GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #3

Assigned: 29-Aug-03
Due Date: Week of 8-Sept-03

Reading: In *Signal Processing First*, Chapter 3, Sections 1-3.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 3.1*:

Consider the signal given by

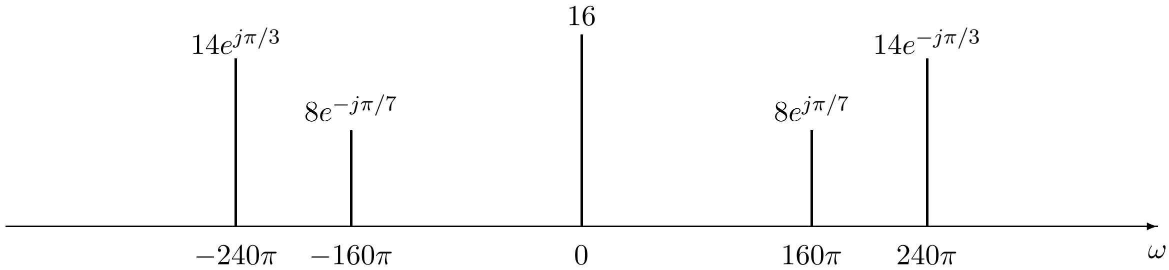
$$x(t) = 4\cos(\pi t - \pi/8) - \cos(\sqrt{3}\pi t + \pi/4) + \sin(3\pi t)$$

(a) Use Euler's formula to expand the terms into sums of complex exponentials. Then plot the spectrum of $x(t)$.

(b) Is $x(t)$ periodic? If so, what is its fundamental period?

PROBLEM 3.2*:

The signal $x(t)$ has the two-sided spectrum representation shown below:



(a) Write an equation for the signal $x(t)$. Make sure to express $x(t)$ as a real-valued signal. Following the conventions of ECE2025, please use $\cos(\cdot)$ functions in your answer, and not $\sin(\cdot)$ functions.

(b) Is $x(t)$ is periodic? If so, what is its fundamental period?

PROBLEM 3.3*:

The distortion created by old tube amplifiers, such as those made by Vox (Queen), Marshall (Led Zeppelin), and Hiwatt (The Who), being overdriven is a characteristic sound of rock music. Suppose we could model such distortion using the cubic function.

$$y(t) = [x(t)]^3.$$

The G string on a guitar vibrates at a frequency of approximately 208 Hz. Guitar strings produce sounds that are much more complicated than sinusoids, but for the purposes of this problem, pretend the guitar string and electric pickup send the amplifier a signal given by $x(t) = 2\cos(416\pi t + \pi/3)$.

(a) Plot the spectrum of $y(t)$. (Hint: Write out the cosine using Euler's formula, and expand out the cubic terms.)

(b) Write $y(t)$ as a sum of cosines. (This is going back the other direction.)

(c) Is $y(t)$ a periodic signal? If so, what is its fundamental frequency?

(Note: The cubic nonlinearity isn't actually a good model for an overdriven amplifier; something like a cube root, instead of a cube, would be more realistic, but that's too hard for us to analyze using the tools of ECE2025. If you'd like to learn more about this stuff, take a look at the UIUC's PHYCS 398EMI course page, created by Prof. Steven Errede: wug.physics.uiuc.edu/courses/phys398emi, on the "Physics of Electronic Musical Instruments." He has two whole lectures on the theory of distortion! How cool is that?)

PROBLEM 3.4*:

A piano derives some of the richness of its sounds from multiple strings being hit by the same hammer for a particular note. A piano tuner will get the strings in tune by listening for the "beats," which are the formed by two sounds which are similar but slightly out of tune with one another. Piano strings produce sounds which are far from sinusoidal, but like in the previous problem, let's pretend they produce cosine waves.

Middle C on a piano is approximately 262 Hz. Suppose one string is tuned to 262 Hz, and another (improperly tuned) string is tuned to 268 Hz. Suppose both strings have the same volume, and the sound we hear is

$$x(t) = \cos(2\pi \cdot 262t) + \cos(2\pi \cdot 268t).$$

(a) Using the discussion on pp. 40-41 of *Signal Processing First*, what *center frequency* will a listener perceive if both string struck are at the same volume?

(b) What will be the time interval between the nulls of the low-frequency beats? (See Figure 3.3 in the text.)

(c) Is $x(t)$ periodic? If so, what is its fundamental frequency? What harmonics do the first and second terms of $x(t)$ correspond to?

PROBLEM 3.5*:

Do problem 3.6 on p. 65 of the text.

PROBLEM 3.6:

Try problem 3.19 on p. 70 of the text; it offers an excellent review for the quiz.