GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2003
Problem Set #11

Assigned: 7-Nov-03
Due Date: Week of 17-Nov-03

Reading: In *Signal Processing First*, Chapter 10 on *Frequency Response* and Sections 1 through 5 of Chapter 11 on the *Continuous-Time Fourier Transform.*

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading.

A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 11.1*:

Do problem P-10.3 on pp. 302-303 of *Signal Processing First.* (If you think about it a bit, you'll find it makes a very subtle connection with the discrete-time stuff we did earlier in the class.)

PROBLEM 11.2:

Try problem P-10.9 on pp. 305-306 of *Signal Processing First.* We really like putting problems like that on quizzes and exams.

PROBLEM 11.3*:

Suppose a continuous-time LTI system has an impulse response $h(t)$ specified by $h(t) = \text{sign}(\sin(\pi t))u(t+2)$, where $\text{sign}(a)$ is defined to be 1 if a is positive, -1 if a is negative, and 0 otherwise.

(a) Draw a carefully labeled sketch of $h(t)$.

(b) Notice that for all a,

$$\int_{-\infty}^{a} h(t)dt < \infty$$

A fellow student tells you that this implies that $h(t)$ is the impulse response of a stable system. Is your fellow student correct? If not, what is wrong with his reasoning?

(c) Suppose the system represented by the impulse response $h(t)$ is cascaded with another system with frequency response $G_1(j\omega) = e^{-j\omega c}$.

For what values (notice I made that plural) of c is the complete cascaded system causal?

(d) Now suppose the system represented by the impulse response $h(t)$ is cascaded with another system with impulse response $g_2(t) = \delta(t) + \delta(t+1)$. Sketch $h_{cas}(t) = h(t) * g_2(t)$, which is the impulse reponse of the complete system.

PROBLEM 11.4*:

Suppose a system has an impulse response given by

$$h(t) = \frac{\sin(400\pi t)}{\pi t} \tag{1}$$

(a) Find the frequency response (or, equivalently, the Fourier transform) $H(j\omega)$. (Hint: check out pp. 316-317 of *Signal Processing First*.)

(b) Consider the *half-rectified* sinusoid with fundamental period T_0 specified by

$$x(t) = \begin{cases} \sin(\omega_0 t) & \text{for } 0 \le T < T_0/2 \\ 0 & \text{for } T_0/2 \le t < T_0 \end{cases},$$

where $\omega_0 = 2\pi/T_0$. According to the scribbles at `users.ece.gatech.edu/lanterma/rect.html`, the Fourier series coefficients of $x(t)$ are given by

$$a_k = \begin{cases} -\frac{1}{\pi(k^2-1)} & \text{for } k \text{ even} \\ -j\frac{k}{4} & \text{for } |k| = 1 \\ 0 & \text{otherwise} \end{cases}$$

If this periodic signal $x(t)$, with $\omega_0 = 150\pi$, is input to an LTI system with the impulse response $h(t)$ given in (1), what is resulting the output $y(t)$? (Be sure to write your answer entirely using "real" quantities, i.e., don't leave any complex exponentials in your answer.)

PROBLEM 11.5*:

Suppose a system has an impulse response given by

$$h(t) = [u(t+100) - u(t-100)]$$

(a) Find the frequency response (or, equivalently, the Fourier transform) $H(j\omega)$. (Hint: check out p. 315 of *Signal Processing First.*

(b) If the input to this system is

$$x(t) = 3 + \cos(0.01\pi t) + \frac{1}{4}\cos(0.02\pi t) + \frac{1}{9}\cos(0.03\pi t) + \frac{1}{16}\cos(0.04\pi t),$$

find the output $y(t)$. Explain your reasoning. Be clever.

(c) If the input to this system is

$$x(t) = \frac{1600}{\sqrt{2}}\cos(0.0025\pi t) + 800\delta(t-50),$$

find the output $y(t)$. (Remember, one of the most important concepts in ECE2025 – and, in fact, most of electrical engineering – is to know when to use the time domain way of thinking and when to use the frequency domain way of thinking. You may often find yourself needing to use different domains on different parts of a problem. Some philosophy about this appears in Section 10.5 of *Signal Processing First.*)

PROBLEM 11.6*:

During their investigations into the paranormal, Agent Fox Mulder and Agent Dana Scully of the FBI discover a continuous-time LTI system that they believe is of extraterrestrial origin. They know that it has a one-sided decaying exponential impulse respone given by $h(t) = \exp(-at)u(t)$; the mystery lies in exactly what a is.

Undaunted, the agents take the alien LTI system to the FBI lab. "We must find out what a is," Mulder drones. "Put an impulse $\delta(t)$ into the system so we can find out what the impulse response is."

The lab tech shakes his head in shame. "Sorry, sir," he apologizes. "Due to budget cuts, the only working piece of signal generation equipment we have is a sinusoidal generator, and even that has a broken knobs, so it's stuck generating a cosine wave with phase 0, an amplitude of 200, and a frequency of 40 radians."

"Are you sure that's 40 radians? Why not 40π radians?" Scully asks.

A voice from the sky, which sounds remarkably like Aaron, says "Yes, it's 40 radians. I'm making it exactly 40 radians so the math in part (a) works out nicely."

The lab tech attaches his rather limited sinusoidal generator to the input of the mysterious alien system, and exclaims, "Ah ha! When given this input, the output is a cosine wave with amplitude 4, a frequency of 40 radians, and a phase of..."

"Ssssh!" the voice from the sky says. "Don't give away the phase! That's part (b) of the problem!"

(a) Help Mulder and Scully by finding the value of a using the information given in the problem. (Hint: check out Example 11-7 on pp. 324-325 of *Signal Processing First*.)

(b) Find the phase of the output cosine the tech almost gave away. (I generally try to set up my homework and quiz problems so that you can do them without a calculator, but for this one, you'll need to go ahead and pull out your calculator.)

(c) Does the function $x(t) = 2^t$ have a Fourier transform? If so, find what it is. If not, explain why. (This subpart doesn't actually have anything to do with the rest of the problem, I just couldn't think of where else to put it.)