GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #5

Assigned: 19-Sept-03
Due Date: Week of 29-Sept-03

Reading: In *Signal Processing First*, Chapter 4 on *Sampling*.

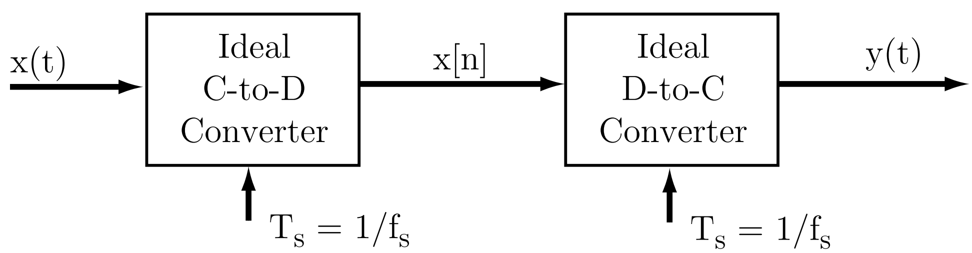
$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 5.1*:

(Hint: parts (c) and (d) don't take very much work at all.)



Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(800\pi t + \pi/5) + \cos(1500\pi t - \pi/3) + \cos(2200\pi t + \pi/2).$$

(a) Given that $f_s = 1000$ Hz, plot the frequency spectrum for $x[n]$ for $-\pi \leq \hat{\omega} \leq \pi$.

(b) Given the same $x(t)$ and f_s in part (a), what is $y(t)$?

(c) Now suppose $x(t) = 4 - 3\cos(6000\pi t) - \cos(12000\pi t)$ and $f_s = 3000$ Hz. What is $y(t)$? (You may be able to see the answer almost immediately.)

(d) Now suppose that $x(t)$ is given by

$$x(t) = [3 - 32\cos(500\pi t + \pi/3)]\sin(3500\pi t).$$

What is the *minimum* sampling rate f_s that can be used in the above system so that $y(t) = x(t)$? (This shouldn't take much work; by this point, you should be able to look at the numbers in the problem and tell right away what the highest frequency in $x(t)$ is without having to work out all the details.)

PROBLEM 5.2*:

The discrete-time signal $x[n] = 3\cos(0.3\pi n + \pi/4)$ is obtained by sampling a continuous-time signal of the form $x(t) = 3\cos(2\pi f t + \phi)$ at a rate of $f_s = 16000$ Hz.

(a) Give three different sets of parameters f and ϕ for the continuous-time signal which all yield the given $x[n]$. The parameter f must satisfy $0 < f < 20000$ Hz in all three cases.

(b) For each case, indicate if aliasing has occured, and if aliasing has occured, indicate if "folding" has occured (i.e. positive and negative frequencies have been flipped.)

PROBLEM 5.3*:

Do problem P-4.15 on p. 99 of *Signal Processing First.* It may help to read Section 4.3.

PROBLEM 5.4:

Try Problem 5.2 from Homework #5 from the Fall 2001 offering of 2025, available from the WORD section on WebCT.

PROBLEM 5.5*:

(a) Sketch the instantaneous frequency vs. time of the signal given by

$$x = \cos(2\pi \cdot 1000t^2)$$

for $0 \leq t \leq 6$.

(b) Now consider the following bit of MATLAB code.

```
fs = 8000;
t = 0:(1/fs):6;
x = cos(2*pi*1000*t .^ 2);
specgram(x,[],fs);
soundsc(x,fs);
```

Sketch what you think the spectrogram generated by this code would look like.

(c) Describe the sound you would hear.

(d) On your sketch for part (b), mark the frequency range where a "folding" effect is taking place (i.e., negative and positive frequencies are getting switched.)

You are certainly welcome to check your answers using MATLAB, but try the problem without MATLAB first. Warning: Turn down the volume a bit before trying the `soundsc` command; the sound can be quite piercing! I didn't notice I had the volume turned up on my Mac all the way, and I just woke up my wife who was sleeping in the other room. She wasn't happy about that. The next morning she told me she thought I was communicating with aliens from outer space or something.

PROBLEM 5.6*:

Consider the system in Problem 5.1 above. Now suppose suppose $x(t) = \cos(2\pi f t + \phi)$, where f is an arbitrary positive frequency in Hertz.

(a) Suppose that $f_s = f$, i.e. we're sampling at the same frequency as the cosine wave. Find $y(t)$. (Hint: it will be a *constant* with respect to t that is a function of the phase ϕ).

(b) Suppose that $f_s = 2f$, i.e. we're now sampling at *exactly* the Nyquist rate. Draw the spectrum of the sampled signal $x[n]$. Show that $y(t)$ can be written as $y(t) = g(\phi)\cos(2\pi f t)$, where $g(\phi)$ is a function of ϕ that you should find. You will see that $g(\phi)$ could be either positive or negative; one consequence of this is that the phase of $y(t)$ is either zero or π. (Something very interesting happens in this case; you'll find the spectral lines at $\hat{\omega} = \pi$ and $\hat{\omega} = -\pi$ fold and land *on top of* each other!)