GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #8

Assigned: 17-Sept-03
Due Date: Week of 27-Oct-03

Reading: In *Signal Processing First*, Chapter 7 on z-Transforms.

$\Longrightarrow$ Please check the "Bulletin Board" **daily**. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 8.1:

Go to the WORD section of WebCT, and try parts (a) through (d) of Problem 8.3 on HW #7 from the Fall 2001 offering of ECE2025. Make sure you're comfortable going back and forth between $h[n]$ and $H(z)$ representations of FIR filters before continuing with this problem set, or else you'll be wasting a lot of your time.

PROBLEM 8.2*:

Do parts (c) through (f) of Problem 7-6 on p. 192 of *Signal Processing First*. (Just read and believe parts (a) and (b), but don't do them.)

PROBLEM 8.3:

Try the parts we left out of the previous problem, i.e., try parts (a) and (b) of Problem 7-6 on pp. 192 of *Signal Processing First*. This is a highly instructive theoretical exercise.

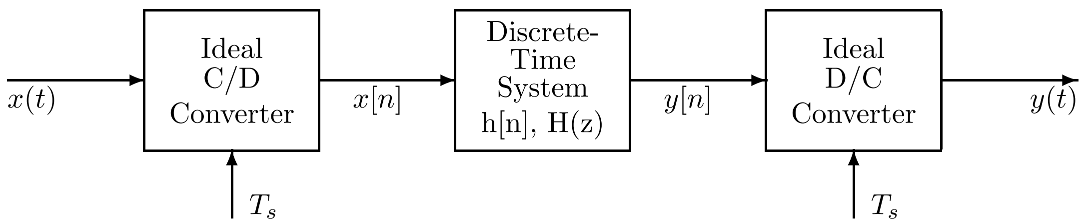
PROBLEM 8.4*:

Do Problem 7-14 on pp. 194 of *Signal Processing First.*

PROBLEM 8.5*:

(Note: This problem covers a lot of concepts. Once you do part (a), you will find that parts (b) through (e) will help you review for the quiz.)

This problem will explore the following system:



The LTI discrete-time system in the middle has the system function

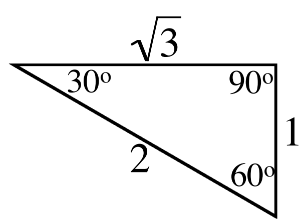
$$H(z) = z^{-2}\left(1 - \frac{1}{\sqrt{3}}z^{-1}\right)$$

(a) Evaluate $H(z)$ at $z = e^{j\hat{\omega}}$ to find the frequency response $H(e^{j\hat{\omega}})$. You may leave your expression in terms of complex exponentials. (In this problem, you will find it convenient *not* to simplify it any further. You'll see why when we get to the hint.)

(b) Suppose $x(t) = \sqrt{3}\cos(500\pi t)$ and the sampling rate $T_s = 0.001$. Find $x[n]$.

(c) Find $y[n]$.

Hint: Remember the 30°-60°-90° triangle from your high school geometry class:



(d) Find $y(t)$. Assume that the sampling rate of the ideal D/C converter is also $T_s = 0.001$.

(e) Now suppose $x(t) = \cos(2000\pi t)$ and $T_s = 0.001$. What is $y(t)$?

(f) Plot the poles and zeros of $H(z)$.

PROBLEM 8.6*:

Factor the following z-transforms and plot their poles and zeros. On parts (a) and (b), you may want to consult some references on polynomial division. See, for instance:

```
www.sci.wsu.edu/~kentler/Fall97_101/Chapter5/lpd_1.html
www.austin.cc.tx.us/~powens/+ColAlg/Html/04-3/04-3.html
www.purplemath.com/modules/polydiv.htm
jwilson.coe.uga.edu/EMT668/EMAT6680.2002/Rouhani/IU/module1.html
```

Also, see the scan of the handwritten examples attached to this homework writeup.

(a) $H(z) = 1 - (1/2)z^{-1} + (1/2)z^{-2} - z^{-3}$. When you see something of this form, with alternating signs, a good trick is to first try factoring out $(1 - z^{-1})$.

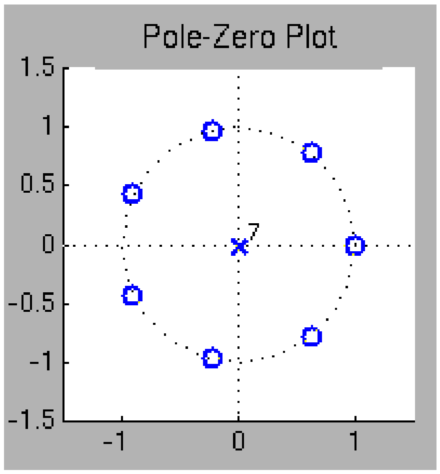
(b) $H(z) = 1 + (1/2)z^{-1} + (1/2)z^{-2} + z^{-3}$. When you see something of this form, a good trick is to first try factoring out $(1 + z^{-1})$.

(c) $H(z) = 1 - 9z^{-1} + 26z^{-2} - 24z^{-3}$. That's just too hard to try without the help of a computer! So check out the `roots` command in MATLAB.

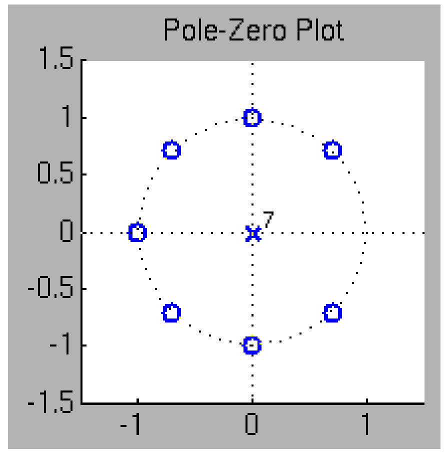
Note: on parts (a) and (b), you can use MATLAB or a fancy calculator to check your answer, but show that you know how to the factoring relying too much on electronic help!

PROBLEM 8.7*:

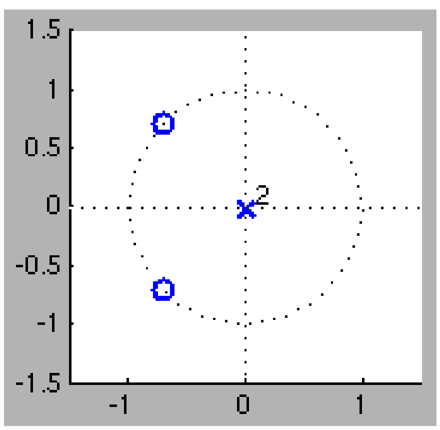
Consider the following four pole-zero plots, which were made by taking screenshots of the PezDemo GUI. (Sorry my screenshots are a little sloppy; I made them in a bit of a hurry so they aren't all uniform in size.)



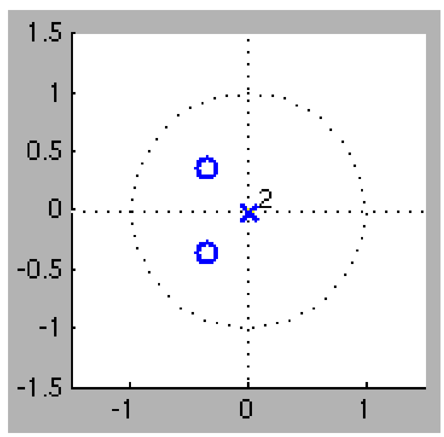
(Fred)



(Wilma)



(Barney)



(Betty)

(a) For each of the pole-zero plots, specify the corresponding causal FIR filter coefficients b_k. Here's some further information and hints. (In particular, the first three are common filters that have special forms.)

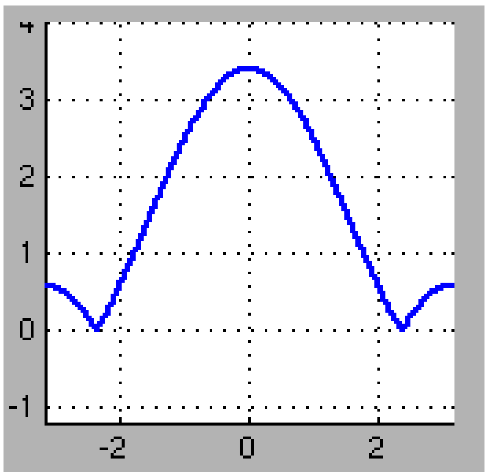
Fred: $b_0 = 1$, and the zeros are equally spaced around the unit circle. See Sec. A-6.1 on pp. 439–440.

Wilma: $b_0 = 1$, and the zeros are equally spaced around the unit circle, but $z = 1$ is missing. See Sec. 7-7.1 on pp. 181–183.

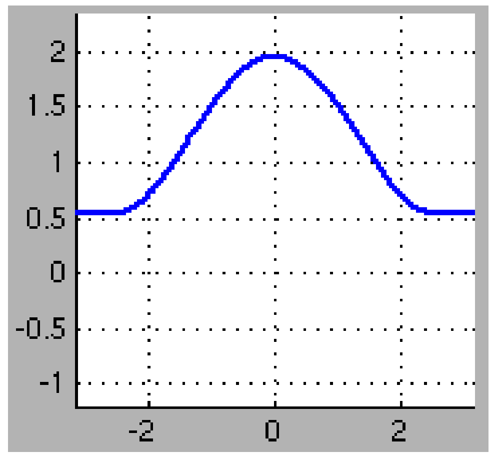
Barney: $b_0 = 1$, and the zeros are on the unit circle at angles $\pm 135°$. See Sec. 7-6.4 on p. 179; pay particular attention to the right column.

Betty: The zeros are at $(1/2)\exp(j\pi 3/4)$ and $(1/2)\exp(-j\pi 3/4)$, and $b_0 = 1$. Unlike the first three, there's no trick to this one. You just have to multiply things out.

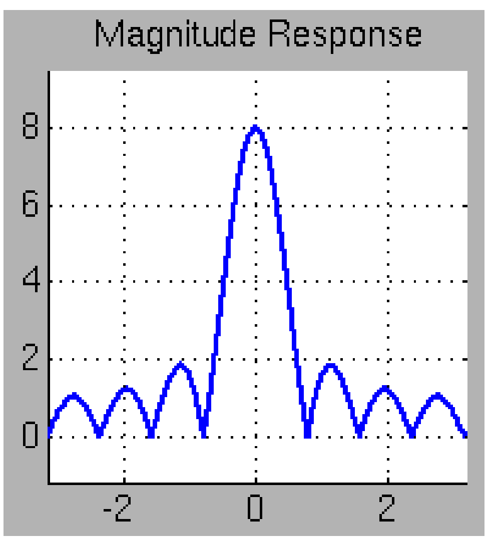
(b) Consider the following screenshots of magnitude responses, also made with the PezDemo GUI. For each of the four screenshots, specify which pole-zero plot (Fred, Wilma, Barney, or George) it corresponds to. Briefly explain your reasoning. (It can be something as simple as "looks kind of like Figure such-and-such in the book", or "the zeros are in the correct location", or "I matched the other three, so by process of elimination it must be this one.")



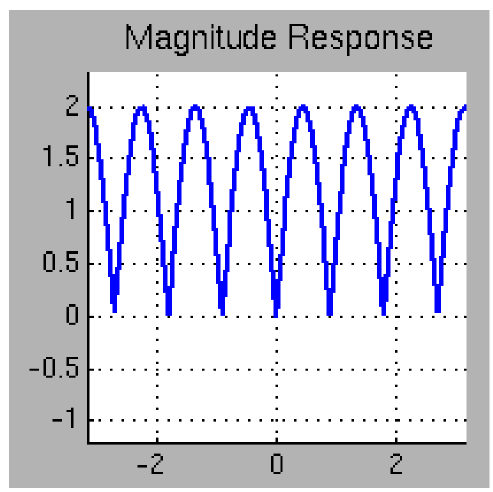
(Mad Murdock)



(B.A. Baracus)



(Faceman Peck)



(Hannibal Smith)

Aaron's notes on polynomial division for factoring

Ex. 7-7 on p. 174 considers

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

The text says "$H_1(z) = (1 - z^{-1})$ is a factor of $H(z)$. The other factor can be obtained by division

$$H_2(z) = \frac{H(z)}{H_1(z)} = \frac{H(z)}{1 - z^{-1}} = 1 - z^{-1} + z^{-2}$$ "

The text just assumes you know how to do this polynomial division. On the next page I spell it out in detail.

$$\begin{array}{r} 1 - z^{-1} + z^{-2} \\ 1 - z^{-1} \overline{) \; 1 - 2z^{-1} + 2z^{-2} - z^{-3}} \end{array}$$

$$-1 + z^{-1}$$

$$-z^{-1}$$

$$z^{-1} - z^{-2}$$

$$z^{-2} - z^{-3}$$

$$-z^{-2} - z^{-3}$$

$$\emptyset$$

Another helpful example:

$$\begin{array}{r} 1 + z^{-1} + z^{-2} \\ 1 + z^{-1} \overline{) \; 1 + 2z^{-1} + 2z^{-2} + z^{-3}} \end{array}$$

$$-1 - z^{-1}$$

$$z^{-1} + 2z^{-2}$$

$$-z^{-1} - z^{-2}$$

$$z^{-2} + z^{-3}$$

$$-z^{-2} - z^{-3}$$

$$\emptyset$$

(Note: on your problem, the signs may follow a different pattern!)