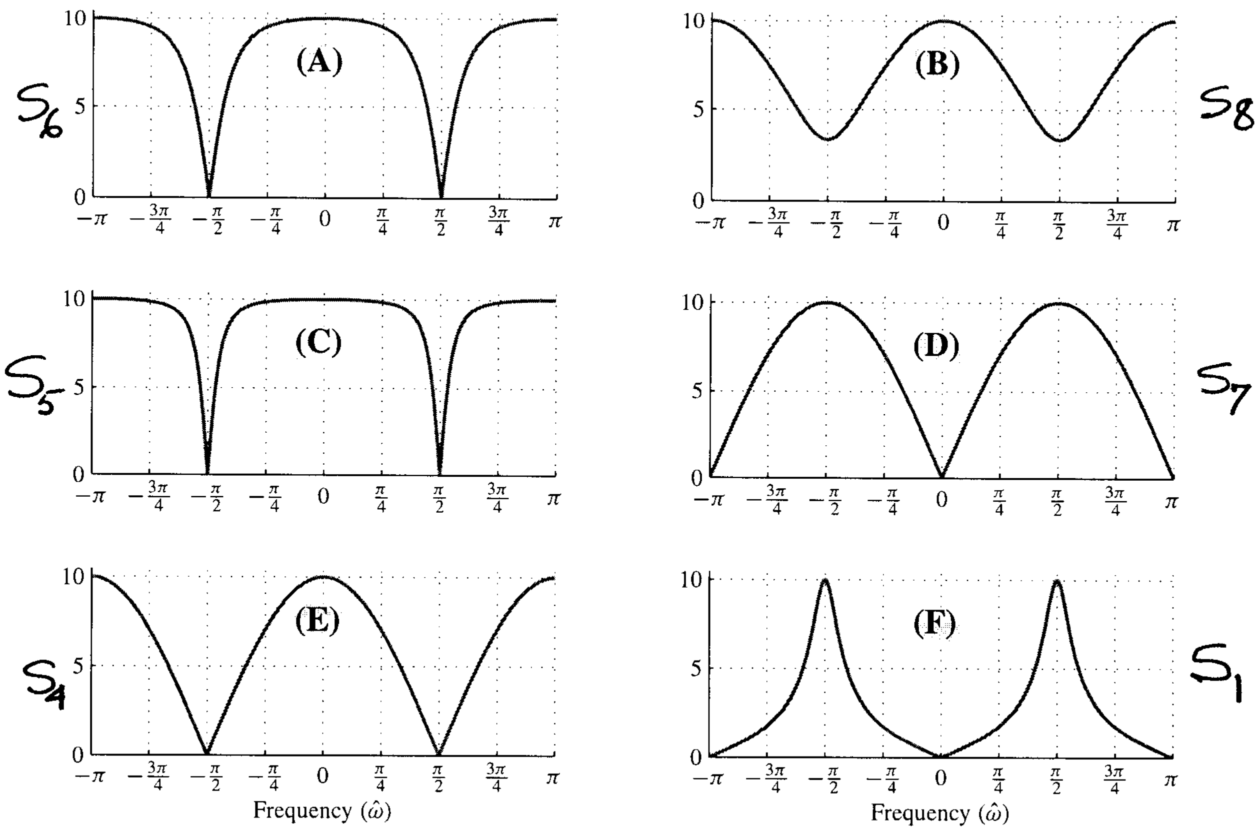
$\mathcal{S}_6$: $y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$ $\dfrac{7.5(1+z^{-2})}{1+.5z^{-2}} \Rightarrow$ zeros: $\pm j$ poles: $\pm j\sqrt{.5}$

$\mathcal{S}_7$: `filter([5, 0, -5], 1, xx)` FIR $\Rightarrow 5(1-z^{-2}) \Rightarrow$ zeros: ± 1

$\mathcal{S}_8$: $y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$ FIR $\Rightarrow \frac{20}{3}(1+\frac{1}{2}z^{-2}) \Rightarrow$ zeros: $\pm j\sqrt{.5} \approx \pm j0.707$

[8] These same systems are also used in the next problem.

PROBLEM spr-03-F.2:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. ***Mark your answers on the answer sheet provided.***
Note: the frequency axis is $\hat{\omega}$.

$\mathcal{S}_1$: `filter([1.5, 0, -1.5], [1, 0, 0.7], xx)` BPF, peaks at $\hat{\omega}=\pm\pi/2$ zeros at $\hat{\omega}=0,\pi$

$\mathcal{S}_2$: $y[n] = 0.7y[n-1] - 0.5y[n-2] + \frac{22}{3}x[n] - \frac{22}{3}x[n-1] + \frac{22}{3}x[n-2]$ Notch at $\pi/3$

$\mathcal{S}_3$: $H(z) = \dfrac{2.5 - 2.5z^{-2}}{1 - 0.7z^{-1} + 0.5z^{-2}}$ BPF, peaks at $\hat{\omega} = \pm\pi/3$

$\mathcal{S}_4$: $y[n] = 5x[n] + 5x[n-2]$ FIR, zeros at $\hat{\omega}=\pm\pi/2$

$\mathcal{S}_5$: $H(z) = \dfrac{8.5 + 8.5z^{-2}}{1 + 0.7z^{-2}}$ Notch at $\pm\pi/2$, narrower notch than S_6 because pole closer to U.C.

$\mathcal{S}_6$: $y[n] = -0.5y[n-2] + 7.5x[n] + 7.5x[n-2]$ Notch at $\hat{\omega}=\pm\pi/2$

$\mathcal{S}_7$: `filter([5, 0, -5], 1, xx)` FIR, zeros at $\hat{\omega}=0,\pi$

$\mathcal{S}_8$: $y[n] = \frac{20}{3}x[n] + \frac{10}{3}x[n-2]$ FIR, zeros near $\hat{\omega}=\pm\pi/2$, but not on U.C.

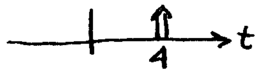
PROBLEM spr-03-F.3:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers on the answer sheet provided.*** (The operator $*$ denotes convolution.)

(a) $x(t) = \delta(t+2) * e^{-t+1}u(t-1) * \delta(t-1)$ $= e^{-t}u(t) \rightarrow \frac{1}{1+j\omega}$

(b) $x(t) = u(t) - u(t-8)$ Rect. pulse → sinc $e^{-j4\omega}\frac{\sin(4\omega)}{\omega/2}$

(c) $x(t) = e^{-(t-4)}\int_{-\infty}^{0}\delta(\tau-4)d\tau$



$\Rightarrow x(t)=0 \longrightarrow X(j\omega)=0$

(d) $x(t) = u(t-3) - u(t-5)$ $e^{-j4\omega}\frac{\sin(\omega)}{\omega/2}$

(e) $x(t) = -e^{-t}u(t) + \delta(t)$ $-\frac{1}{1+j\omega} + 1 = \frac{-1+1+j\omega}{1+j\omega} = \frac{j\omega}{1+j\omega}$

(f) $x(t) = \delta(t) - \delta(t-8)$ $1 - e^{-j8\omega} = e^{-j4\omega}(e^{j4\omega} - e^{-j4\omega}) = 2je^{-j4\omega}\sin(4\omega)$

(g) $x(t) = \cos(\pi t)\delta(t-4)$ $= \cos(4\pi)\delta(t-4) = \delta(t-4) \longrightarrow e^{-j4\omega}$

(h) $x(t) = \int_{-\infty}^{t} e^{-t+\tau}\delta(\tau-4)d\tau$ $= \int_{-\infty}^{t} e^{-t+4}\delta(\tau-4)d\tau = e^{-(t-4)}u(t-4) \rightarrow \frac{e^{-j4\omega}}{1+j\omega}$

Each of the time signals above has a Fourier transform that might be in the list below.

[1] $X(j\omega) = 2e^{-j4\omega}\frac{\sin(\omega)}{\omega}$ (d)

[2] $X(j\omega) = \frac{\sin(4\omega)}{\omega/2}$

(g) is not in the list

[3] $X(j\omega) = e^{-j4\omega}[\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)]$

[4] $X(j\omega) = e^{-j4\omega}[u(\omega) - u(\omega-8)]$

[5] $X(j\omega) = \frac{j\omega}{1+j\omega}$ (e)

[6] $X(j\omega) = \frac{1}{1+j\omega}$ (a)

[7] $X(j\omega) = \frac{e^{-j4\omega}}{1+j\omega}$ (h)

[8] $X(j\omega) = 2e^{-j4\omega}\frac{\sin(4\omega)}{\omega}$ (b)

[9] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$ (f)

[10] $X(j\omega) = 0$ (c)

[None] $X(j\omega)$ not in the list above.

PROBLEM spr-03-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

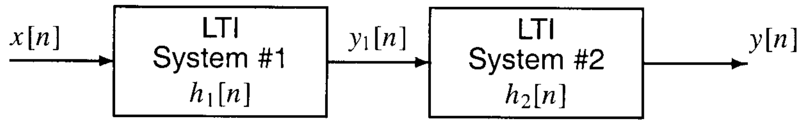


Figure 1: Cascade connection of two discrete-time LTI systems.

Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{5z^{-3} - 5z^{-4}}{1 - 3z^{-1}}$$

but System #2 is unknown.

(a) Determine whether or not System #1 is stable. Give a reason to support your answer.

pole at $z=3$ is outside unit circle $\Rightarrow$ NOT stable

(b) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $y_1[n]$.

$x[n] = u[n] \Rightarrow X(z) = \frac{1}{1-z^{-1}}$

$Y_1(z) = H_1(z)X(z) = \frac{5z^{-3}(1-z^{-1})}{1-3z^{-1}} \cdot \frac{1}{1-z^{-1}} = \frac{5z^{-3}}{1-3z^{-1}}$

Invert using $a^n u[n] \rightarrow \frac{1}{1-az^{-1}}$ and use shift property

$y_1[n] = 5\,(3)^{n-3}u[n-3]$

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n-5]$$

From this information, determine the system function $H_2(z)$ for the second system. SIMPLIFY.

$x[n] = \delta[n] \Rightarrow X(z) = 1$

$y[n] = \delta[n-5] \Rightarrow Y(z) = z^{-5}$

$Y(z) = H_1(z)H_2(z)X(z)$ ← solve for $H_2(z)$

$z^{-5} = H_1(z)H_2(z)\,1$

$H_2(z) = \frac{z^{-5}}{H_1(z)} = \frac{z^{-5}}{\frac{5z^{-3}-5z^{-4}}{1-3z^{-1}}} = \frac{z^{-5}(1-3z^{-1})}{5z^{-3}(1-z^{-1})}$

$= \frac{1}{5}\frac{z^{-2}(1-3z^{-1})}{1-z^{-1}}$

PROBLEM spr-03-F.5:

A few quick questions:

(a) Consider the following MATLAB program:

```
nn = 0:16000;
yy = pi*cos(1.6*pi*nn);
soundsc(yy,48000)
```

$\hat{\omega} = 1.6\pi$ which "aliases" to $1.6\pi - 2\pi = -0.4\pi$

← f_s

Neglecting the end effects in the convolution, the frequency ***in hertz*** for the output signal produced by the soundsc( ) statement, i.e., the frequency that you hear.

Frequency = 9600 Hz

$\pi\cos(-0.4\pi n) = \pi\cos(0.4\pi n)$

$\omega_{ANALOG} = \hat{\omega} f_s = (0.4\pi)48000$
$= 2\pi(9600)$

(b) Evaluate $|H(e^{j\hat{\omega}})|^2$, where $H(e^{j\hat{\omega}}) = 3je^{-j(3\hat{\omega}^2+\hat{\omega})}\sin(2\hat{\omega})$.

$|j| = 1$ and $|e^{-j\theta}| = 1$

$|H(e^{j\hat{\omega}})|^2 = [3\sin(2\hat{\omega})]^2 = 9\sin^2(2\hat{\omega})$

(c) Determine the ***minimum period*** (in seconds) of the following signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin^2(k/4)}{k^2} e^{j\pi kt/4}$$

Period = 8 secs.

$\omega_0 = \pi/4 = 2\pi(1/8) = 2\pi/T$
$\Rightarrow T = 8$ secs

(d) Solve the following relationship for A and ϕ

$$12\cos(3t) + A\cos(3t+\phi) = 9\cos(3t - \pi/2)$$

A = 15

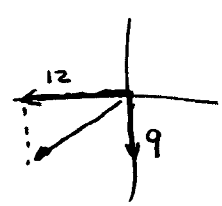
ϕ = −2.498 rads

Convert to phasors:

$12e^{j0} + Ae^{j\varphi} = 9e^{-j\pi/2}$

$Ae^{j\varphi} = -12 + 9e^{-j\pi/2}$

$= 15e^{-j2.5}$

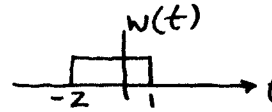


-2.5 rads $= -0.795\pi = -143.13°$

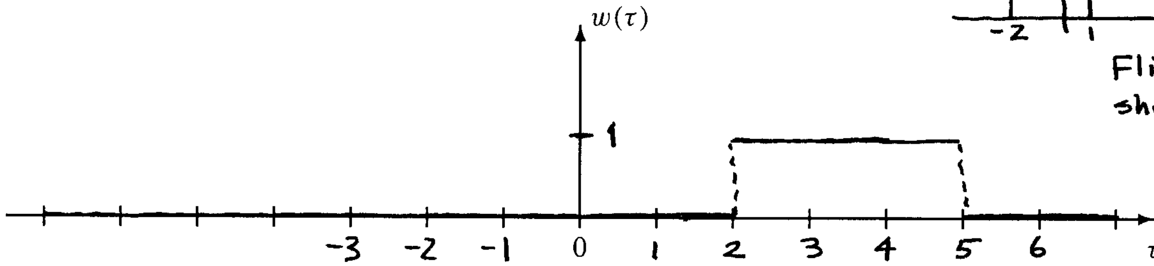
↗ -2.498

PROBLEM spr-03-F.6:

(a) Assume that $w(t) = u(t+2) - u(t-1)$. Plot $w(3-\tau)$ as a function of τ.



Flip and shift by 3

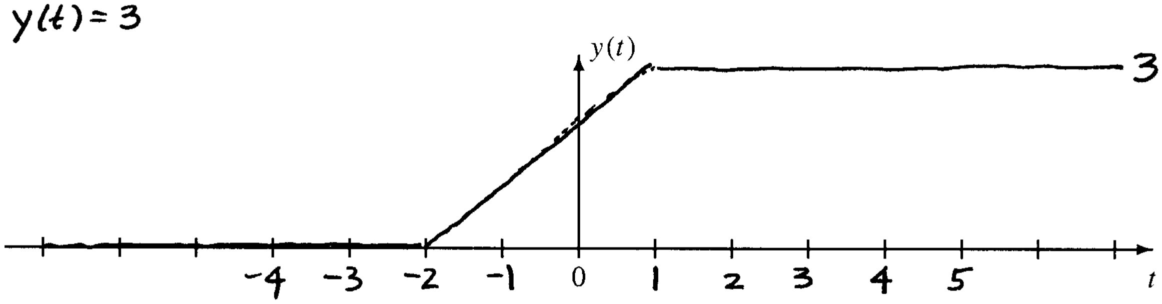


(b) Perform the convolution $y(t) = w(t) * u(t)$ where $u(t)$ is the unit-step signal. Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.

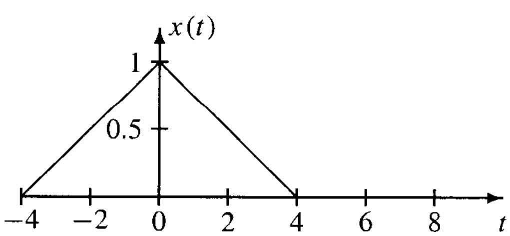
$$y(t) = u(t+2) * u(t) - u(t-1) * u(t)$$
$$= (t+2)u(t+2) - (t-1)u(t-1)$$

Starts up at $t=-2$
Levels off at $t=1$

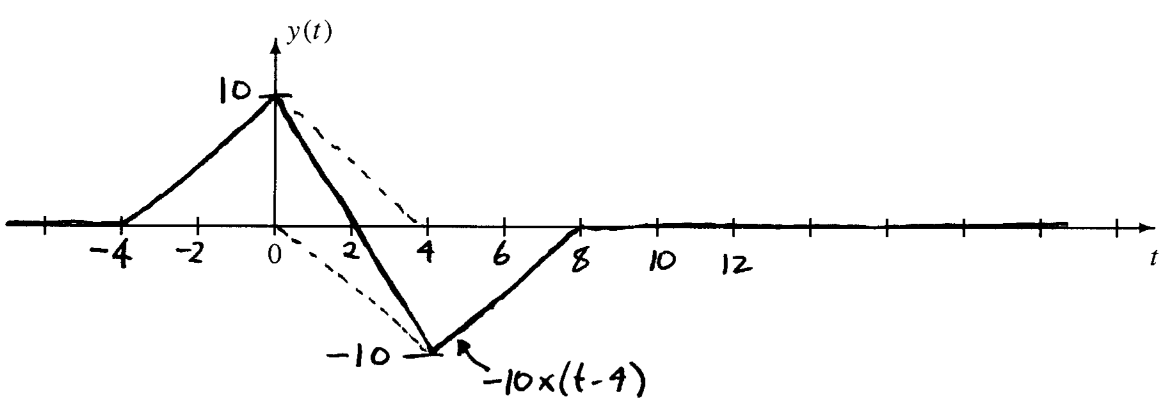
at $t \to \infty$
$y(t) = 3$



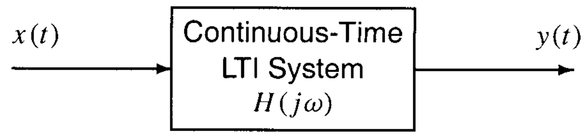
(c) If the input to an LTI system is $x(t)$ below, determine the output signal $y(t) = x(t) * h(t)$ when $h(t) = 10\delta(t) - 10\delta(t-4)$. Give your answer as a carefully labeled sketch showing numerical values for the amplitudes and along the time axis.



$$y(t) = [10\delta(t) - 10\delta(t-4)] * x(t)$$
$$= 10x(t) - 10x(t-4)$$



PROBLEM spr-03-F.7:



The periodic input to the above system is defined by the equation:

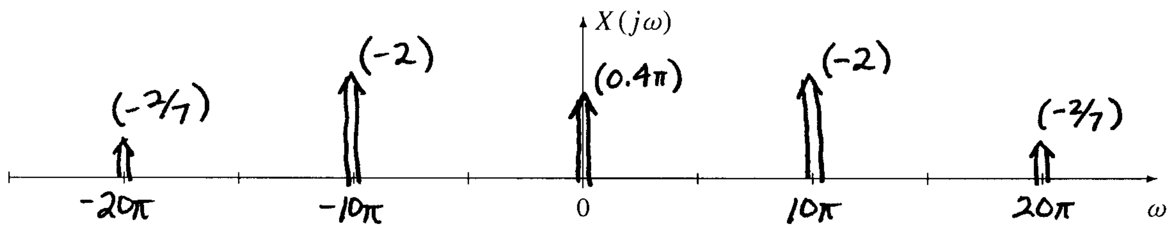
$$x(t) = \sum_{k=-2}^{2} a_k e^{j10\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1-2k^2} & k \neq 0 \\ 0.2 & k = 0 \end{cases}$$

$k=1 \rightarrow \frac{1/\pi}{1-2} = -1/\pi$

$k=2 \rightarrow \frac{1/\pi}{1-8} = -1/7\pi$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot with numerical values to receive full credit.

$$X(j\omega) = \sum_{k=-2}^{2} 2\pi a_k \delta(\omega - 10\pi k)$$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \frac{j32\omega}{\omega^2 - 100\pi^2 + j10\omega}$$

Determine the maximum magnitude of $H(j\omega)$, and the frequency where the maximum occurs.

max $|H(j\omega)| = 3.2$

at $\omega =$ 10π rad/s

$H(j\omega)$ is maximum at $\omega = 10\pi$ because the real part of the denominator cancels out

$$H(j10\pi) = \frac{j32(10\pi)}{j10(10\pi)} = 3.2$$

(c) For $x(t)$ given above, the output signal can be written as

$$y(t) = B_0 + B_1\cos(\omega_0 t + \psi_1) + B_2\cos(2\omega_0 t + \psi_2)$$

where $B_k \geq 0$. Determine the values of the parameters B_0 and B_1.

$\omega_0 = 10\pi$ rad/s

$B_0 = 0$

$B_1 = 2.037$

$H(j0) = 0 \Rightarrow B_0$

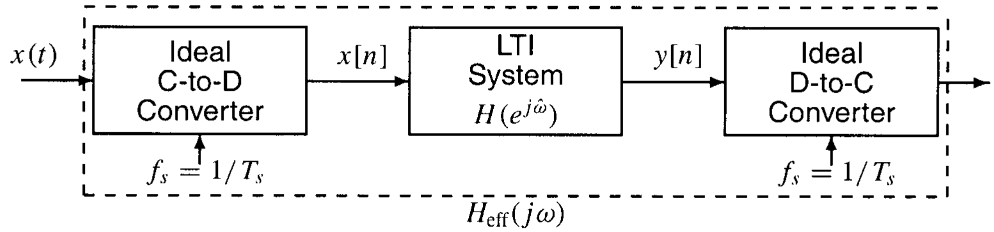
$b_k = a_k H(j\omega_0 k)$

$b_1 = a_1 H(j10\pi) = 3.2 a_1 = -3.2/\pi$

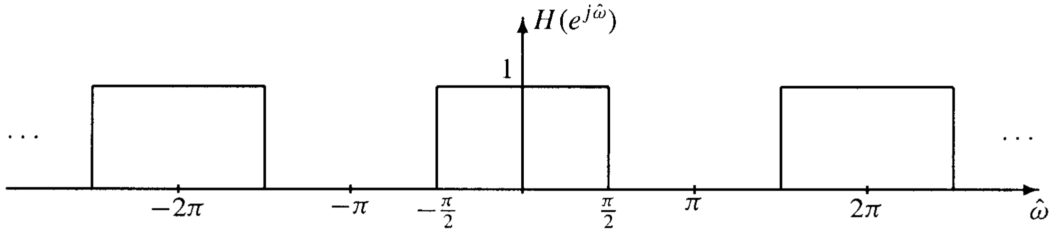
$B_1 = 2|b_1| = 6.4/\pi = 2.037$

PROBLEM spr-03-F.8:

Consider the following system for discrete-time filtering of a continuous-time signal:



Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot:

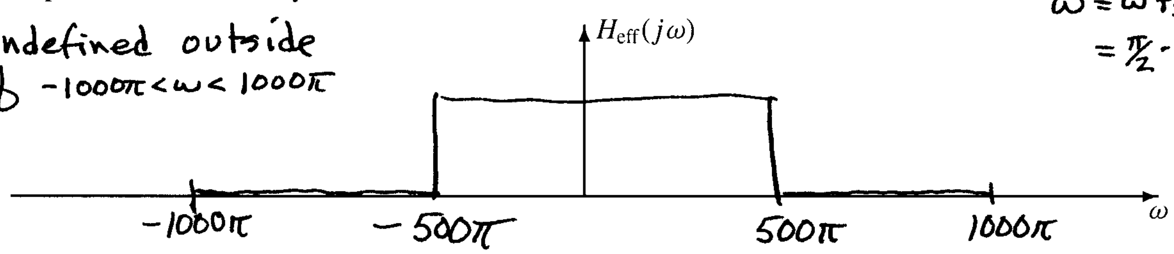


(a) Now, if f_s = **1000 samples/sec**, make a carefully labeled plot of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system.

$\omega = \hat{\omega} f_s$

$= \frac{\pi}{2} \cdot 1000$

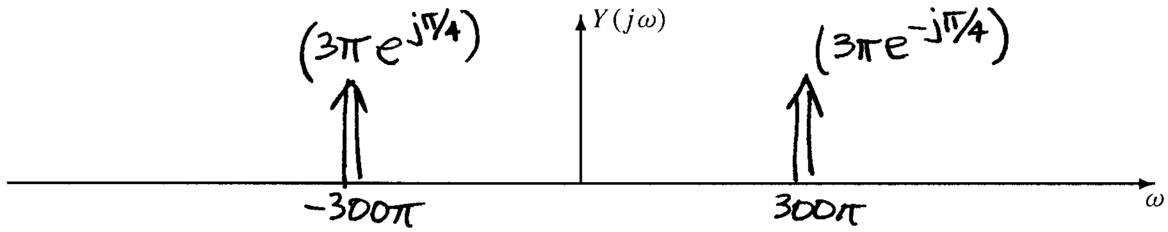
Undefined outside of $-1000\pi < \omega < 1000\pi$



(b) Assume that the input signal $x(t)$ is a sum of cosines:

$$x(t) = 9\cos(850\pi t + \pi/3) + 3\cos(1700\pi t + \pi/4)$$

For this input signal, determine the Fourier transform of the output signal $y(t)$ when the sampling rate is f_s = **1000 samples/sec.** Make a plot of $Y(j\omega)$.



There is aliasing

$\hat{\omega} = .3\pi \rightarrow \omega = \hat{\omega} f_s = 300\pi$

$\omega = 1700\pi \longrightarrow \hat{\omega} = \omega / f_s = \frac{1700\pi}{1000} = 1.7\pi + 2\pi\ell$

use $\ell = -1$ $\quad \hat{\omega} = -0.3\pi \Rightarrow x[n] = 3\cos(-0.3\pi n + \pi/4)$

$\omega = 850\pi \rightarrow \hat{\omega} = \frac{850\pi}{1000} = 0.85\pi$ ← No aliasing, but this component is in the stopband of the digital filter