GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

# ECE 2025 Spring 2004
# Problem Set #5

Assigned: 6-Feb-04
Due Date: Week of 16-Feb-04

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Reading: In *SP First*, Chapter 4: *Sampling and Aliasing*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

**ALL** of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

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**Your homework is due in recitation at the beginning of class.** After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

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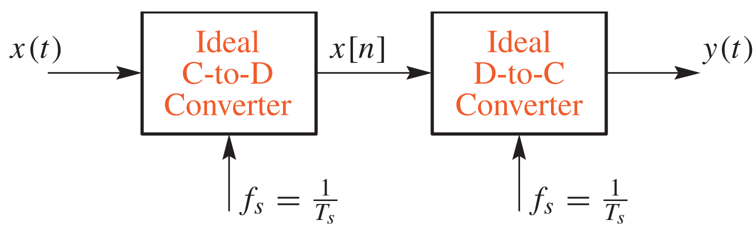



Figure 1: Ideal sampling and reconstruction systems. An ideal C-to-D converter samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

## PROBLEM 5.1*:

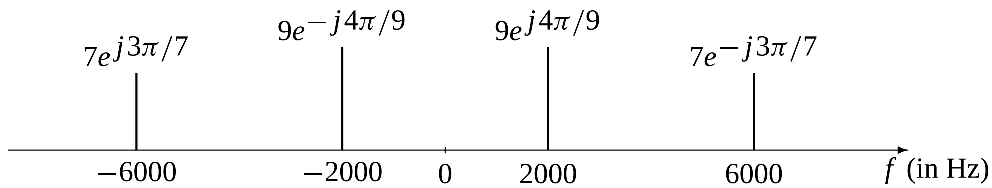
Consider the ideal sampling and reconstruction system shown in Fig. 1.

(a) Suppose that the discrete-time signal $x[n]$ in Fig. 1 is given by the formula

$$x[n] = 7\cos(0.2\pi n + \pi/4)$$

If the sampling rate of the C-to-D converter is $f_s = 8000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency between 40000 and 48000 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/8000$ secs.

(b) Now if the input $x(t)$ to the system in Fig. 1 has the two-sided spectrum representation shown below, what is the *minimum* sampling rate $f_s$ such that the output $y(t)$ is equal to the input $x(t)$?

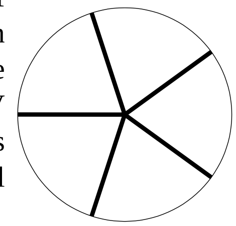


(c) Using the signal $x(t)$ from part (b), determine the spectrum for $x[n]$ when $f_s = 8000$ samples/sec. Simplify your as much as possible and make a plot for your answer, but label the frequency, and complex amplitude (magnitude and phase) of each spectral component.

## PROBLEM 5.2*:

When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the car's hubcaps have a spoked pattern. Both of these are due to the 30 frames/sec sampling used in transmitting TV images.

In the figure to the right, a five-spoked wheel is shown. Assume that the diameter of this wheel is 0.6 meters, which is nearly the tire diameter of a typical automobile. In addition, assume that the wheel is rotating clockwise, so that if attached to a car, the car would be traveling to the right *at a constant speed.* However, when seen on TV the spoke pattern of the car wheel appears to backwards (i.e., CCW) at 2 revolutions per second. How fast is the car traveling (in kilometers per hour)? Derive a general equation that will make it easy to give all possible answers.



## PROBLEM 5.3*:

Shown in Fig. 1 above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [5 + 4\cos(800\pi t + \pi/3)]\cos(2000\pi t)$$

(a) Use Euler's formulas for the cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this *continuous-time* signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the *minimum* sampling rate $f_s$ that can be used in the system of Fig. 1 so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 1000$ samples/sec. Your plot should include labels on the frequency axis ($\hat{\omega}$), as well as the amplitude and phase of each spectrum component.

(d) Determine the output signal $y(t)$ when both the C-to-D and D-to-C converters are operating at $f_s = 1000$ samples/sec.

## PROBLEM 5.4*:

Chirps and FM signals are very useful signals for probing the behavior of sampling and reconstruction systems (such as Fig. 1). Consider the following MATLAB code:

```
%-- make an FM signal and display its spectrogram
%--
fs = 5000;   %-- or fs = 8000;
tt = 0:1/fs:1.3;
psi = 5000*pi*tt + 200*cos(6*pi*tt);
xx = cos(psi);
plotspec(xx+j*1e-11,fs,128)  %-- specgram could be used here
grid on, shg
```

(a) The MATLAB code can interpreted as equivalent to the system in Fig. 1. Determine the mathematical expressions for $x(t)$ and $x[n]$, the signals at the input and output of the C-to-D converter. Write your answer assuming that $f_s$ is a parameter whose value is not yet assigned.

(b) When using the spectrogram, it turns out that you are essentially calculating the spectrogram of the output signal, $y(t)$. If the sampling rate is $f_s = 8000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Use mathematics to determine the analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction**, and then draw a graph of what the spectrogram should look like. Comment on whether or not the sampling theorem is satisfied when $f_s = 8000$ Hz. *Hint:* this could be checked in MATLAB by using the code above.

(c) Change the sampling frequency to $f_s = 5000$ Hz and repeat everything in the previous part. Explain how aliasing and/or folding affects the result and make the spectrogram look different.

## PROBLEM 5.5*:

In all parts below, the sampling rates of the C/D and D/C converters are **NOT equal,** and the input to the ideal C/D converter is

$$x(t) = 9e^{j(44\pi t - \pi/9)} + 4e^{j(16\pi t + \pi/4)}$$

(a) If the sampling rate of the C-to-D converter is $f_s = 20$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-\pi \le \hat{\omega} \le \pi$.

(b) Using the result of part (a), determine the value for the sampling rate of the ideal D-to-C converter so that the the output $y(t)$ contains two spectrum lines, one at 16 Hz and the other at 64 Hz. In addition, draw the spectrum for this continuous-time output signal.

**PROBLEM 5.6**:
Assume that $x(t)$ is the input to an ideal C-to-D converter; $x[n]$ is its output, and $y(t)$ is the output of an ideal D-to-C converter when $x[n]$ is the input (as in Fig. 1).

(a) Suppose that the input $x(t)$ is given by

$$x(t) = 3 + 4\cos(2\pi(2000)t - \pi) + 5\cos(2\pi(7000)t - 3\pi/4)$$

Determine the spectrum for $x[n]$ when $f_s = 8000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, making sure to label the frequency, amplitude and phase of each spectral component.

(b) Using the discrete-time spectrum for $x[n]$ from part (a), determine the analog frequency components in the spectrum of the output $y(t)$ when the sampling rate of the D-to-C converter is $f_s = 8000$ Hz.

(c) It is possible to choose a sampling rate so that the output is a constant. Determine the *largest* value of $f_s$ for which $y(t)$ will be a constant. Furthermore, determine the numerical value of the constant.

**PROBLEM 5.7**:
A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.1$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} 1 - (.5)^n & 0 \le n \le 4 \\ (15/16)(.5)^{n-4} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus $n$.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its nonzero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its nonzero region.