GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2004
Problem Set #4

Assigned: 30-Jan-04
Due Date: Week of 9-Feb-04

Quiz #1 will be held in lecture on Monday 2-Feb-04. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$**, both sides)**

Reading: In *SP First*, Chapter 3: *Spectrum Representation*, all.

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

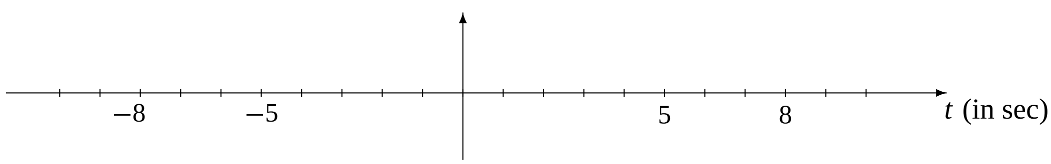
ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 4.1*:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} e^t & \text{for } -1 \le t \le 1 \\ 0 & \text{for } 1 < t < 4 \end{cases}$

(a) Assume that the period of $x(t)$ is 5 sec. Draw a plot of $x(t)$ over the range $-8 \le t \le 8$ sec.



(b) Determine the DC value of $x(t)$ from the Fourier series integral.

(c) Write the Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. Set up all the specifics of the integral (e.g., limits of integration, integrand), but do not evaluate the integral.

PROBLEM 4.2*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$.

(a) One way to write the chirp is to use the following MATLAB code:

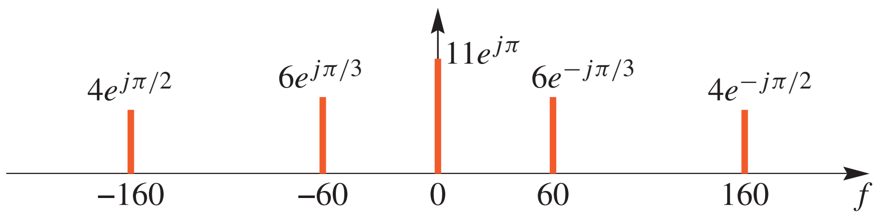
```
tt = 0:0.0001:1.5;
aa = 300*pi;
bb = 700*pi;
cc = 0.3*pi;
psi = aa*tt.*tt + bb*tt + cc;
xx = real( exp(j*psi) );
```

Using the values in the MATLAB code and the signal defined by `xx` and the *angle function* `psi`, draw a sketch of the instantaneous frequency as time goes from $t = 0$ to $t = 1.5$ sec. Label the axes to make it clear whether frequency is in hertz or rad/s.

(b) Write MATLAB code (similar to that above) for a chirp that starts at $f_1 = 2000$ Hz and ends at $f_2 = 200$ Hz over a duration of 1.8 secs.

PROBLEM 4.3*:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$. *The frequency axis has units of Hz.*



(a) Determine the fundamental frequency f_0 (in Hz) of this signal.

(b) Determine the period T_0 of $x(t)$.

(c) Determine the DC value of this signal.

(d) A periodic signal of this type can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $\quad k = 0, \pm1, \pm2, \pm3, \ldots$, determine the indices for which the coefficients are nonzero. List these nonzero Fourier series coefficients and their values in a table.

PROBLEM 4.4:
A signal $x(t)$ is periodic with period $T_0 = 5$. Therefore, it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/5)kt}$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{5} \int_{-1}^{1} e^t e^{-j(2\pi/5)kt} dt$$

(a) In the expression for a_k above, the integral and its limits effectively define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-8 \leq t \leq 8$ seconds. Label it carefully.

(c) Determine a_0, the DC value of $x(t)$ found in part (a).

PROBLEM 4.5*:
A periodic signal $x(t)$ is described over one period $-1 \leq t < 4$ by the equation

$$x(t) = \begin{cases} e^t & \text{for } -1 \leq t \leq 1 \\ 0 & \text{for } 1 < t < 4 \end{cases}$$

The period of this signal is $T_0 = 5$ sec.

(a) Set up the *Fourier analysis* integral for determining a_k for $k \neq 0$. (Insert proper limits and integrand.)

(b) Evaluate the integral in part (a) and obtain an expression for a_k that is valid for all k.

(c) Make a plot of the spectrum over the range $-2.5\omega_0 \leq \omega \leq 2.5\omega_0$, where ω_0 is the fundamental frequency of the signal in rad/s. Use MATLAB or a calculator to determine the complex numerical values (in polar form) for each of the Fourier coefficients corresponding to this range of frequencies.

PROBLEM 4.6*:

The periodic signal $x(t)$ described over one period $-1 \le t < 4$ by the equation

$$x(t) = \begin{cases} e^t & \text{for } -1 \le t \le 1 \\ 0 & \text{for } 1 < t < 4 \end{cases}$$

can be represented by a Fourier series.

(a) If we subtract a constant value of two from $x(t)$, we obtain a new signal $w(t) = x(t) - 2$. Make a plot of the periodic signal $w(t)$ over the time interval $-8 \le t < 8$.

(b) The new signal $w(t)$ can also be represented by a Fourier series, $w(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$, because it is periodic with period T_0. Explain how b_0 and b_k are related to a_0 and a_k.
Hint: You should not have to evaluate any new integrals explicitly to answer this question.

(c) Sketch the waveform of another new signal $y(t) = 3x(-t)$ over the time interval $-8 \le t < 8$.

(d) Determine the spectrum for the signal $y(t)$ defined in part (c), and make a plot of its spectrum over the range $-2.5\omega_0 \le \omega \le 2.5\omega_0$, where ω_0 is the fundamental frequency of the signal in rad/s. Label all the frequencies and complex amplitudes in the spectrum.
Hint to avoid integrals: Does flipping $x(t)$, flip the a_k's ? If you denote the coefficients in the Fourier series for $y(t)$ as c_k, then

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

By substituting the Fourier series expansion for $x(t)$ into the definition of $y(t)$, you should be able to find a simple relationship between c_k and a_k, the Fourier series coefficients of $x(t)$.

PROBLEM 4.7:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. In musical notation the tones are called notes; the names of the notes in the octave starting with middle-C and ending with high-C are:

note name	C	$C^{\#}$	D	$E^{\flat}$	E	F	$F^{\#}$	G	$G^{\#}$	A	$B^{\flat}$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones in the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.