GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #9

Assigned: 24-Oct-03
Due Date: Week of 3-Nov-03

Reading: In *Signal Processing First*, Chapter 8 on *IIR Filters*.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. **Special note for this assignment: You MUST do Problem 9.1, or else the grader will not bother grading the rest of your problems.** This shouldn't be too much of a burden, as Problem 9.1 is not that hard – but you must take it to heart!

A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 9.1*:

In this class, we've tried to build ideas slowly over time, introducing complex ideas by showing the simplest cases first. When we discussed the z-transform of FIR filters, we noted that when you take the inverse z-transform of a system function like

$$H(z) = 1 - z^{-1},$$

you get an impulse response like

$$h[n] = \delta[n] - \delta[n-1].$$

Unfortunately, this leads some people to believe that

$$H(z) = \frac{1}{1 - z^{-1}}$$

has an inverse z-transform given by

$$h[n] = \frac{1}{\delta[n] - \delta[n-1]}. \quad \longleftarrow \text{NOT TRUE!!! BAD! AWFUL!}$$

Such an expression, with a delta function in the *denominator*, is a mathematical abomination. It should be cast into the fires of Mount Doom. The correct inverse z-transform, of course, is the unit step function

$$h[n] = u[n]. \quad \leftarrow \text{TRUE AND GOOD}$$

Notice that $1/(\delta[n] - \delta[n-1])$ yields total nonsense for most n. For $n = 0$, you get $1/1 = 1$ which is at least not insane; likewise, for $n = 1$, you get -1, but, for $n \neq 0, 1$, you get $1/0 = \infty$ which is completely crazy.

Why might people think the equation above labeled "NOT TRUE!!! BAD! AWFUL!" above could at all be reasonable? What would possess them, year after year, to write such things $1/(\delta[n]-\delta[n-1])$ as answers on quizzes, even when we jump up and down in class repeatedly telling them not to? Well, recall that inverse z-transforms are linear, and hence they commute over addition:

$$Z^{-1}\{F(z)+G(z)\} = Z^{-1}\{F(z)\}+Z^{-1}\{G(z)\}. \quad \longleftarrow \text{TRUE, GOOD, AND WONDERFUL}$$

Alas, this seems to lead some people to think that inverse z-transforms commute over division, as in

$$Z^{-1}\left\{\frac{F(z)}{G(z)}\right\} = \frac{Z^{-1}\{f[n]\}}{Z^{-1}\{g[n]\}}, \quad \longleftarrow \text{NOT TRUE!!! BAD! AWFUL!}$$

which is blatantly wrong.

That all said, this first problem is very simple. All you need to do is write "I have read Problem 9.1 carefully. I promise I will not put delta functions in the denominator." Then sign your name.

PROBLEM 9.2*:

Consider an LTI system with input/output relationship given by

$$y[n] = 0.3y[n-1] + x[n-3] + 0.6x[n-4]$$

(a) Find its system function $H(z)$.

(b) Plot the poles and zeros of this system.

(c) Use z-transform concepts to find the output $y[n]$ if the system is given an input $x[n] = (-0.6)^n u[n]$.

(d) Derive a formula for the frequency response of this system.

(e) Plot the magnitude of the frequency response versus $\hat{\omega}$.

PROBLEM 9.3*:

Consider an LTI system with the system function

$$H(z) = \frac{1-0.2z^{-2}+0.3z^{-5}}{1+0.5z^{-3}-0.9z^{-7}}$$

(a) Find the input/output equation in terms of an input $x[n]$ and an output $y[n]$ for the system implementing this $H(z)$.

(b) Suppose we want to implement this system in MATLAB using the command:

```
yy = filter(bb,aa,xx)
```

What should the variables `bb` and `aa` contain?

PROBLEM 9.4*:

(a) Find the inverse z-transform of

$$H(z) = \frac{(1 - \frac{\sqrt{3}}{4}z^{-1})(2 - 3z^{-1})}{1 - \frac{\sqrt{3}}{2}z^{-1} + \frac{1}{4}z^{-2}} = (2 - 3z^{-1})\frac{(1 - \frac{\sqrt{3}}{4}z^{-1})}{1 - \frac{\sqrt{3}}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

Hint: To begin with, ignore the $(2 - 3z^{-1})$ factor in the numerator. Use the material on the slide with the title "2nd ORDER Z-transform PAIR" from Lecture 17 on the remaining part. I've picked the numbers to make things work out nicely. Then, you should be able to handle the $(2 - 3z^{-1})$ business by just using the linearity and delay properties of the z-transform. It is not easy to simplify the answer, so you can leave the answer as the difference of two terms.

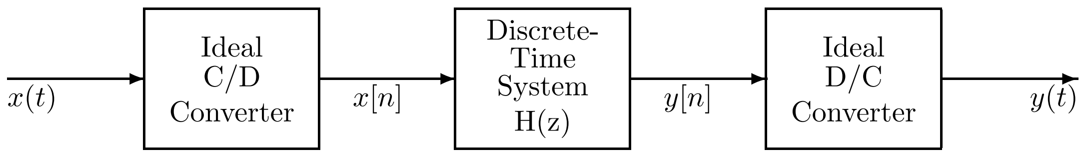
(b) Plot the poles and zeros of $H(z)$.

(c) Use your answer in part (b) to determine whether or not $H(z)$ represents a stable system.

PROBLEM 9.5*:

In addition to dealing with some the new concepts in Chapter 8, this problem will help you review some concepts from previous homeworks. We'd like to do this since these concepts, in various forms, will come up over and over again throughout the rest of the class!

This problem will explore the following system:



Both the C/D and the D/C run at the same sampling rate, denoted T_s. The LTI discrete-time system in the middle has the system function

$$H(z) = \frac{z^{-2}}{1 - \frac{1}{\sqrt{3}} z^{-1}}$$

(a) Find $h[n]$, the impulse response of the system. (Recall Problem 9.1, and stop yourself if you find yourself writing a delta function in the denominator!!!)

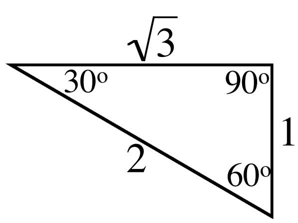
(b) Plot the poles and zeros of $H(z)$.

(c) Evaluate $H(z)$ at $z = e^{j\hat{\omega}}$ to find the frequency response $H(e^{j\hat{\omega}})$. (You will find it most convenient to leave this answer in terms of complex exponentials, and not try to simplify it any further right away. This will make more sense when you see the hint below.)

(d) Suppose $x(t) = 2\cos(500\pi t)$ and $T_s = 0.001$. Find $x[n]$.

(e) Find $y[n]$.

Hint: Remember the 30°-60°-90° triangle from your high school geometry class:



(f) Find $y(t)$.

(g) Now suppose $x(t) = (\sqrt{3} - 1)\cos(2000\pi t)$ and $T_s = 0.001$. What is $y(t)$?

PROBLEM 9.6*:

For some reason, authors of introductory computer science texts are fascinated with the computation of Fibonacci numbers. The first two Fibonacci numbers are 1 and 1. After that, you get a Fibonacci number by summing the previous two Fibonacci numbers, yielding a sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, etc. The computer scientists never explain why anyone should care what Fibonacci numbers are, but they then proceed to obsess over computing them, and then are surprised when students reading their books get bored silly.

A formal mathematical definition of Fibonacci numbers (labelling the first one as 0, the second one as 1, etc.) looks like:

$$\text{fib}(n) = \begin{cases} 1 & \text{if } n = 0 \text{ or } n = 1 \\ \text{fib}(n-1) + \text{fib}(n-2) & \text{otherwise} \end{cases}$$

If you suffered through CS1321, you could write this as the following Scheme code:

```
(define (fib n)
  (if (n < 2)
      1
      (+ (fib (- n 1)) (fib (- n 2)))))
```

(If you have no idea what Scheme is, that's cool. It's not at all relevant to this problem.)

Anyway, in this problem we'll see how to use z-transforms to write a *closed-form solution* for the Fibonacci numbers. (We won't claim to know why anyone would want them, though...)

(a) Consider the LTI system defined by the input/output relationship

$$y[n] = x[n] + y[n-1] + y[n-2]$$

(b) By brute force, compute $y[n]$ for $n = 0, 1, 2$ and 3 if $x[n] = \delta[n]$. Assume the "at rest" initial condition. This should convince you that the impulse response of this system consists of the Fibonacci numbers.

(c) Find the system function $H(z)$.

(d) Plot the poles and zeros of this system.

(e) Is the system stable? (You can find the answer one of two ways: you can either consider the location of the poles, or you can think about the Fibonacci numbers for about three seconds.)

(f) Compute the impulse response $h[n]$ by taking the inverse z-transform of $H(z)$. In other words, obtain an $h[n]$ that is a closed-form expression for the Fibonacci numbers. This may require you to do a "partial fraction expansion."

$$H(z) = \frac{K_1}{1 - p_1 z^{-1}} + \frac{K_2}{1 - p_2 z^{-1}}$$

where p_1 and p_2 are the poles of the system. Don't be scared of partial fraction expansions. They're not really that hard.

To check your work, you may want to compute $h[n]$ explicitly for the first few n from your answer, and note that they correspond to the Fibonacci numbers.