GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #6

Assigned: 28-Sept-03
Due Date: Week of 8-Oct-03

Reading: In *Signal Processing First*, Chapter 5 on *FIR Filters*.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 6.1*:

Consider a discrete-time system specified by the input-output relationship:

$$y[n] = 3x[n] - 4x[n-2] + x[n-7]$$

(a) What is the impulse response $h[n]$ of the system?

(b) What are the filter coefficients b_k in the causal FIR representation:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]? \tag{1}$$

(c) What is the order of the filter M, and the length of the filter L?

(d) Find an expression for $y[n]$ if $x[n] = 3$ for all n.

(e) Suppose we instead had a discrete-time system specified by the input-output relationship:

$$y[n] = 9x[n+3] - 3x[n] + 2x[n-4]$$

Could this system be expressed using the form of equation (1)? Explain your reasoning.

PROBLEM 6.2*:

Suppose a sequence specified by

$$x[n] = \begin{cases} n^2 & \text{for } |n| \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

is input to a system with impulse response $h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2]$.

(a) Find the output $y[n]$ by drawing out the convolution table to compute $y[n] = x[n] * h[n]$. You may express your final answer in whatever form is most convenient for you.

(b) Rewrite $x[n]$ using unit step function notation to indicate the limits, i.e. rewrite it as

$$x[n] = n^2(u[\text{something}] - u[\text{something else}])$$

(c) Now compute the convolution $x[n] * \delta[n+7]$. (Notice that it's $+7$, not -7; that's legal to do.) Give your answer both as a carefully labeled plot *and* as a sum of δ-functions. (Hint: You should be able to do this without drawing out a big complicated convolution table!)

PROBLEM 6.3:

Check your answer in the previous problem two different ways:

(a) However you did the convolution table, try it the other way, i.e. if you put x along the top and h along the left side, try putting h along the top and x along the left side, and vice-versa. You should get the same answer.

(b) Use the `conv` command in MATLAB.

PROBLEM 6.4*:
The four subparts of this problem are independent of one another.

(a) Suppose we want to implement a system with impulse response

$$h[n] = -3\delta[n] + 19\delta[n-1] + 21\delta[n-2]$$

Considering the discussion on p. 113 of *Signal Processing First*, draw a *direct-form* implementation of this system in block-diagram notation.

(b) Suppose we want to implement a system with an input/output relationship given by

$$y[n] = 5x[n] - 7x[n-1] - 2x[n-2]$$

Considering the discussion on p. 115 of *Signal Processing First*, draw a *transposed-form* implementation of this system in block-diagram notation.

(c) Is a system with the impulse response $h[n] = \sin(\pi n/10)u[n-5]$ causal? Briefly explain.

(d) Is a system with the impulse response $h[n] = (0.4)^n u[n+3]$ causal? Briefly explain.

PROBLEM 6.5*:
Do P-5.12 on p. 128 of *Signal Processing First.* To do this problem, forget about convolution for a moment, and think in terms of the basic properties of *linearity* and *time-invariance.* They are your friends.

PROBLEM 6.6*:
Do P-5.18 on p. 129 of *Signal Processing First.*

PROBLEM 6.7:
Try P-5.15 on p. 129. It's a rather deep problem.