GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2004
Problem Set #6

Assigned: 13-Feb-04
Due Date: Week of 23-Feb-04

Reading: In *SP First*, Chapter 5: *FIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 6.1*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = -x[n] - 2x[n-1] - 3x[n-2] - 4x[n-3]$$

(a) Determine the impulse response $h[n]$ for this system.

(b) Determine the filter coefficients b_k in the causal FIR representation: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

(c) Determine the *order* of the filter (M), and the *length* of the filter (L).

(d) Make a plot of the shifted unit-step signal $s[n] = -u[n-20]$; plot enough to show its essential behavior.

(e) Use convolution to determine the output due to the input

$$x[n] = u[n-1] - u[n-20] = \begin{cases} 1 & n = 1, 2, 3, 4, 5, \ldots, 19 \\ 0 & \text{otherwise} \end{cases}$$

Use the convolution table, but look for patterns. Plot the output sequence $y[n]$ for $-3 \le n \le 25$.

PROBLEM 6.2*:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n]\,x[n-1]$ (Multiplier)

(b) $y[n] = e^{jx[n-3]}$ (Complex Exponential)

(c) $y[n] = x[n^2 - 1]$ (Time Distortion)

PROBLEM 6.3*:

Signal Processing First, Chapter **5**, Problem **15**, page 129.

PROBLEM 6.4*:

Signal Processing First, Chapter **5**, Problem **18**, page 129.

PROBLEM 6.5*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

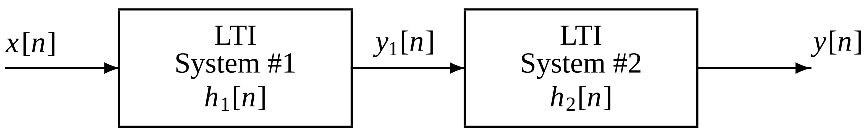

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = x[n-3] - x[n-1]$$

and System #2 is a modified *first-difference* filter described by the impulse response:

$$h_2[n] = \delta[n-1] - \delta[n]$$

(a) Determine the impulse response sequence, $h_1[n]$, of the first system. Plot $h_1[n]$ versus n.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) If the input signal is

$$x[n] = n^2 \left(u[n] - u[n-10]\right)$$

determine the output signal, $y[n]$, and make a plot of $y[n]$ versus n. Indicate regions where the output is zero; and regions where it is nonzero.

PROBLEM 6.6:

Consider a system defined by $\quad y[n] = \sum_{k=0}^{M} b_k x[n-k]$

(a) What is the filter length?

(b) Suppose that the input $x[n]$ is non-zero only for $10 \leq n \leq 20$ and $M = 9$. Where will the output $y[n]$ first become non-zero? What is the index of the last non-zero value in the output sequence $y[n]$? What is the total length of the input sequence (in samples).

(c) Suppose that the input $x[n]$ is non-zero only for $N_1 \leq n \leq N_2$. What is the length of the input sequence (in samples).

(d) For the input in (b) and the above system, show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \leq n \leq N_4$ and determine N_3 and N_4.

(e) What is the length of the output sequence (in samples)?

Hint: Draw a sketch similar to Fig. 5.5 (on p. 105) to illustrate the zero regions of the output signal.