GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2004

Problem Set #3

Assigned: 16-Jan-04

Due Date: Week of 26-Jan-04

Quiz #1 will be held in lecture on Monday 2-Feb-04. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.

Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$**, both sides)**

Reading: In *SP First*, Chapter 3: *Spectrum Representation*, Sections 3-1, 3-2 and 3-3.

There is a web site for *SP First* text: `www.ece.gatech.edu/˜spfirst`

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

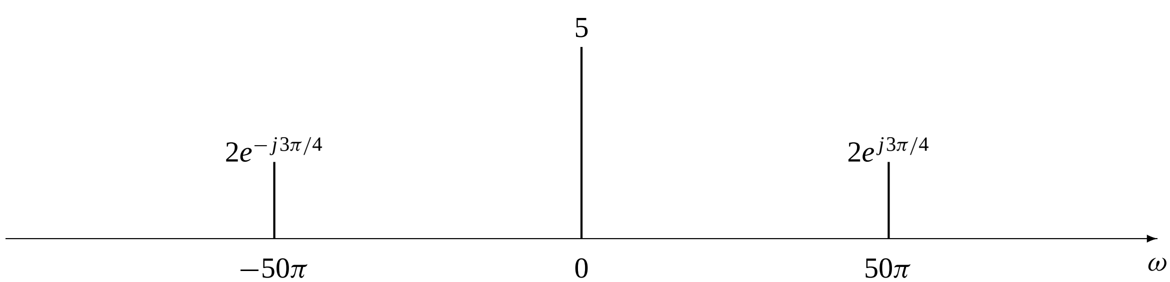
ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 3.1*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of the signal: $y(t) = x^2(t - 0.01)$.

(c) Plot the spectrum of the *real-valued* signal: $z(t) = 2x(t)\sin(200\pi t)$.

PROBLEM 3.2*:

The two-sided spectrum of a signal $x(t)$ is given in the following table:

Frequency (rad/sec)	Complex Amplitude
$-\omega_5$	X_{-5}
$-\pi$	$-3 - j\sqrt{3}$
0	B
ω_2	X_2
$2.5\omega_2$	$2e^{j\pi/6}$

where $\omega_5 > \omega_2$, and $B > 0$.

(a) If $x(t)$ is a *real* signal, determine the numerical values of the parameters: X_{-5}, X_2, ω_2 and ω_5.

(b) Write an expression for $x(t)$ involving only real numbers and cosine functions, so that the DC value of $x(t)$ is equal to 4.

(c) Determine the *fundamental period* of $x(t)$, i.e., the minimum $T > 0$ such that $x(t+T) = x(t)$.

PROBLEM 3.3*:

A piano derives some of the richness of its sounds from multiple strings being hit by the same hammer for a particular note. Usually three strings are used for each note. Piano strings produce sounds which are not perfect sinusoids, but let's pretend they produce cosine waves.

The note A-440 (the A above Middle C) on a piano should be 440 Hz. Suppose that the three strings for A-440 are tuned to 436 Hz, 440 Hz and 444 Hz, and that all three strings produce exactly the same volume when the A-440 key is struck. The resulting sound that we hear will be the sum of three sinusoids:

$$x(t) = \cos(2\pi(436)t + \phi_1) + \cos(2\pi(440)t + \phi_2) + \cos(2\pi(444)t + \phi_3)$$

where the phases ϕ_1, ϕ_2, and ϕ_3 depend on how the 3 strings are struck with the hammer.

(a) Consider the simplest case where all the phases are the same, $\phi_1 = \phi_2 = \phi_3 = \pi/4$. Show that the signal $x(t)$ can be written as a sinusoid at the desired frequency of 440 Hz, multiplied by another function. In other words,

$$x(t) = e(t)\cos(2\pi(440)t + \pi/4)$$

Find $e(t)$ as a simple real-valued function.
Hint: Use a derivation that writes $x(t)$ as the real part of the sum of three complex exponentials.

(b) The signal $e(t)$ is usually called the *envelope* because its frequency is low and and it causes the amplitude of $x(t)$ to go up and down slowly. Determine the time interval between the maximal peak locations of the low-frequency envelope.

PROBLEM 3.4*:

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal*. The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB could be written as:

$$x(t) = (v(t) + A)\cos(2\pi(750 \times 10^3)t)$$

where A is a constant.

Note: The constant A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $|v(t)|$.

(a) Voice-band signals tend to contain frequencies less than 4000 Hz (4 kHz). Suppose that $v(t)$ is a 3000 Hz sinusoid, $v(t) = 2\cos(2\pi(3000)t + 0.5\pi)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a) and assume that $A = 2.5$.
Hint: Express both $v(t)$ and the cosine as a sum of complex exponentials, and then multiply.

PROBLEM 3.5*:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the formula for a time signal from each of the spectrum plots.