GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #2

Assigned: 22-Aug-03
Due Date: Week of 1-Sept-03

Reading: In *Signal Processing First*, all of Chapter 2 on *Sinusoids*.

Note: If your recitation meets on a Labor Day, then turn your homework in during your lab period instead.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 2.1*:

Each of the following signals may be simplified, and expressed as a single sinusoid of the form: $A\cos(\omega t + \phi)$. For each signal, draw a vector diagram of the complex amplitudes (phasors), and use vector addition to estimate the amplitude A and phase ϕ of the sinusoid. Then use your calculator or MATLAB and the phasor addition theorem to find the exact values for A and ϕ.

(a) $x_a(t) = 2\cos(100\pi t + 3\pi/4) + \sqrt{2}\cos(100\pi t)$

(b) $x_b(t) = 4\cos(2000\pi t + 7\pi) + 5.5\cos(2000\pi t - 2.5\pi) - 6\cos(2000\pi t - 3\pi/4)$

(c) $x_c(t) = 50\cos(120\pi t - \pi/6) + 50\cos(120\pi t - 5\pi/6) + 50\sin(120\pi t + \pi)$

(Note: be sure to notice the minus sign in the third term in part (b) and the sin in part (c)).

PROBLEM 2.2*:

Complex exponentials obey the expected rules of algebra when doing operations such as integrals, derivatives, and time-shifts. Consider the complex signal $z(t) = Ze^{j40\pi t}$ where $Z = 2e^{j\pi/3}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j40\pi t}$, i.e., $\frac{d}{dt}z(t) = Qe^{j40\pi t}$. Determine the value for the complex amplitude Q.

(b) Plot both Z and Q in the complex plane. How much greater (or smaller) is the angle of Q than the angle of Z?

(c) Compare $\Re e\{\frac{d}{dt}z(t)\}$ to $\frac{d}{dt}\Re e\{z\}$ for the given signal $z(t)$. Do you think that this would be true for any complex exponential signal?

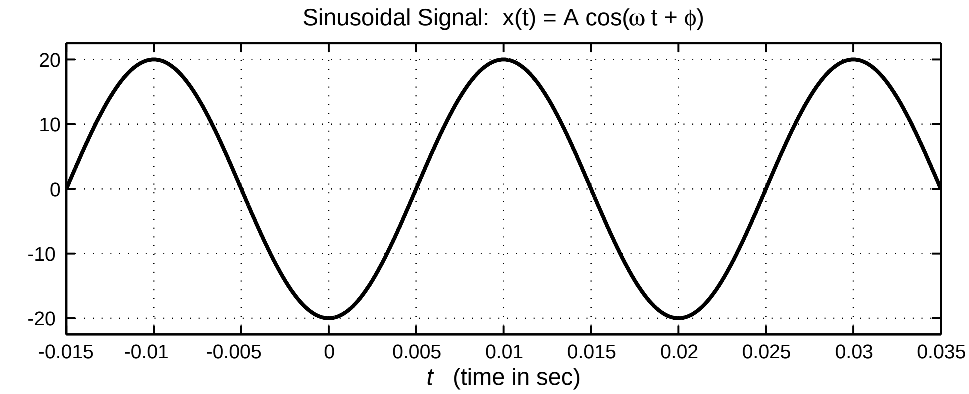
(d) Evaluate the definite integral of $z(t)$ over the range $-0.05 \leq t \leq 0.05$:

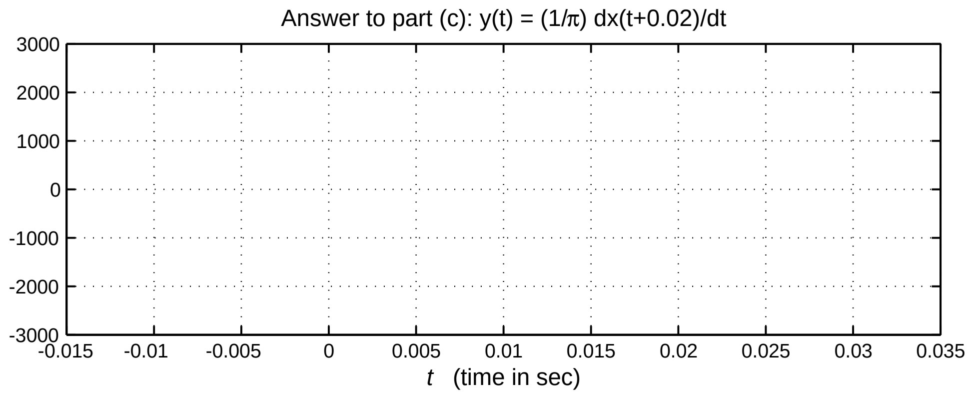
$$\int_{-0.05}^{0.05} z(t)dt = \,?$$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(e) Show that the time-shifted version of $z(t)$ can be represented as a new complex exponential $Pe^{j40\pi t}$, i.e., $z(t-t_d) = Pe^{j40\pi t}$. Determine the value for the complex amplitude P. Use your result to determine how t_d be should chosen so that $z(t-t_d) = z(t)$.

PROBLEM 2.3*:





(a) The above figure shows a plot of a sinusoidal wave $x(t)$. From the plot, determine the values of A, ω_0, and $-\pi < \phi \leq \pi$ in the representation

$$x(t) = A\cos(\omega_0 t + \phi)$$

Where appropriate, be sure to indicate the units of the sinusoidal signal parameters.

(b) The signal $x(t)$ in part (a) can be written as the real part of a complex exponential. Determine Z for the complex signal $z(t) = Ze^{j\omega_0 t}$ such that $x(t) = \Re e\{z(t)\}$.

(c) Sketch the signal $y(t) = \frac{1}{\pi}\frac{d}{dt}[x(t + 0.02)]$, where $x(t)$ is the signal from part (a). Use the axes provided above or make your own axes covering the same time interval.

PROBLEM 2.4*:

Suppose that MATLAB is used to plot a sinusoidal signal. The following MATLAB code generates the signal and makes the plot. Draw a sketch of the plot that will be done by MATLAB. Determine the amplitude (A), phase (ϕ), and period of the sinusoid and label the period on your plot.

```
Fo = 25;td=.001
dt = 0.0001;
tt = -.04 : dt : .04;
Z =8;
xx = real( Z*exp( j*2*pi*Fo*(tt + td) ) );
%
plot( tt, xx ),   grid
title( 'SECTION of a SINUSOID' ),    xlabel('TIME   (sec)')
```

PROBLEM 2.5*:

If you think about it for a bit, Euler's formula, which says that $\exp(j\theta) = \cos(\theta) + j\sin(\theta)$, is really freaky. From calculus, you may remember the following Taylor series formulas:

$$\begin{aligned}
\cos(a) &= 1 - \frac{a^2}{2!} + \frac{a^4}{4!} - \frac{a^6}{6!} + \cdots \\
\sin(a) &= a - \frac{a^3}{3!} + \frac{a^5}{5!} - \frac{a^7}{7!} + \cdots \\
\exp(a) &= 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \frac{a^4}{4!} + \frac{a^5}{5!}
\end{aligned}$$

If Taylor series gave you nightmares in calculus, that's O.K. You don't need to understand where those formulas come from; you can just take our word for it.

Show that Euler's formula is correct using these Taylor series formulas. (Hint: plug in $j\theta$ for a in the Taylor series formula for $\exp(a)$ and see what happens!)

Contemplating this deeply may yield expansion of consciousness.