

PROBLEM spr-02-Q.3.1:(Circle exactly one answer⁷ for each system, S_i)

S_1 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_2 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_3 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_4 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_5 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_6 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_7 :	#1	#2	#3	#4	#5	#6	#7	#8	#9
S_8 :	#1	#2	#3	#4	#5	#6	#7	#8	#9

PROBLEM spr-02-Q.3.2:(Circle exactly one answer for each system, S_i)

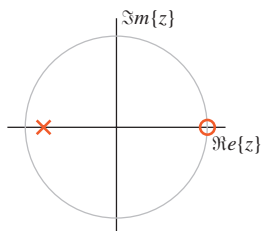
S_1 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_2 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_3 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_4 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_5 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_6 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_7 :	(A)	(B)	(C)	(D)	(E)	(F)	None
S_8 :	(A)	(B)	(C)	(D)	(E)	(F)	None

PROBLEM spr-02-Q.3.4:(Circle exactly one answer for each part)

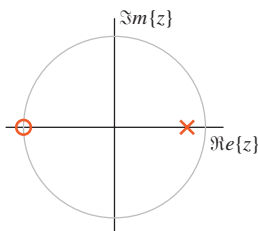
(a)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(b)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(c)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(d)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(e)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(f)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(g)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
(h)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

⁷If more than one answer is circled, the response will be considered wrong and will receive no credit.

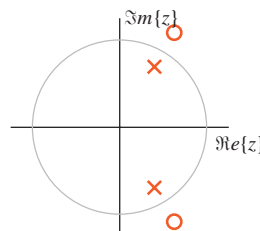
PROBLEM spr-02-Q.3.1:



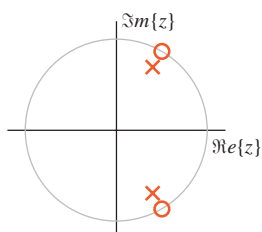
Pole-Zero Plot #1



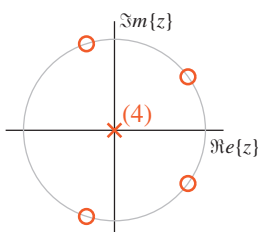
Pole-Zero Plot #2



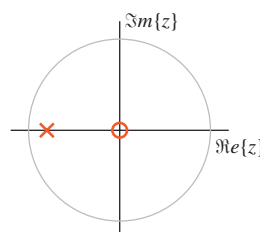
Pole-Zero Plot #3



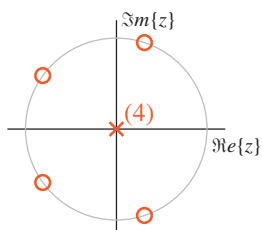
Pole-Zero Plot #4



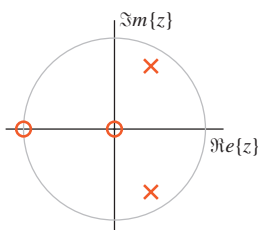
Pole-Zero Plot #5



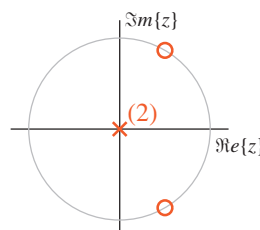
Pole-Zero Plot #6



Pole-Zero Plot #7



Pole-Zero Plot #8



Pole-Zero Plot #9

For each of systems below⁸ determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

Note: the unit circle is shown for reference.

$$S_1 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}}$$

$$S_2 : y[n] = 0.8y[n - 1] + x[n] + x[n - 1]$$

$$S_3 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$S_4 : y[n] = -0.8y[n - 1] + 2x[n]$$

$$S_5 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

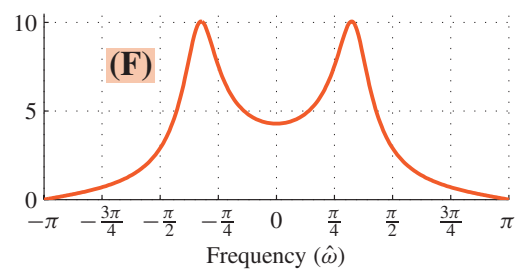
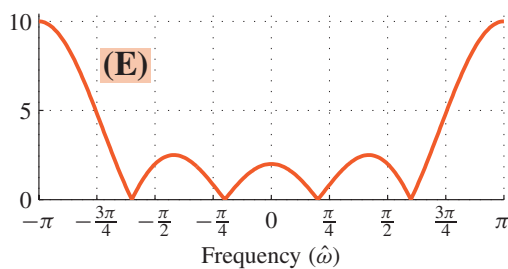
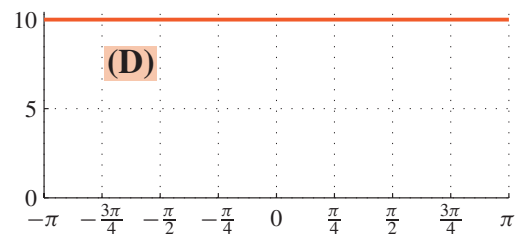
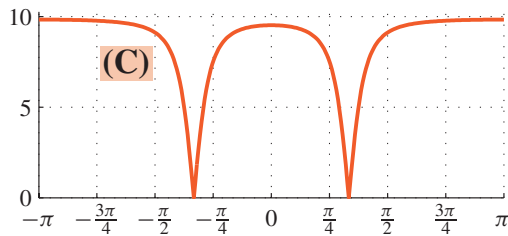
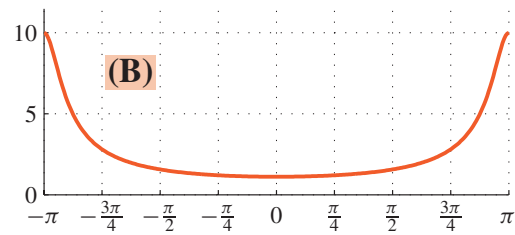
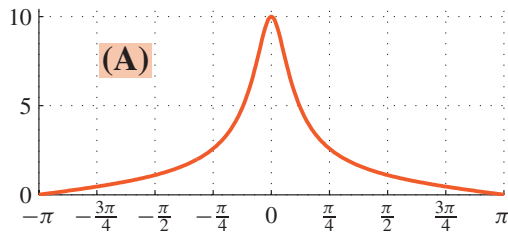
$$S_6 : y[n] = 2x[n] + 2x[n - 1] + 2x[n - 2] + 2x[n - 3] + 2x[n - 4]$$

$$S_7 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_8 : y[n] = 0.8y[n - 1] - 0.64y[n - 2] + 1.8x[n] + 1.8x[n - 1]$$

⁸These same systems are also used in the next problem.

PROBLEM spr-02-Q.3.2:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**

Note: the frequency axis is $\hat{\omega}$.

$$S_1 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}}$$

$$S_2 : y[n] = 0.8y[n - 1] + x[n] + x[n - 1]$$

$$S_3 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$S_4 : y[n] = -0.8y[n - 1] + 2x[n]$$

$$S_5 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_6 : y[n] = 2x[n] + 2x[n - 1] + 2x[n - 2] + 2x[n - 3] + 2x[n - 4]$$

$$S_7 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_8 : y[n] = 0.8y[n - 1] - 0.64y[n - 2] + 1.8x[n] + 1.8x[n - 1]$$

PROBLEM spr-02-Q.3.3:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

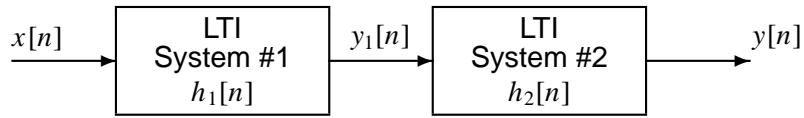


Figure 1: Cascade connection of two discrete-time LTI systems.

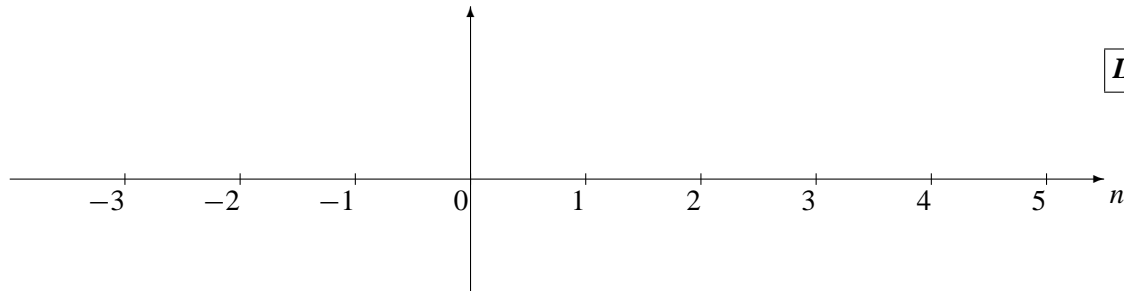
- (a) Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{5 - 3z^{-1}}{1 - 0.6z^{-1}}$$

and System #2 is described by the impulse response

$$h_2[n] = -5\delta[n - 2] + 8\delta[n - 3] - 3\delta[n - 4]$$

Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a *plot*.



- (b) Determine the output, $y[n]$, of the overall cascade system when the input, $x[n]$, is a *unit-step* signal.
Hint: The output, $y[n]$, will be finite-length.

PROBLEM spr-02-Q.3.4:

For each of the following expressions, select the correct match from the second list below.

Write your answers on the answer sheet provided. (The operator * denotes convolution.)

(a) $u(t - 1) * u(t - 2)$

(b) $e^{-t}u(t) * \delta(t - 3)$

(c) $\int_{-\infty}^0 \delta(t - 3)dt$

(d) $u(3)$

(e) $\frac{d}{dt} \{e^{-t}u(t - 3)\}$

(f) $e^{-t}u(t)\delta(t - 3)$

(g) $\delta(t - 1) * \delta(t - 2)$

(h) $e^{-t}u(t) * u(t - 3)$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $\delta(t - 3)$

[2] 1

[3] e^{-3}

[4] $-e^{-t}u(t - 3)$

[5] $u(t - 3)$

[6] $-e^{-t}u(t - 3) + e^{-3}\delta(t - 3)$

[7] $(t - 3)u(t - 3)$

[8] $(1 - e^{-t+3})u(t - 3)$

[9] $e^{-(t-3)}u(t - 3)$

[10] $e^{-3}\delta(t - 3)$

[11] 0

PROBLEM spr-02-Q.3.1:(Circle exactly one answer⁷ for each system, S_i)

S_1 :	<input checked="" type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_2 :	<input type="radio"/> #1	<input checked="" type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_3 :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input checked="" type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_4 :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input checked="" type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_5 :	<input type="radio"/> #1	<input type="radio"/> #2	<input checked="" type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_6 :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input checked="" type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_7 :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input checked="" type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input type="radio"/> #8	<input type="radio"/> #9
S_8 :	<input type="radio"/> #1	<input type="radio"/> #2	<input type="radio"/> #3	<input type="radio"/> #4	<input type="radio"/> #5	<input type="radio"/> #6	<input type="radio"/> #7	<input checked="" type="radio"/> #8	<input type="radio"/> #9

PROBLEM spr-02-Q.3.2:(Circle exactly one answer for each system, S_i)

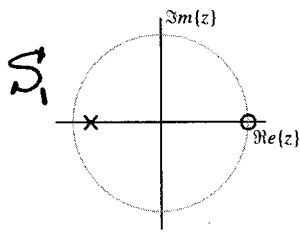
S_1 :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input checked="" type="radio"/> None
S_2 :	<input checked="" type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
S_3 :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input checked="" type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
S_4 :	<input type="radio"/> (A)	<input checked="" type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
S_5 :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input checked="" type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
S_6 :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input checked="" type="radio"/> None
S_7 :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input checked="" type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input type="radio"/> (F)	<input type="radio"/> None
S_8 :	<input type="radio"/> (A)	<input type="radio"/> (B)	<input type="radio"/> (C)	<input type="radio"/> (D)	<input type="radio"/> (E)	<input checked="" type="radio"/> (F)	<input type="radio"/> None

PROBLEM spr-02-Q.3.4:(Circle exactly one answer for each part)

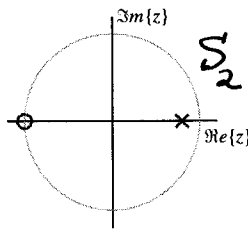
(a)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input checked="" type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> [11]
(b)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input checked="" type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> [11]
(c)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input checked="" type="radio"/> [11]
(d)	<input type="radio"/> [1]	<input checked="" type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> [11]
(e)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input checked="" type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> [11]
(f)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input checked="" type="radio"/> [10]	<input type="radio"/> [11]
(g)	<input checked="" type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> [11]
(h)	<input type="radio"/> [1]	<input type="radio"/> [2]	<input type="radio"/> [3]	<input type="radio"/> [4]	<input type="radio"/> [5]	<input type="radio"/> [6]	<input type="radio"/> [7]	<input checked="" type="radio"/> [8]	<input type="radio"/> [9]	<input type="radio"/> [10]	<input type="radio"/> [11]

⁷If more than one answer is circled, the response will be considered wrong and will receive no credit.

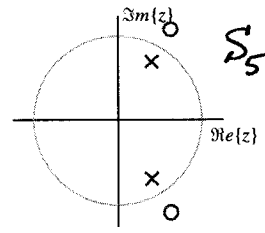
PROBLEM spr-02-Q.3.1:



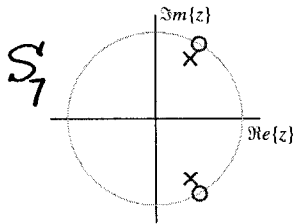
Pole-Zero Plot #1



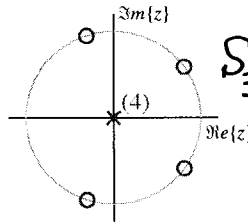
Pole-Zero Plot #2



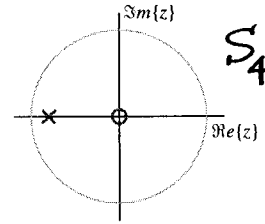
Pole-Zero Plot #3



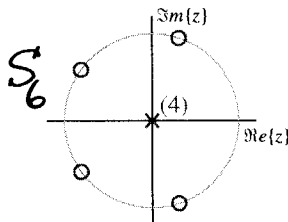
Pole-Zero Plot #4



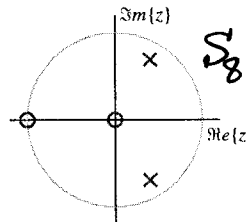
Pole-Zero Plot #5



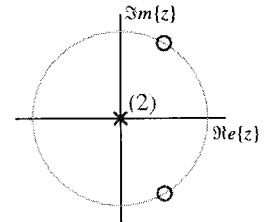
Pole-Zero Plot #6



Pole-Zero Plot #7



Pole-Zero Plot #8



Pole-Zero Plot #9

For each of systems below⁸ determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. **Mark your answers on the answer sheet provided.**

Note: the unit circle is shown for reference.

$$S_1: H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}} \quad \begin{array}{l} \text{zero at } z=1 \\ \text{pole at } z=-0.8 \end{array}$$

$$S_2: y[n] = 0.8y[n-1] + x[n] + x[n-1] \quad \frac{1+z^{-1}}{1-0.8z^{-1}}$$

$$S_3: H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}) \quad \text{FIR, 4 zeros}$$

$$S_4: y[n] = -0.8y[n-1] + 2x[n] \quad \frac{2}{1+0.8z^{-1}}$$

$$S_5: H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}} \quad \leftarrow \text{ALL-pass, zeros outside U.C.}$$

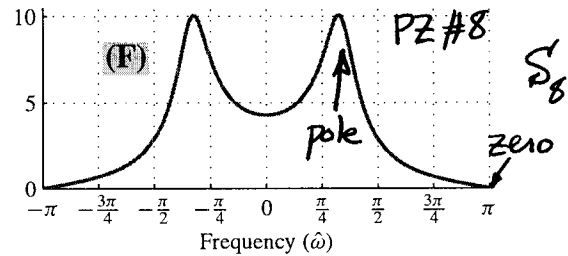
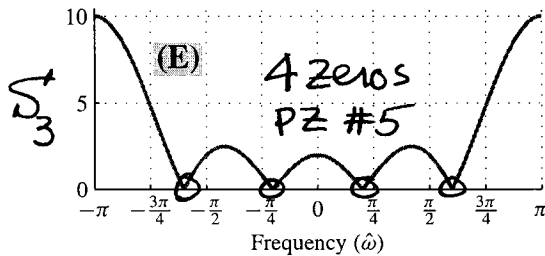
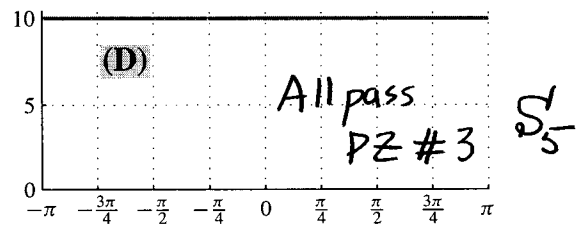
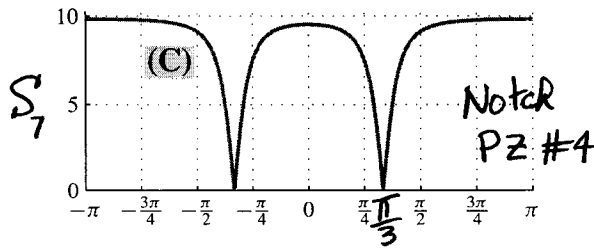
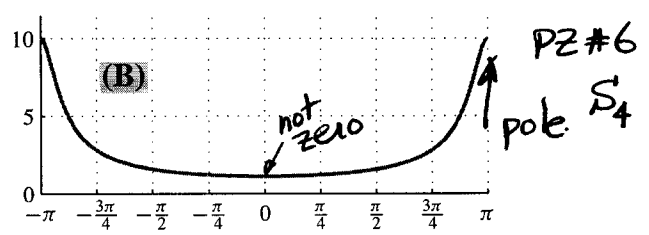
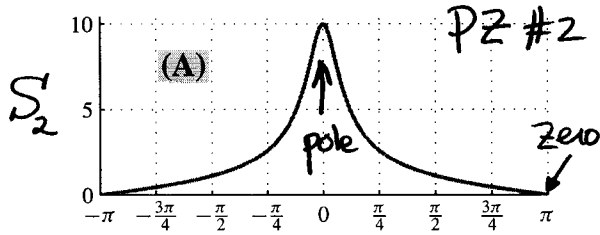
$$S_6: y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4] \quad \text{FIR, 4th order}$$

$$S_7: H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}} \quad \text{Notch, zeros on U.C.}$$

$$S_8: y[n] = 0.8y[n-1] - 0.64y[n-2] + 1.8x[n] + 1.8x[n-1] \\ \frac{1.8(1+z^{-1})}{1-0.8z^{-1}+0.64z^{-2}}$$

⁸These same systems are also used in the next problem.

PROBLEM spr-02-Q.3.2:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. **Mark your answers on the answer sheet provided.**
 Note: the frequency axis is $\hat{\omega}$.

$$S_1 : H(z) = \frac{1 - z^{-1}}{1 + 0.8z^{-1}}$$

$$S_2 : y[n] = 0.8y[n - 1] + x[n] + x[n - 1]$$

$$S_3 : H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$S_4 : y[n] = -0.8y[n - 1] + 2x[n]$$

$$S_5 : H(z) = \frac{6.4 - 8z^{-1} + 10z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_6 : y[n] = 2x[n] + 2x[n - 1] + 2x[n - 2] + 2x[n - 3] + 2x[n - 4]$$

$$S_7 : H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$S_8 : y[n] = 0.8y[n - 1] - 0.64y[n - 2] + 1.8x[n] + 1.8x[n - 1]$$

PROBLEM spr-02-Q.3.3:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

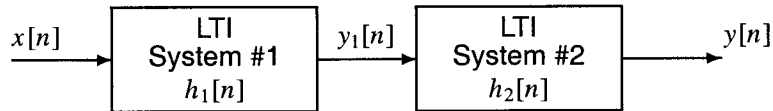


Figure 1: Cascade connection of two discrete-time LTI systems.

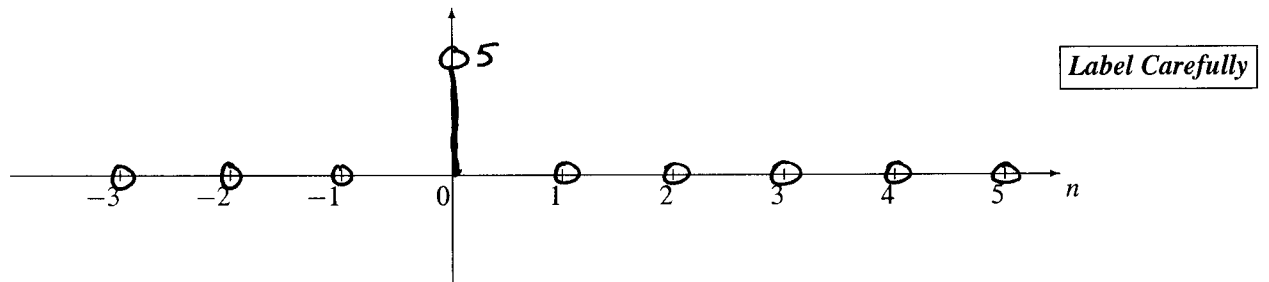
- (a) Suppose that System #1 is an IIR filter described by the system function:

$$H_1(z) = \frac{5 - 3z^{-1}}{1 - 0.6z^{-1}} = 5 \Rightarrow \text{[Handwritten scribbles]} \Rightarrow h_1[n] = 5\delta[n]$$

and System #2 is described by the impulse response

$$h_2[n] = -5\delta[n - 2] + 8\delta[n - 3] - 3\delta[n - 4]$$

Determine the impulse response sequence, $h_1[n]$, of the first system. Give your answer as a *plot*.



$$H_1(z) = \frac{5}{1 - 0.6z^{-1}} - \frac{3z^{-1}}{1 - 0.6z^{-1}} \Rightarrow h_1[n] = 5(0.6)^n u[n] - 3(0.6)^{n-1} u[n-1]$$

$$h_1[0] = 5$$

$$h_1[2] = 5(0.6)^2 - 3(0.6) = 0$$

$$h_1[1] = 5(0.6) - 3 = 0$$

- (b) Determine the output, $y[n]$, of the overall cascade system when the input, $x[n]$, is a *unit-step* signal.

Hint: The output, $y[n]$, will be finite-length.

$$Y(z) = \frac{1}{1-z^{-1}} (5) (-5z^{-2} + 8z^{-3} - 3z^{-4})$$

$$= \frac{-5z^{-2}(5 - 8z^{-1} + 3z^{-2})}{1-z^{-1}}$$

$$= \frac{-5z^{-2}(1-z^{-1})(5-3z^{-1})}{(1-z^{-1})} = -25z^{-2} + 15z^{-3}$$

$$y[n] = -25\delta[n-2] + 15\delta[n-3]$$

PROBLEM spr-02-Q.3.4:

For each of the following expressions, select the correct match from the second list below.

Write your answers on the answer sheet provided. (The operator * denotes convolution.)

(a) $u(t-1) * u(t-2) = (t-3)u(t-3)$

(b) $e^{-t}u(t) * \delta(t-3) = e^{-(t-3)}u(t-3)$

(c) $\int_{-\infty}^0 \delta(t-3)dt = 0$

(d) $u(3) = 1$

(e) $\frac{d}{dt} \{e^{-t}u(t-3)\} = -e^{-t}u(t-3) + e^{-t}\delta(t-3)$
↑ eval at t=3

(f) $e^{-t}u(t)\delta(t-3)$ (eval) $e^{-3}\delta(t-3)$

(g) $\delta(t-1) * \delta(t-2) = \delta(t-3)$

(h) $e^{-t}u(t) * u(t-3) = (1 - e^{-(t-3)})u(t-3)$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $\delta(t-3)$ (g)

[2] 1 (d)

[3] e^{-3}

[4] $-e^{-t}u(t-3)$

[5] $u(t-3)$

[6] $-e^{-t}u(t-3) + e^{-3}\delta(t-3)$ (e)

[7] $(t-3)u(t-3)$ (a)

[8] $(1 - e^{-t+3})u(t-3)$ (h)

[9] $e^{-(t-3)}u(t-3)$ (b)

[10] $e^{-3}\delta(t-3)$ (f)

[11] 0 (c)