GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2004
Problem Set #7

Assigned: 20-Feb-04
Due Date: Week of 1-March-04

Quiz #2 will be given on 5-March-2004.
Coverage will include HW #4, #5, #6 and #7. Solutions for this homework will be posted on 4-March.

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 7.1*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

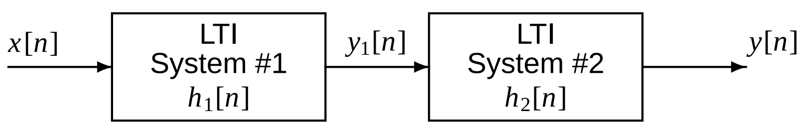


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the difference equation

$$y_1[n] = \tfrac{1}{2}x[n] + x[n-1] + \tfrac{1}{2}x[n-2]$$

and System #2 is described by the impulse response

$$h_2[n] = \tfrac{1}{2}\delta[n-1] + \delta[n-2] + \tfrac{1}{2}\delta[n-3],$$

(a) Determine the frequency response sequence, $H_1(e^{j\hat{\omega}})$, of the first system.

(b) Determine the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system.

(c) Plot the magnitude and phase of the frequency response of the overall cascaded system.

(d) If the input to this system is

$$x[n] = 5 + 3\cos(\tfrac{1}{3}\pi(n-1))$$

use the frequency response to determine the output signal, $y[n]$, over the range $-\infty \leq n \leq \infty$. Express the answer as a simple formula.

PROBLEM 7.2*:
Consider the linear time-invariant system given by the difference equation

$$y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] + x[n-4] - x[n-5] = \sum_{k=0}^{5} (-1)^k x[n-k]$$

(a) Find an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(b) Show that your answer in (a) can be simplified and expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(3(\hat{\omega}-\pi))}{\sin(\frac{1}{2}(\hat{\omega}-\pi))}\, e^{-j2.5(\hat{\omega}-\pi)}$$

Hint: use the fact that $(-1)^k = e^{\pm j\pi k}$.

(c) Sketch the frequency response (magnitude and phase) versus $\hat{\omega}$ from the formula above. Notice that you get a *Dirichlet* shape, but its peak is no longer centered at $\hat{\omega} = 0$. You might want to check your plot by doing it in MATLAB with `freekz( )` or `freqz( )`.

(d) Suppose that the input signal is

$$x[n] = 7 + 9\cos(\hat{\omega}_0 n) \qquad \text{for} \quad -\infty < n < \infty$$

Find all possible non-zero frequencies $0 < \hat{\omega}_0 < \pi$ for which the output $y[n]$ is zero for all n. In other words, the sinusoid and DC are removed by the filter.

PROBLEM 7.3*:
A discrete-time system is defined by the impulse response:

$$h[n] = \tfrac{1}{2}\delta[n-4] + \delta[n-5] + \tfrac{1}{2}\delta[n-6]$$

(a) When the input is the signal, $x_1[n] = 3 + (-1)^n$, the output is $y_1[n] = 6$. Determine the output when the input is

$$x_2[n] = \begin{cases} c_1 & \text{for } n \text{ even} \\ c_2 & \text{for } n \text{ odd} \end{cases}$$

where c_1 and c_2 are constants. Use linearity and time invariance to simplify your work.

(b) Obtain an expression for the frequency response of this system, defined in part (a).

(c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency.
Hint: Use symmetry to simplify your expression before determining the magnitude and phase.

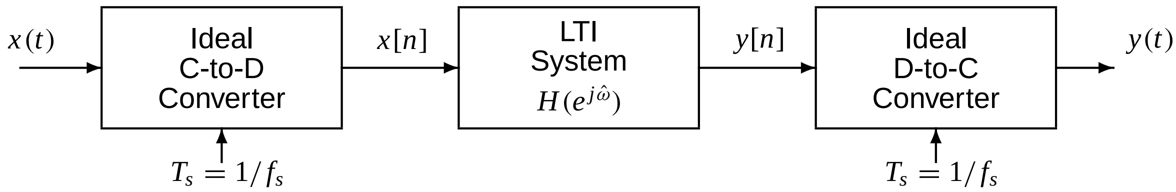
(d) For the system above, determine the output $y_1[n]$ when the input is

$$x_1[n] = 3(-1)^n + 2\cos(0.625\pi n)$$

Hint: Use the frequency response and superposition to solve this problem.

PROBLEM 7.4*:

Consider the following system for discrete-time filtering of a continuous-time signal:



In this problem, assume that the impulse response of the discrete-time system is

$$h[n] = \tfrac{1}{2}\delta[n-4] + \delta[n-5] + \tfrac{1}{2}\delta[n-6]$$

(a) Determine the frequency response formula, $H(e^{j\hat{\omega}})$, for the LTI system.

(b) For a sampling rate of $f_s = 800$ samples/sec, determine the frequency of an input sinusoid of the form $x(t) = \cos(\omega t)$ such that the resulting output will be $y(t) = \cos(\omega t + \phi)$, i.e., the output amplitude and frequency are the same as the input.

(c) In this part, assume that the input is

$$x(t) = 99 + 88\cos(500\pi t) \quad \text{for} \quad -\infty < t < \infty$$

For a sampling rate of $f_s = 800$ samples/sec, determine the output $y(t)$ for $-\infty < t < \infty$.

PROBLEM 7.5*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(e^{j\hat{\omega}}) = \left(1 + e^{-j2\hat{\omega}}\right)\left(1 + e^{-j4\pi/3}e^{-j\hat{\omega}}\right)\left(1 + e^{-j2\pi/3}e^{-j\hat{\omega}}\right) \qquad (1)$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.
Hint: Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.

(b) Determine the impulse response of this system, and make a stem plot. Notice that $h[n]$ is finite length.

(c) If the input is a complex exponential of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for which values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?
Hint: In this part, the answer is easy to obtain if you use the factored form of Eq. (1).

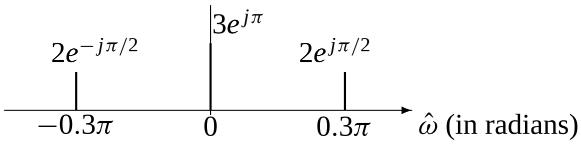
(d) Use superposition to determine the output of this system when the input is

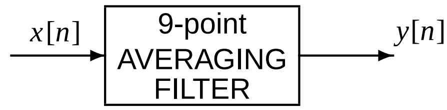
$$x[n] = 3 + 7\delta[n-1] + 13\cos(0.5\pi n - \pi/4) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results. This is what it means to apply the principle of *Superposition*.

PROBLEM 7.6:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.

PROBLEM 7.7:

A discrete-time system is defined by the input/output relation

$$y[n] = \begin{cases} 1 & \text{if } |x[n]| \geq 0.5 \\ 0 & \text{if } |x[n]| < 0.5 \end{cases}$$

(a) For the system above, determine the output $y_1[n]$ when the input is

$$x_1[n] = \cos(0.5\pi n)$$

(b) Explain why the result from part (a) proves that the system is not an LTI system.

(c) Is the system linear? or time-invariant? or neither?