GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #10

Assigned: 31-Oct-03
Due Date: Week of 10-Nov-03

Reading: In *Signal Processing First*, Chapter 9 on *Continuous-Time Signals and LTI Systems.*

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading.

A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 10.1*:

The first two subparts are just continuous-time variations of some discrete-time questions which showed up on Quiz 2.

(a) Make a plot of $\delta(t-3) * [u(t+1) - u(t-4)]$, where $*$ represents continuous-time convolution.

(b) Simplify $\delta(t-3)[u(t+1) - u(t-4)]$, and make a plot of it.

(c) Compute the convolution $y(t) = \sin(100\pi t) * \delta^{(1)}(t)$, where $\delta^{(1)}(t)$ is the strange "doublet" from Section 9-5.2 of *Signal Processing First.*

(d) Compute $\delta(n+7) * \delta(t-3) * \delta(t-11)$. (Hint: see Section 9-6 of *Signal Processing First.*)

(e) Simplify

$$x(t) = \int_{-\infty}^{t} \delta(\tau+3) d\tau$$

(f) Simplify

$$x(t) = \int_{-\infty}^{-2} t\delta(\tau+3) d\tau$$

(g) Simplify

$$x(t) = \int_{-\infty}^{-4} t\delta(\tau+3) d\tau$$

(h) Compute and simplify
$$\frac{d}{dt}\left\{\frac{2}{\sqrt{3}}\cos(2t)u(t-\pi/6)\right\}.$$

PROBLEM 10.2*:

One of the lectures slides claims that the system specified by $y(t) = [A + x(t)]\cos(\omega_c t)$ is neither linear nor time-invariant. That's true for *interesting* A and ω_c, but... let's think about that claim more critically.

(a) What constraints would we need to put on A and/or ω_c to make the system *linear*?

(b) What constraints would we need to put on A and/or ω_c to make the system *time-invariant*?

PROBLEM 10.3*:

Consider the functions $x(t) = 2[u(t-2)-u(t-4)]$ and $h(t) = 3e^{-0.4(t-1)}[u(t-1)-u(t-6)]$

(a) To get your mind thinking in convolutionland, draw three different plots with "τ" on the horizontal axis.

(i) On the first graph, plot $x(\tau)$.

(ii) On the second graph, plot $h(-\tau)$. (Don't worry about getting the decay rate in your sketch exactly right; these sketches are just meant to aid your understanding.)

(iii) On the third graph, plot $h(2-\tau)$. This should look like your plot in (ii), just shifted to the *right*.

(b) Compute the convolution[1] $y(t) = x(t) * h(t)$; let $h(t)$ be the signal that you "flip and shift." Your answer will consist of five different cases. Doing the calculus is actually the easy part; the tricky part is figuring out the limits of the integrals and figuring out what the different cases are. Convolutions are difficult to do, so read Section 9-7 of *Signal Processing First* if you are having trouble.

[1]Some textbooks would write this like $y(t) = (x * h)(t)$; Aaron prefers that kind of notation, but will use $x(t) * h(t)$ to stay consistent with *Signal Processing First*. The trouble with $y(t) = x(t) * h(t)$ is that it might mislead you to thinking that you plug in a specific t, find $x(t)$ and $y(t)$, to get two numbers and then do a point operation on those two numbers; but in reality, convolution is acting on the entire functions $x(\cdot)$ and $h(\cdot)$, and we're evaluating that the result of that convolution at a particular point. $y(t) = (x * h)(t)$ makes it clearer that convolution is acting on all of x and all of h. The *SPF* notation does have the advantage that it lets you write things like "$y(t) = x(t) * \delta(t-4)$." In Aaron's notation, you'd have to scribble something like "$y(t) = (x * h)(t)$ where $h(t) = \delta(t-4)$", which is more cumbersome to write.

PROBLEM 10.4:

Check your work in part (b) of the previous problem a couple of ways:

(a) Redo the convolution in the previous problem, but this time let $x(t)$ be the function that you "flip and shift." This is a good way of checking your work, as you should get the same answer!

(b) Play around with the `cconvdemo` MATLAB GUI. This would be good preparation for the lab assignment on using the continuous convolution GUI.

PROBLEM 10.5*:

Do Problem 9.14 on p. 281 of *Signal Processing First.* We have often put problems like this on quizzes, since we can't ask you to do a full complicated five-region convolution problem under the time constraints of a 50 minute exam.

PROBLEM 10.6*:

Do Problem 9.25 on p. 284 of *Signal Processing First.* This problem is rather easy if you attack it the right way.