GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #7

Assigned: 3-Oct-03
Due Date: Week of 13-Oct-03

Note: This problem set writeup was **updated on 8-Oct-03**.

Reading: In *Signal Processing First*, Chapter 6 on *Frequency Response of FIR Filters*.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 7.1:

Turn to p. 142 of *Signal Processing First*. Check out Example 6-9 in the right column. Put a piece of paper over the bottom half of the page to hide everything from the line "In order to get the output singal, we must..." downward. Try to compute the output $y[n]$ on your own, then check the hidden text to see if you were able to do it correctly.

PROBLEM 7.2*:

Suppose an LTI system has a frequency response of the form $H(e^{j\hat{\omega}}) = B\sin(2\hat{\omega})e^{j(\frac{\pi}{2}-C\hat{\omega})}$.

(a) Suppose input signal is

$$x[n] = \frac{1}{\sqrt{3}}\cos\left(\frac{\pi}{6}n + \frac{\pi}{4}\right),$$

and the output signal is

$$y[n] = \frac{1}{8}\cos\left(\frac{\pi}{6}n - \frac{\pi}{4}\right).$$

Find B and C, where B is a positive real number and C is an integer between 0 and 11. (If you are having trouble with this, first try the suggested optional Problem 7.1).

(b) Using your answers for B and C from part (a), find the impulse response $h[n]$. (Hint: first use Euler's formula to rewrite the sine using complex exponentials. Recall that $e^{j\pi/2} = j$; that j will cancel with the j that appears in the denominator when you apply Euler's formula.)

PROBLEM 7.3*:

On the CD which comes with *Signal Processing First,* there's a really cool example showing the different ways you can combine FIR filters. This problem is just a way of encouraging you to look at this example and contemplate its coolness. (If you don't have the CD, that's OK; you can get the same material at the website users.ece.gatech.edu/spfirst. We have given out the username and password in lecture, and will do so again over the next several lectures.)

If you click on the folder icon for Chapter 6: Frequency Response of FIR Filters, then click on "Demos," then click on "Cascading FIR Filters," you'll see a page containing two sections.

(a) Check out the first section on "Cascading Two FIR Filters." What type of filter is H_1? What type of filter is H_2? You can find the filter types by clicking on the appropriate "Click Here" links and reading the plot titles, which have the form "FREQUENCY RESPONSE of (filter type goes here)" and "IMPULSE RESPONSE of (filter type goes here)."

(b) Now check out the section section on "Adding the Outputs of two FIR Filters." What type of filter is H_1? What type of filter is H_2? Do $x[n]$ and $y[n]$ look very similar or very different to you?

(You could just go in and snarf the answers you need for this problem without paying attention to what the demo is trying to say – but please, while you're there, do read the text and look at the pictures and think about what's there!)

PROBLEM 7.4*:

Some researchers need to compute derivatives of a measured signal. In ECE2040, you'll learn how to hook capacitors and resistors together to make circuits that to a decent job of approximating derivatives of continuous signals represented as voltages. In ECE2025, we have to deal with sampled versions of those signals (say, sampled with a period of T_s) which we store in memory. If we want to compute derivatives of the original continuous signal, we have to compute approximations to those derivatives using the samples that we have.

Some discussion from a web page I ran across[1] while Googling on the topic of *numeric differentiation* suggests approximating second derivatives using a five-point sum:

$$\frac{\partial^2 x(t)}{\partial t^2} \approx y(t) = \frac{-x(t+2T_s) + 16x(t+T_s) - 30x(t) + 16x(t-T_s) - x(t-2T_s)}{12T_s^2}.$$

for $t = T_s n$, where n is an integer. Converting to our discrete-time notation gives us:

$$y[n] = \frac{-x[n+2] + 16x[n+1] - 30x[n] + 16x[n-1] - x[n-2]}{12T_s^2}.$$

To make the math work out nicely, suppose $T_s = 1/\sqrt{12}$. (I'm just having us do this so the denominator will disappear.)

(a) Find the impulse response $h[n]$ of this system.

(b) Is the system causal?

(c) Find the frequency response $H(e^{j\hat{\omega}})$ of this system. Express your answer using complex exponentials.

(d) Use Euler's formula to rewrite $H(e^{j\hat{\omega}})$ using cosines.

(e) Now let's get some practice sketching filter curves. Sketch $H(e^{j\hat{\omega}})$ for $-\pi \leq H(e^{j\hat{\omega}}) \leq \pi$. Do this in two steps:

  (i) Evaluate $H(e^{j\hat{\omega}})$ for $\hat{\omega} = 0,\ \pi/2,\ -\pi/2$, and π. (Hint: they turn out to be integers.) This gives you some "dots" to help you plot $H(e^{j\hat{\omega}})$.

  (ii) To draw a nice curve that connects the dots, notice that one of the cosines in part (b) has much greater magnitude than the other cosine, so your smooth curve connecting the dots should look something like a single period of a cosine (although you shouldn't expect it to be exactly a cosine.)

(f) Plot the magnitude of $H(j\hat{\omega})$.

(g) Plot the phase of $H(j\hat{\omega})$. (This should be *very simple* curve.)

(Note: in this problem, it turned out that $H(j\omega)$ was real, so we could visualize it easily. In general, it is complex, and plotting the magnitude and phase can be trickier.)

[1] See http://www-ncce.ceg.uiuc.edu/tutorials/pde/html/node2.html. Well, you don't have to see it; there's nothing on that web page that will help you with this problem. I just wanted to give credit where it is due.

PROBLEM 7.5:

Use MATLAB to check the validity of your sketches in the previous problem.

PROBLEM 7.6*:

On p. 140 of *Signal Processing First*, the authors (McClellan, Schafer, and Yoder, here denoted MSY) compute the magnitude of the frequency response of a *first difference* system the usual way: add the sum the square of the real part to the square of the imaginary part, and then take a square root. Let's try another trick for getting the same answer:

(a) Compute the product $H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$, where $H(e^{j\hat{\omega}}) = 1 - e^{j\hat{\omega}}$. (Don't get confused: that *superscript* star represents conjugation, not convolution.)

(b) Take the square root of your answer in (a). Make sure that it is the same as the MSY's result on p. 140.

(Note: The trick used in this problem comes in handy when you just want the magnitude. Notice by using it, we avoiding having to expand out $H(e^{j\hat{\omega}}) = 1 - \cos\hat{\omega} + j\sin\hat{\omega}$ as MSY do. Of course, if you want the phase, then it's hard to avoid that path.)

PROBLEM 7.7*:

Suppose a discrete-time LTI system has frequency response

$$H(e^{j\hat{\omega}}) = 16\frac{\sin(4\hat{\omega})}{\sin(\hat{\omega}/2)}e^{-j6.5\hat{\omega}}$$

Draw a carefully labeled sketch of its impulse response $h[n]$. (Hint: Check out p. 145 of the text. Note it doesn't exactly match Equation 6.25, so you have to think a bit.)