GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 20-June-03 COURSE: ECE 2025

NAME: Key
LAST, FIRST

STUDENT #: ____________

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L01:Tues-10:00am (D. Taylor)

L02:Tues-12:00am (T. Michaels)

L03:Tues-2:00pm (D. Taylor)

L04:Tues-4:00pm (D. Taylor)

L06:Mon-4:00pm (T. Michaels)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- Unless stated otherwise, JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive full credit for any answer. (The exception on this exam is question #1, which is intended to test your intuition.)
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem Q2.1:

(a) Circle "Yes" or "No" to indicate whether each given system is linear, time-invariant, and/or causal. **You do not need to give any reasons. Don't be afraid to use your intuition! Do not spend too much time on this question; try not to get bogged down here. If you don't know, guess!**

(i) $y[n] = 3x[n^2]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(ii) $y[n] = 1492x[n-3] + \sqrt{3x[n-5]}$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(iii) $y[n] = e^5 x[n-3] + \pi x[n-5]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(iv) $y[n] = \exp(-5x[n-3])$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(v) $y[n] = e^{5(n-3)} x[n-3] + \pi x[n-5]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(vi) $y[n] = 1492x[n-3] + \sqrt{3}\, x[n-5]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(vii) $y[n] = 42x[n+3] - 13x[n+5]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(viii) $y[n] = 5x[-n^2]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(ix) $y[n] = 6.02 \times 10^{23} x[n-3] + \pi x[n-5]$
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(x) $y[n] = 6.02 \times 10^{23} x[n-3] \times \pi x[n-5]$ (Be sure to notice the difference between this and part (ix)).
Linear: Yes / No Time-Inv.: Yes / No Causal: Yes / No

(b) Suppose we have a linear, time-invariant system with impulse response $h[n] = \cos(0.2\pi n)u[n]$. Is the system an FIR filter?

No!

Problem Q2.2:

A periodic signal, $x(t)$, is given by

$$x(t) = 5 + 3\sin(2000\pi t) + \cos(5000\pi t + \pi/5).$$

(a) What is the *fundamental* period of $x(t)$?

$$\omega_0 = 1000\pi$$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{1000\pi} = \boxed{\frac{1}{500}}$$

$$\text{or } = \boxed{0.002}$$

(b) Find the Fourier series coefficients a_k of $x(t)$ for $-6 \leq k \leq 6$. Simplify all expressions as much as possible.

$$x(t) = 5 + 3\cos(2000\pi t - \tfrac{\pi}{2}) + \cos(5000\pi t + \tfrac{\pi}{5})$$

$$= 5 + \tfrac{3}{2} e^{-j\frac{\pi}{2}} e^{j2000\pi t} + \tfrac{3}{2} e^{j\frac{\pi}{2}} e^{-j2000\pi t} + \tfrac{1}{2} e^{j\frac{\pi}{5}} e^{j5000\pi t} + \tfrac{1}{2} e^{-j\frac{\pi}{5}} e^{-j5000\pi t}$$

$a_0 = 5$

$a_2 = \tfrac{3}{2} e^{-j\frac{\pi}{2}}$ $\quad a_{-2} = \tfrac{3}{2} e^{j\frac{\pi}{2}}$

$a_5 = \tfrac{1}{2} e^{j\frac{\pi}{5}}$ $\quad a_{-5} = \tfrac{1}{2} e^{-j\frac{\pi}{5}}$

$a_k = 0$ for other k

Problem Q2.3:

(a) (This question requires you to do a little bit of thinking; but once you see the answer, it will seem obvious!) Suppose we have a linear, time-invariant system with impulse response $h[n]$. We input a signal

$$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2].$$

to the system, which results in the output

note pattern

$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] - \delta[n-3] - 2\delta[n-4] - 3\delta[n-5].$$

What is $h[n]$? (Hint: it will only have two terms.)

$h[n] = \delta[n] - \delta[n-3]$

(b) What does the variable y contain after executing the following piece of MATLAB code?

```
x = [3 -1 2]
y = conv(x, [0 0 0 1 2 -1])
```

```
    1  2 -1
    3 -1  2
   ---------
    3  6 -3
      -1  2  1
          2  4 -2
   ---------------
    3  5 -3  5 -2
```

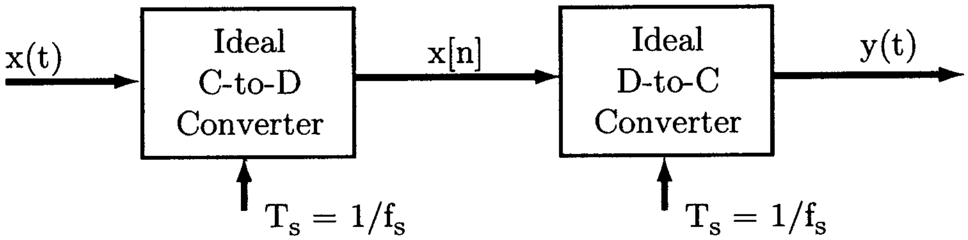
OR

```
    3 -1  2
    1  2 -1
   ---------
    3 -1  2
       6 -2  4
         -3  1 -2
   ---------------
    3  5 -3  5 -2
```

$y = [0\ 0\ 0\ 3\ 5\ -3\ 5\ -2]$

Problem Q2.4:

Hint on this problem: parts (c) and (d) don't actually require much work at all.



(a) Suppose that the continuous-time input $x(t)$ to the above system is given as

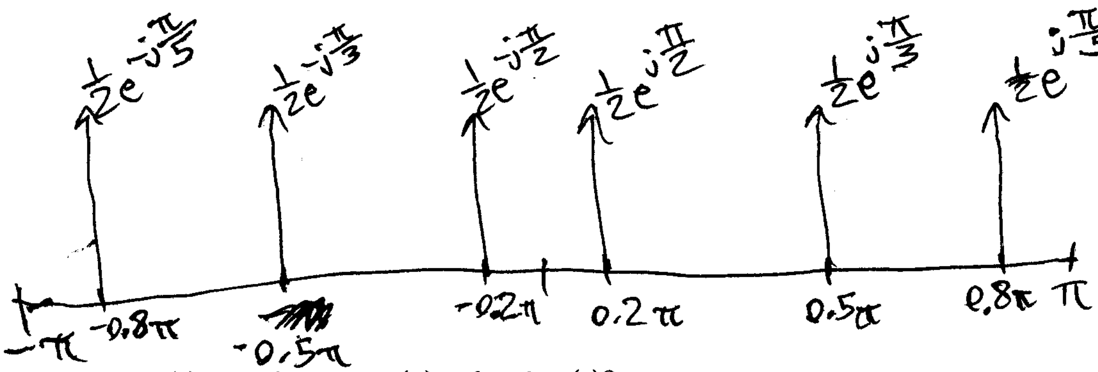
$$x(t) = \cos(800\pi t + \pi/5) + \cos(1500\pi t - \pi/3) + \cos(2200\pi t + \pi/2).$$

Given that $f_s = 1000$ Hz, plot the frequency spectrum for $x[n]$ for $-\pi \leq \hat{\omega} \leq \pi$.

$\frac{800\pi}{1000} = 0.8\pi$, $\frac{1500\pi}{1000} = 1.5\pi$, $\frac{2200\pi}{1000} = 2.2\pi$

$1.5\pi - 2\pi = -0.5\pi$ $2.2\pi - 2\pi = 0.2\pi$

folding!



(b) Given the same $x(t)$ and f_s in part (a), what is $y(t)$?

$$y(t) = \cos\left(800\pi t + \frac{\pi}{5}\right) + \cos\left(500\pi t + \frac{\pi}{3}\right) + \cos\left(200\pi t + \frac{\pi}{2}\right)$$

$0.5 \times 1000 = 500$
$0.2 \times 1000 = 200$

Aliases to DC!

(c) Now suppose $x(t) = 4 - 3\cos(6000\pi t) - \cos(12000\pi t)$ and $f_s = 3000$ Hz. What is $y(t)$?

$$x[n] = 4 - 3\cos\left(\frac{6000\pi n}{1000}\right) - \cos\left(\frac{12000\pi n}{1000}\right)$$

$$= 4 - 3\cos(2\pi \cdot 3 \cdot n) - \cos(2\pi \cdot 6 \cdot n)$$

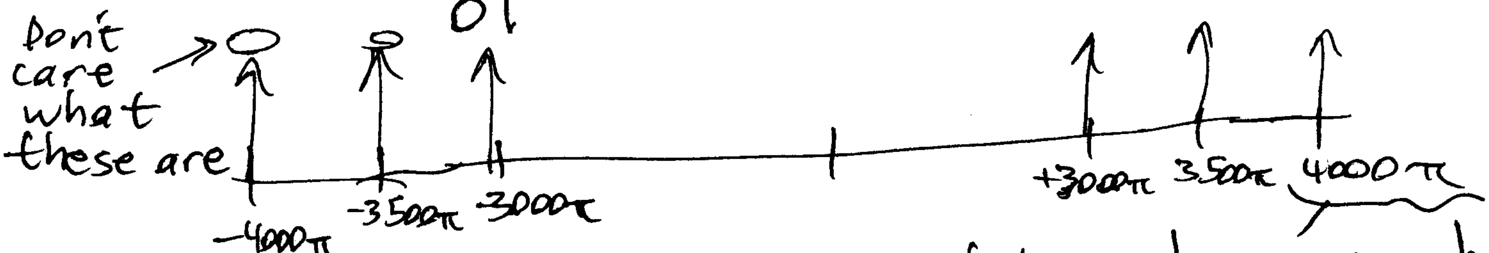
$$= 4 - 3 - 1 = \boxed{\varnothing}$$

(d) One last question. Now suppose that $x(t)$ is given by

$$x(t) = [3 - 32\cos(500\pi t + \pi/3)]\sin(3500\pi t).$$

What is the *minimum* sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

Spectrum of has the form



We only care about the highest freq.

$$f_{max} = \frac{4000\pi}{2\pi} = 2000 \text{ Hz}$$

need $f_s > 2f_{max} = \boxed{4000 \text{ Hz}}$