GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 3/14/03 COURSE: ECE-2025

NAME: ______________________________ GT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L01:Tues-9:30am (McLaughlin)		L02:Thur-9:30am (Barry)
	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
	L05:Tues-1:30pm (Li)		
L11:M-3pm (McClellan)	L07:Tues-3pm (Li)	L12:W-3pm (Hayes)	L08:Thur-3pm (Williams)
	L09:Tues-4:30pm (Zhou)	L14:W-4:30pm (Hayes)	
	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM SPR-02-Q.2.1:

A signal $x(t)$ is periodic with period $T_0 = 7$ seconds. Therefore it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/7)kt}$$

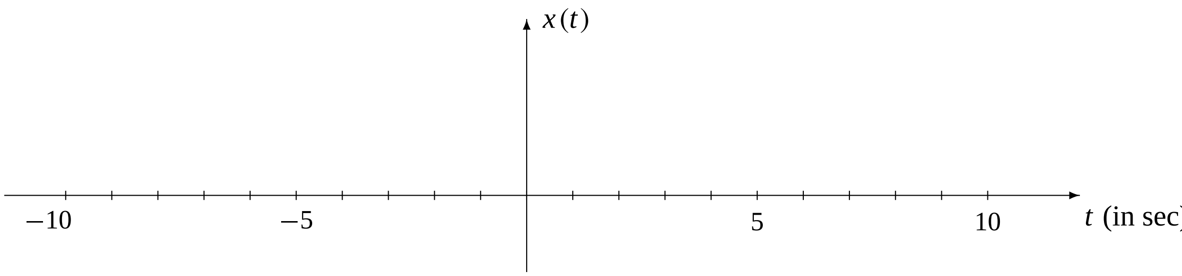
It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{7}\int_{-2}^{2} t^2 \, e^{-j(2\pi/7)kt} dt \qquad (1)$$

NOTE: Parts (c) and (d) of this problem can be worked independently of parts (a) and (b).

(a) In the expression for a_k in Eq. (1) above, the integral and its limits define the signal $x(t)$. Write an equation for $x(t)$ that is valid over one period.

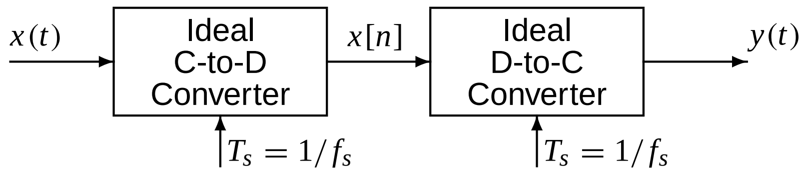
(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.



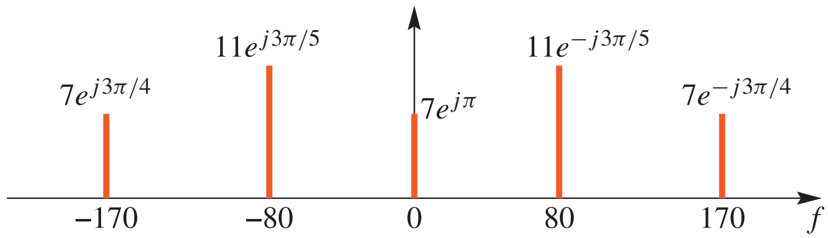
(c) Which value of k in Eq. (1) gives the DC (or average) value of $x(t)$? $k =$

(d) Determine the DC value of $x(t)$. Give your answer as a number.

PROBLEM SPR-02-Q.2.2:



In all parts below, the sampling rates of the C/D and D/C converters are **equal,** and the input to the Ideal C/D converter is a signal $x(t)$ whose spectrum is shown below, where the frequency f is in hertz.



(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$. $f_{\text{Nyquist}} =$ ☐

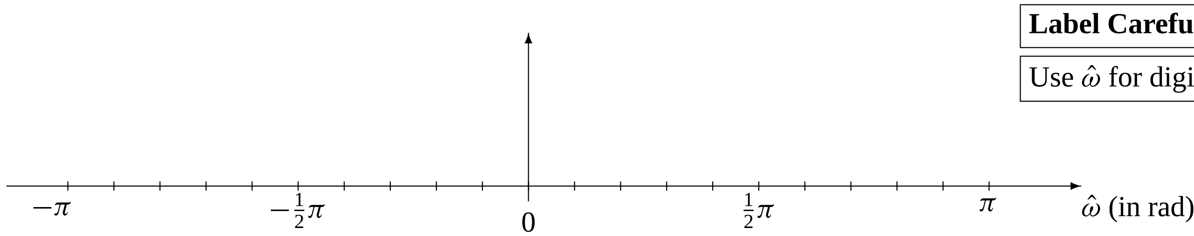
(b) If the sampling rate is $f_s = 200$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.*

$x[n] =$

Plot the spectrum of this signal over the range of frequencies $-\pi \le \hat{\omega} \le \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.

Label Carefully

Use $\hat{\omega}$ for digital freq.



(c) If the sampling rate is $f_s = 170$, list ***all*** frequencies that will be present in the spectrum of the output signal, $y(t)$.

PROBLEM SPR-02-Q.2.3:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

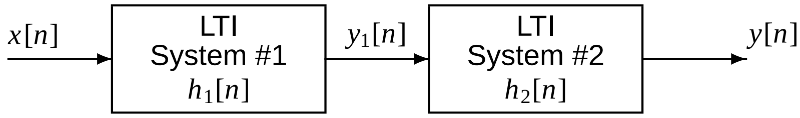


Figure 1: Cascade connection of two LTI systems.

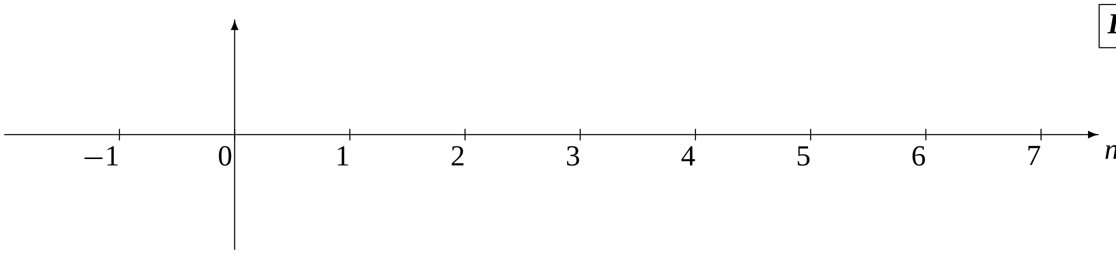
(a) Suppose that System #1 is an FIR filter described by the difference equation

$$y_1[n] = 2x[n-1] + 2x[n-2] + 2x[n-3] + 3x[n-4]$$

and System #2 is described by the impulse response

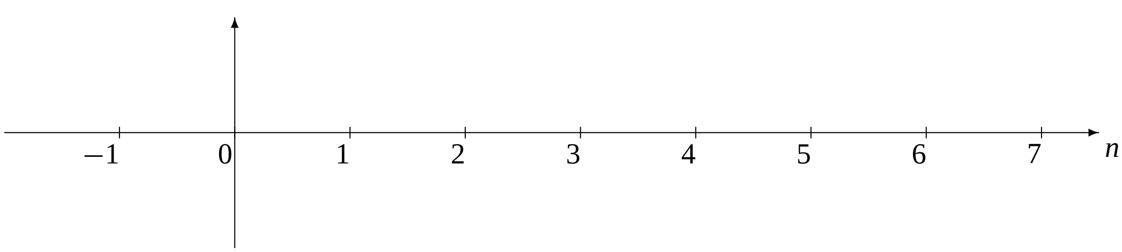
$$h_2[n] = \delta[n-2] - \delta[n-3]$$

Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a ***plot.*** ***Label Carefully***



(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) Although this has nothing to do with the previous parts, make a plot of the signal $s[n] = -u[n-5]$.



PROBLEM SPR-02-Q.2.4:

A discrete-time system is defined by the input/output relation (given as a difference equation)

$$y[n] = -Gx[n-1] + 3Gx[n-2] - Gx[n-3]$$

where G is a real-valued constant to be determined.

(a) Obtain an expression (in terms of G) for the frequency response of this system. Simplify into the "magnitude-phase" form: $H(e^{j\hat{\omega}}) = e^{j\psi(\hat{\omega})}M(\hat{\omega})$, where $M(\hat{\omega})$ and $\psi(\hat{\omega})$ are real.

(b) When the input is the signal, $x_1[n] = (-1)^n$, the output is $y_1[n] = 15(-1)^{n+1}$. Determine the value of G.

$G =$ ☐

(c) For the system above, set $G = 1$ and then determine the output $y_2[n]$ when the input is

$$x_2[n] = 5\cos(0.5\pi n - \pi/4)$$

Fill in the boxes below:

$y_2[n] =$ ☐ $\cos($ ☐ $n +$ ☐ $)$

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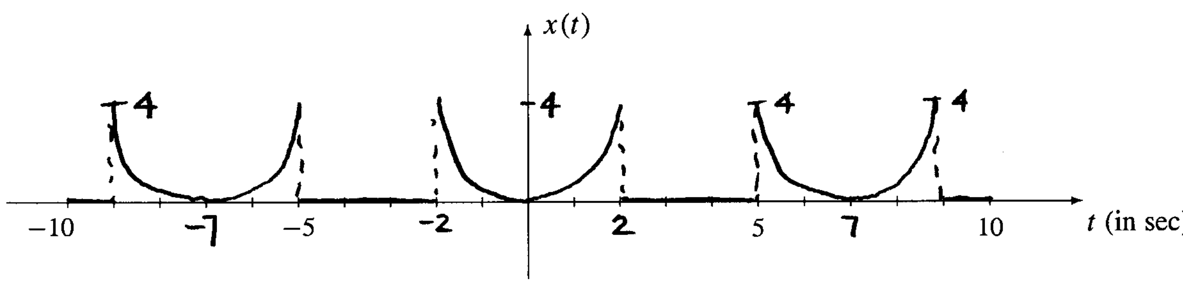
(a) In the expression for a_k in Eq. (1) above, the integral and its limits define the signal $x(t)$. Write an equation for $x(t)$ that is valid over one period.

$$x(t) = \begin{cases} t^2 & \text{for } -2 \le t \le 2 \\ 0 & \text{for } 2 < t \le 5 \end{cases}$$

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-10 \le t \le 10$ seconds. Label it carefully.



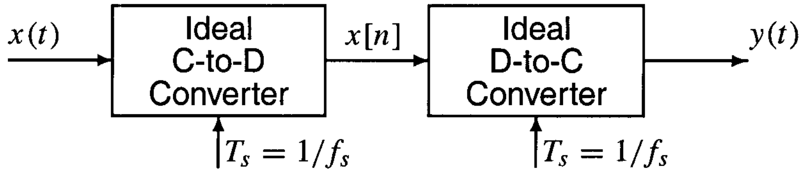
(c) Which value of k in Eq. (1) gives the DC (or average) value of $x(t)$? $k = 0$

(d) Determine the DC value of $x(t)$. Give your answer as a number.

$$DC = a_0 = \frac{1}{7}\int_{-2}^{2} t^2\,dt = \frac{1}{7}\frac{t^3}{3}\Big|_{-2}^{2}$$

$$= \frac{1}{21}\left(8-(-8)\right) = 16/21$$

PROBLEM SPR-02-Q.2.2:



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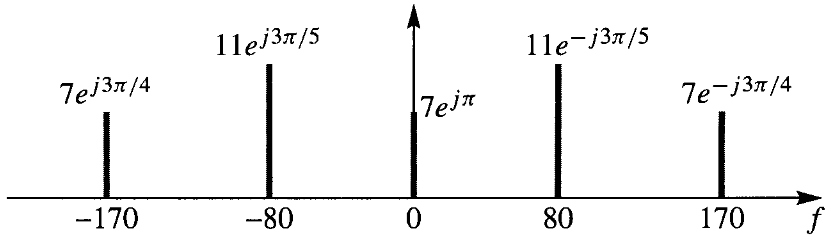


(a) Determine the Nyquist rate (in hertz) for sampling the signal $x(t)$. $f_{\text{Nyquist}} =$ **340** Hz

Sampling Thm $\Rightarrow f_s \geq 2 f_{MAX}$

(b) If the sampling rate is $f_s = 200$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.*

$$x[n] = -7 + 22\cos(0.8\pi n - 3\pi/5) + 14\cos(0.3\pi n + 3\pi/4)$$

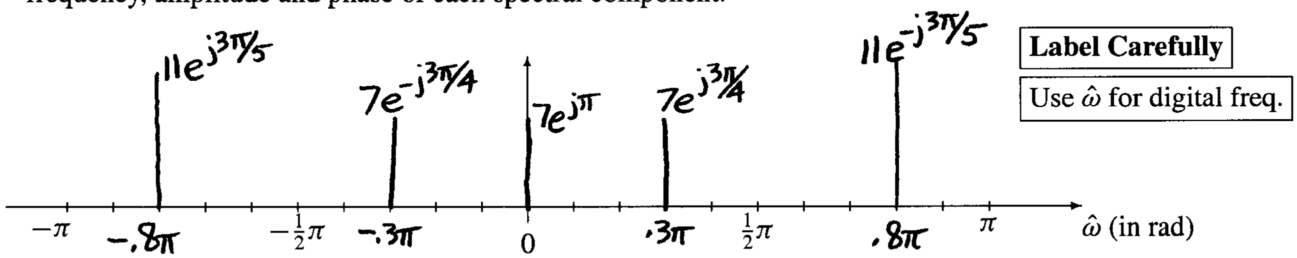
$\hat{\omega} = 2\pi f/f_s$

$f = 80 \rightarrow \hat{\omega} = 2\pi(80/200) = .8\pi$

$f = 170 \rightarrow \hat{\omega} = 2\pi(170/200) = 1.7\pi \leftarrow$ Need to subtract 2π

$\hat{\omega} = -0.3\pi$ Folding

Plot the spectrum of this signal over the range of frequencies $-\pi \leq \hat{\omega} \leq \pi$. Make sure to label the frequency, amplitude and phase of each spectral component.



(c) If the sampling rate is $f_s = 170$, list ***all*** frequencies that will be present in the spectrum of the output signal, $y(t)$. −80Hz, 0, +80Hz

$f = 80 \rightarrow \hat{\omega} = 2\pi\left(\frac{80}{170}\right)$ no aliasing $\rightarrow f = \frac{\hat{\omega}}{2\pi} f_s = 80$ Hz

$f = 170 \rightarrow \hat{\omega} = 2\pi\left(\frac{170}{170}\right)$ alias!

$= 2\pi \longrightarrow \hat{\omega} = 0 \rightarrow f_{out} = 0$ Hz.

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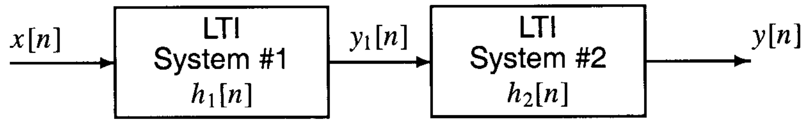

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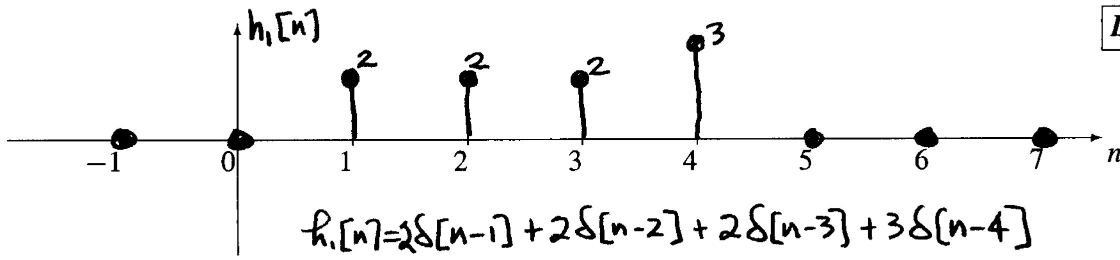


$h_1[n] = 2\delta[n-1] + 2\delta[n-2] + 2\delta[n-3] + 3\delta[n-4]$

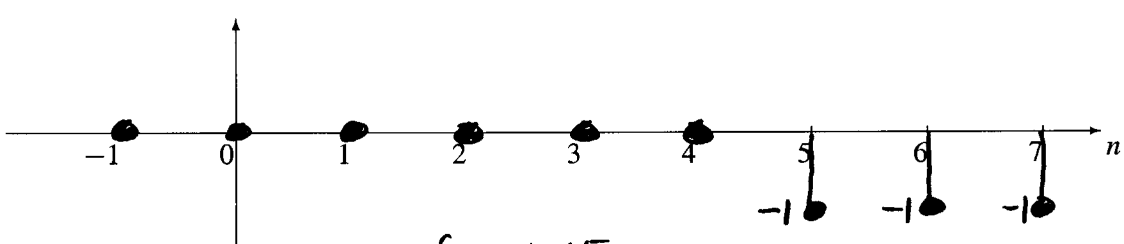
(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

Convolve: (n = 0)

```
0 2 2 2 3
0 0 1 -1
-----------------------
0 0 0 0 0
  0 0 0 0 0
    0 2 2 2 3
      0 -2 -2 -2 -3
-----------------------
n=0 → 0 0 0 2 0 0 1 -3
```

$h[n] = 2\delta[n-3] + \delta[n-6] - 3\delta[n-7]$

(c) Although this has nothing to do with the previous parts, make a plot of the signal $s[n] = -u[n-5]$.



$$-u[n-5] = \begin{cases} 0 & n < 5 \\ -1 & n \geq 5 \end{cases}$$

PROBLEM SPR-02-Q.2.4:
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$$H(e^{j\hat{\omega}}) = \sum_{k=1}^{3} b_k e^{-jk\hat{\omega}}$$

$$= -Ge^{-j\hat{\omega}} + 3Ge^{-j2\hat{\omega}} - Ge^{-j3\hat{\omega}}$$

$$= Ge^{-j2\hat{\omega}}(-e^{j\hat{\omega}} + 3 - e^{-j\hat{\omega}})$$

$$= e^{-j2\hat{\omega}} \cdot G(3 - 2\cos\hat{\omega})$$

(b) When the input is the signal, $x_1[n] = (-1)^n$, the output is $y_1[n] = 15(-1)^{n+1}$. Determine the value of G.

$G = \boxed{-3}$

$$x_1[n] = e^{j\pi n}$$

$$y_1[n] = H(e^{j\pi}) \cdot e^{j\pi n} = 15e^{j\pi(n+1)} = -15e^{j\pi n}$$

$$H(e^{j\pi}) = e^{-j2\pi} \cdot G(3 - 2\cos\pi) = 5G$$

$$5G = -15 \implies G = -3$$

(c) For the system above, set $G = 1$ and then determine the output $y_2[n]$ when the input is

$$x_2[n] = 5\cos(0.5\pi n - \pi/4)$$

Fill in the boxes below:

$y_2[n] = \boxed{15}\cos(\boxed{0.5\pi}\, n + \boxed{3\pi/4})$

Need $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/2$

$$H(e^{j\pi/2}) = e^{-j2\pi/2}(3 - 2\cos(\pi/2)) = 3e^{-j\pi}$$

$\Rightarrow$ Multiply amplitude by 3; add $-\pi$ to the phase

$-\pi - \pi/4 = -5\pi/4$ or $+3\pi/4$