

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING
QUIZ #3

DATE: Apr. 11, 2003

COURSE: ECE-2025

NAME:

LAST,

FIRST

GT #: _____

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

	L01:Tues-9:30am (McLaughlin)		L02:Thur-9:30am (Barry)
	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
	L05:Tues-1:30pm (Li)		
L11:M-3pm (McClellan)	L07:Tues-3pm (Li)	L12:W-3pm (Hayes)	L08:Thur-3pm (Williams)
	L09:Tues-4:30pm (Zhou)	L14:W-4:30pm (Hayes)	
	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- Justify your reasoning CLEARLY to receive partial credit.
Explanations are also required to receive full credit for any answer.
- You must write your answer **on the answer sheet** or in the space provided on the exam paper itself.
Only these answers will be graded. Circle your answers, or write them in the boxes provided.
If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	

PROBLEM Spring-02-Q.3.1:(Circle exactly one answer⁵ for each system, S_i)

S_1 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_2 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_3 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_4 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_5 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_6 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_7 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_8 :	#1	#2	#3	#4	#5	#6	#7	#8	#9

PROBLEM Spring-02-Q.3.2:(Circle exactly one answer for each system, S_i)

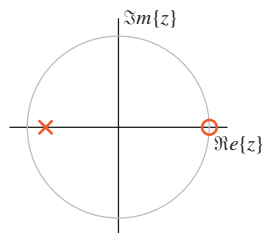
S_1 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_2 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_3 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_4 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_5 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_6 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_7 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_8 :	(A)	(B)	(C)	(D)	(E)	(F)	None

PROBLEM Spring-02-Q.3.4:(Circle exactly one answer for each part)

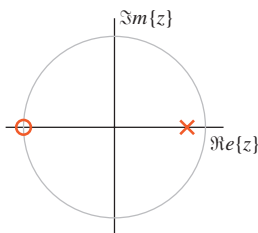
(a)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(d)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(e)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(h)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

⁵If more than one answer is circled, the response will be considered wrong and will receive no credit.

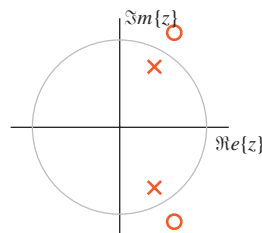
PROBLEM Spring-02-Q.3.1:



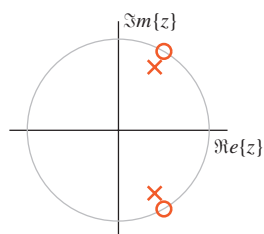
Pole-Zero Plot #1



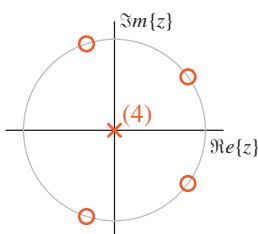
Pole-Zero Plot #2



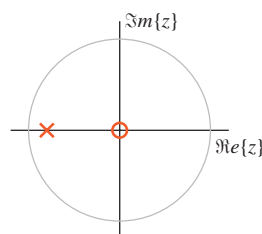
Pole-Zero Plot #3



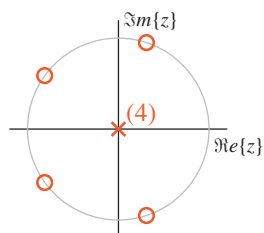
Pole-Zero Plot #4



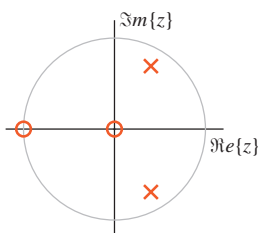
Pole-Zero Plot #5



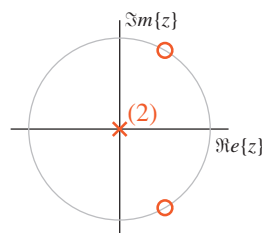
Pole-Zero Plot #6



Pole-Zero Plot #7



Pole-Zero Plot #8



Pole-Zero Plot #9

For each of systems below⁶ determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

Note: the unit circle is shown for reference.

$$\mathcal{S}_1 : y[n] = 0.8y[n-1] - 0.64y[n-2] + 1.8x[n] + 1.8x[n-1]$$

$$\mathcal{S}_2 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$\mathcal{S}_3 : y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4]$$

$$\mathcal{S}_4 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$\mathcal{S}_5 : y[n] = -0.8y[n-1] + 2x[n]$$

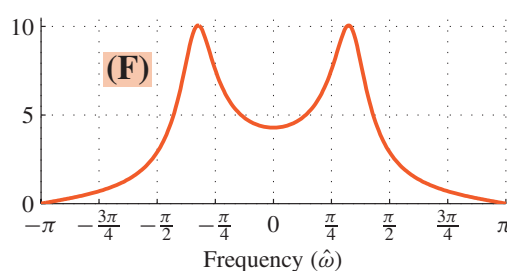
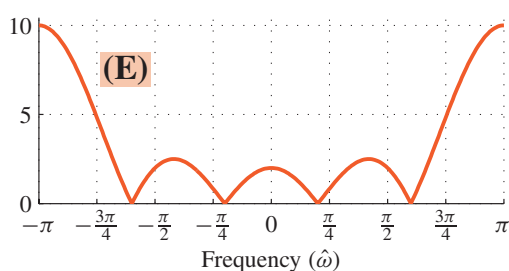
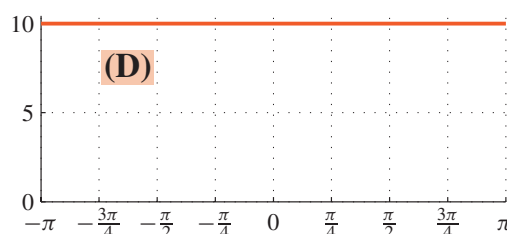
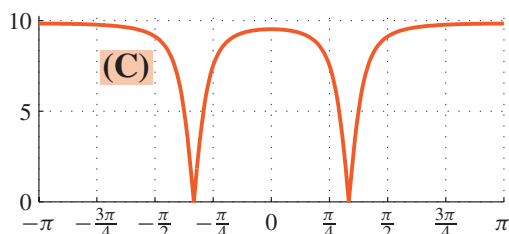
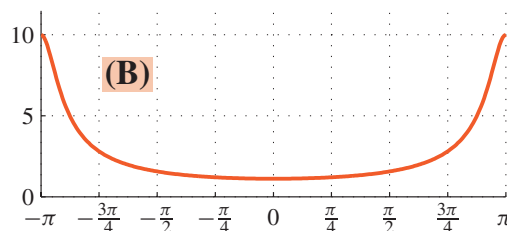
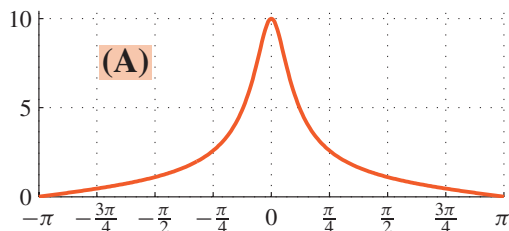
$$\mathcal{S}_6 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$\mathcal{S}_7 : y[n] = 0.8y[n-1] + x[n] + x[n-1]$$

$$\mathcal{S}_8 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}}$$

⁶These same systems are also used in the next problem.

PROBLEM Spring-02-Q.3.2:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**

Note: the frequency axis is $\hat{\omega}$.

$$S_1 : y[n] = 0.8y[n - 1] - 0.64y[n - 2] + 1.8x[n] + 1.8x[n - 1]$$

$$S_2 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_3 : y[n] = 2x[n] + 2x[n - 1] + 2x[n - 2] + 2x[n - 3] + 2x[n - 4]$$

$$S_4 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_5 : y[n] = -0.8y[n - 1] + 2x[n]$$

$$S_6 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$S_7 : y[n] = 0.8y[n - 1] + x[n] + x[n - 1]$$

$$S_8 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}}$$

PROBLEM Spring-02-Q.3.3:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

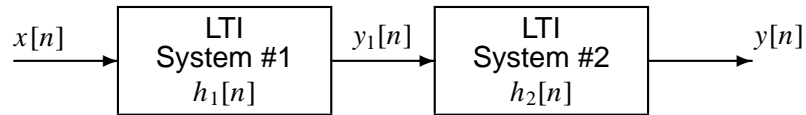


Figure 1: Cascade connection of two discrete-time LTI systems.

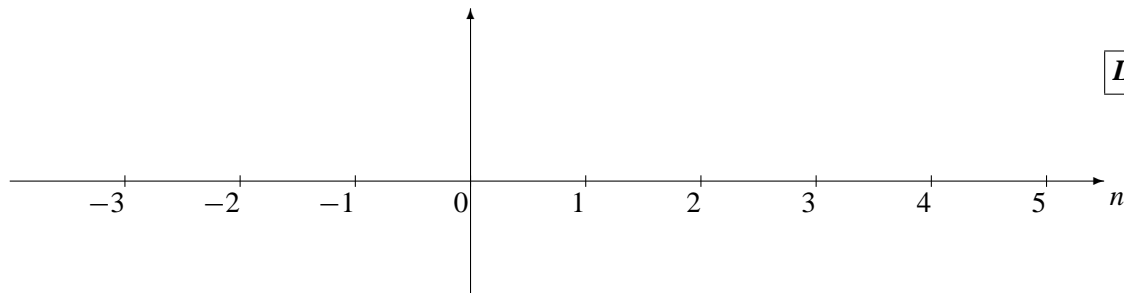
- (a) Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{1 + 2z^{-1}}{1 + 0.5z^{-1}}$$

and System #2 is described by the impulse response

$$h_2[n] = 2\delta[n - 2] - \delta[n - 3] - \delta[n - 4]$$

Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a *plot*.



- (b) Determine the output, $y[n]$, of the overall cascade system when the input, $x[n]$, is a *unit-step* signal.
Hint: The output, $y[n]$, will be finite-length.

PROBLEM Spring-02-Q.3.4:

For each of the following expressions, select the correct match from the second list below.

Write your answers on the answer sheet provided. (The operator * denotes convolution.)

(a) $e^{-t}u(t) * u(t - 3)$

(b) $\delta(t - 1) * \delta(t - 2)$

(c) $e^{-t}u(t)\delta(t - 3)$

(d) $\frac{d}{dt} \{e^{-t}u(t - 3)\}$

(e) $u(3)$

(f) $u(t - 1) * u(t - 2)$

(g) $e^{-t}u(t) * \delta(t - 3)$

(h) $\int_{-\infty}^0 \delta(t - 3)dt$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $-e^{-t}u(t - 3)$

[2] $u(t - 3)$

[3] $-e^{-t}u(t - 3) + e^{-3}\delta(t - 3)$

[4] $(t - 3)u(t - 3)$

[5] $(1 - e^{-t+3})u(t - 3)$

[6] $e^{-(t-3)}u(t - 3)$

[7] $e^{-3}\delta(t - 3)$

[8] 0

[9] $\delta(t - 3)$

[10] 1

[11] e^{-3}

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NAME: Answer Key
LAST, FIRST

GT #: Version - 3

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	L03:Tues-Noon (McLaughlin)		L04:Thur-Noon (Barry)
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	L10:Tues-6pm (Zhou)		RPK:Thur-Late (Tugcu)

-
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PROBLEM Spring-02-Q.3.1:(Circle exactly one answer⁵ for each system, S_i)

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S_3 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_4 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_5 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_6 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_7 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_8 :	#1	#2	#3	#4	#5	#6	#7	#8	#9

PROBLEM Spring-02-Q.3.2:(Circle exactly one answer for each system, S_i)

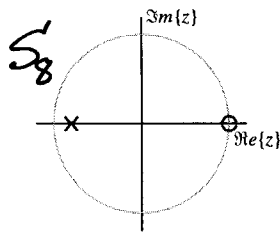
S_1 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_2 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_3 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_4 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_5 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_6 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_7 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_8 :	(A)	(B)	(C)	(D)	(E)	(F)	None

PROBLEM Spring-02-Q.3.4:(Circle exactly one answer for each part)

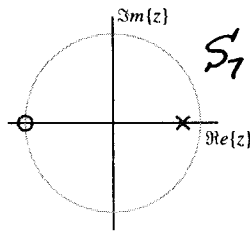
(a)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
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(e)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(h)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

⁵If more than one answer is circled, the response will be considered wrong and will receive no credit.

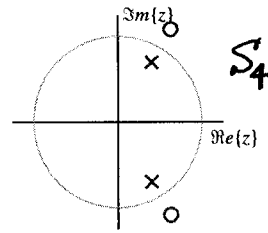
PROBLEM Spring-02-Q.3.1:



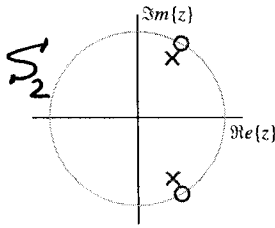
Pole-Zero Plot #1



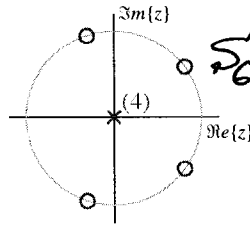
Pole-Zero Plot #2



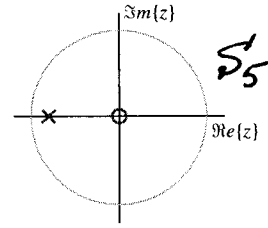
Pole-Zero Plot #3



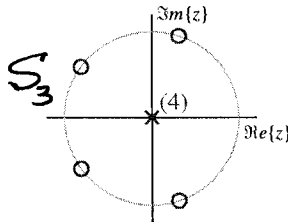
Pole-Zero Plot #4



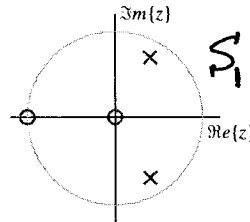
Pole-Zero Plot #5



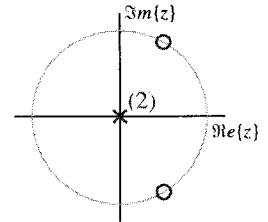
Pole-Zero Plot #6



Pole-Zero Plot #7



Pole-Zero Plot #8



Pole-Zero Plot #9

For each of systems below⁶ determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

Note: the unit circle is shown for reference.

$$S_1 : y[n] = 0.8y[n-1] - 0.64y[n-2] + 1.8x[n] + 1.8x[n-1] \quad \frac{1.8(1+z^{-1})}{1-0.8z^{-1}+0.64z^{-2}}$$

$$S_2 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}} \quad \text{Notch, zeros on U.C.}$$

$$S_3 : y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4] \quad \text{FIR, 4 zeros}$$

$$S_4 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}} \quad \text{All-pass, zeros outside U.C.}$$

$$S_5 : y[n] = -0.8y[n-1] + 2x[n] \quad \frac{2}{1+0.8z^{-1}}$$

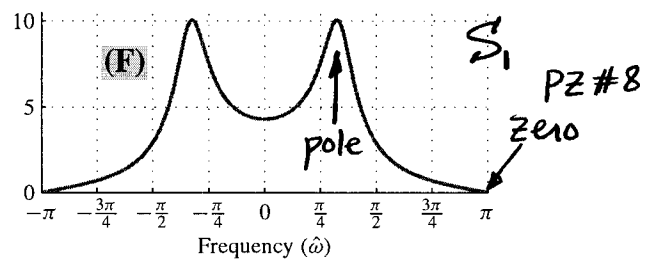
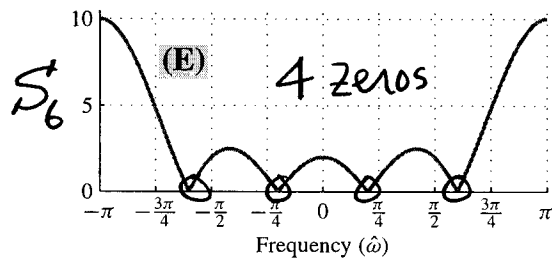
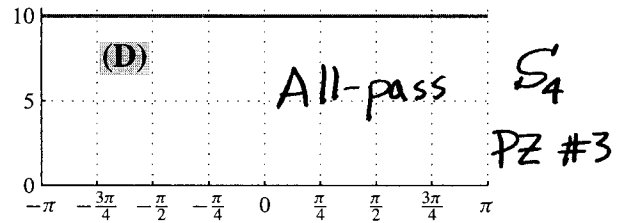
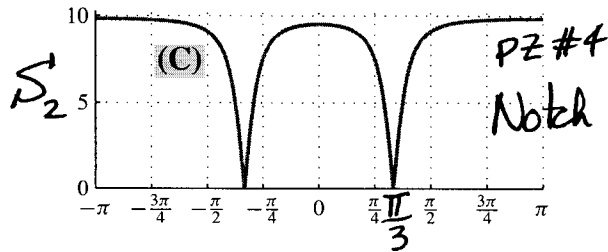
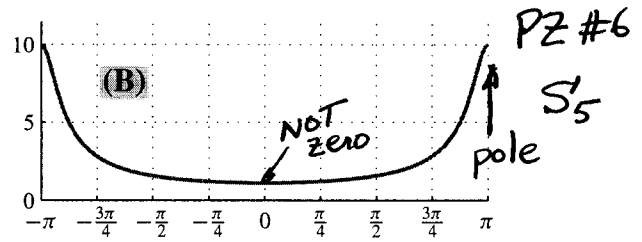
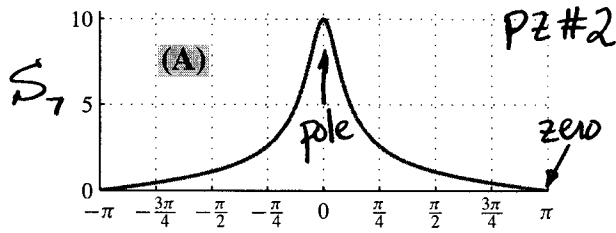
$$S_6 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}) \quad \text{FIR 4 zeros}$$

$$S_7 : y[n] = 0.8y[n-1] + x[n] + x[n-1] \quad \frac{1+z^{-1}}{1-0.8z^{-1}}$$

$$S_8 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}} \quad \text{zero at } z=1, \text{ pole at } z=-0.8$$

⁶These same systems are also used in the next problem.

PROBLEM Spring-02-Q.3.2:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**

Note: the frequency axis is $\hat{\omega}$.

$$S_1 : y[n] = 0.8y[n-1] - 0.64y[n-2] + 1.8x[n] + 1.8x[n-1]$$

$$S_2 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_3 : y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4]$$

$$S_4 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_5 : y[n] = -0.8y[n-1] + 2x[n] \quad \frac{2}{1 + 0.8z^{-1}}$$

$$S_6 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$S_7 : y[n] = 0.8y[n-1] + x[n] + x[n-1] \quad \frac{1+z^{-1}}{1-0.8z^{-1}}$$

$$S_8 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}}$$

PROBLEM Spring-02-Q.3.3:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

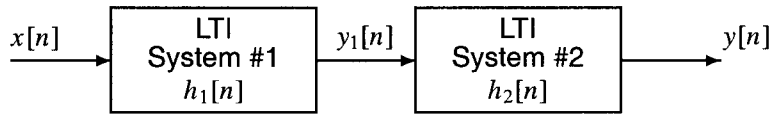


Figure 1: Cascade connection of two discrete-time LTI systems.

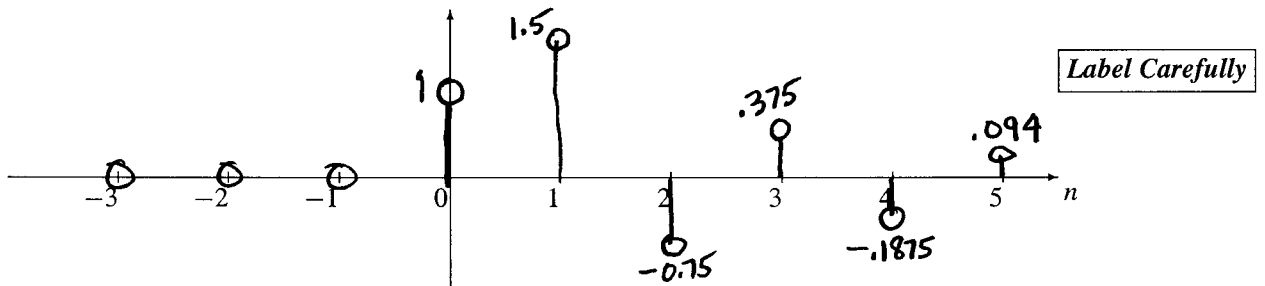
- (a) Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{1 + 2z^{-1}}{1 + 0.5z^{-1}}$$

and System #2 is described by the impulse response

$$h_2[n] = 2\delta[n - 2] - \delta[n - 3] - \delta[n - 4]$$

Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a *plot*.



$$H_1(z) = \frac{1}{1 + 0.5z^{-1}} + \frac{2z^{-1}}{1 + 0.5z^{-1}} \Rightarrow h_1[n] = (-0.5)^n u[n] + 2(-0.5)^{n-1} u[n-1]$$

$$h_1[0] = 1 \quad h_1[2] = (-0.5)^2 + 2(-0.5) = 0.25 - 1 = -0.75$$

$$h_1[1] = (-0.5) + 2 = 1.5 \quad h_1[3] = (-0.5)^3 + 2(-0.5)^2 = -0.125 + 0.5 = 0.375$$

- (b) Determine the output, $y[n]$, of the overall cascade system when the input, $x[n]$, is a *unit-step* signal.

Hint: The output, $y[n]$, will be finite-length.

$$Y(z) = \frac{1}{1 - z^{-1}} \left(\frac{1 + 2z^{-1}}{1 + 0.5z^{-1}} \right) (2z^{-2} - z^{-3} - z^{-4})$$

$$= \frac{(1 + 2z^{-1})(2z^{-2})(1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2})}{1 - 0.5z^{-1} - 0.5z^{-2}} = (1 + 2z^{-1})(2z^{-2})$$

$$= 2z^{-2} + 4z^{-3}$$

$$y[n] = 2\delta[n-2] + 4\delta[n-3]$$

PROBLEM Spring-02-Q.3.4:

For each of the following expressions, select the correct match from the second list below.

Write your answers on the answer sheet provided. (The operator * denotes convolution.)

$$(a) e^{-t}u(t) * u(t-3) = (1 - e^{-(t-3)})u(t-3)$$

$$(b) \delta(t-1) * \delta(t-2) = \delta(t-3)$$

$$(c) e^{-t}u(t)\delta(t-3) \text{ (eval)} \quad e^{-3}\delta(t-3)$$

$$(d) \frac{d}{dt} \{e^{-t}u(t-3)\} = -e^{-t}u(t-3) + e^{-t}\delta(t-3)$$

$$(e) u(3) = 1$$

$$(f) u(t-1) * u(t-2) = (t-3)u(t-3)$$

$$(g) e^{-t}u(t) * \delta(t-3) = e^{-(t-3)}u(t-3)$$

$$(h) \int_{-\infty}^0 \delta(t-3)dt = 0$$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $-e^{-t}u(t-3)$

[2] $u(t-3)$

[3] $-e^{-t}u(t-3) + e^{-3}\delta(t-3)$ (d)

[4] $(t-3)u(t-3)$ (f)

[5] $(1 - e^{-t+3})u(t-3)$ (a)

[6] $e^{-(t-3)}u(t-3)$ (g)

[7] $e^{-3}\delta(t-3)$ (c)

[8] 0 (h)

[9] $\delta(t-3)$ (b)

[10] 1 (e)

[11] e^{-3}