GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2004
Problem Set #9

Assigned: 20-Mar-04
Due Date: Week of 29-Mar-04

Quiz #3 will be given on 9-April. One page ($8\frac{1}{2} \times 11$ in.) of **handwritten** notes allowed.

Reading: In *SP First*, Chapter 9: *Continuous-Time Signals & Systems*
and Chapter 10: *Frequency Response*.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 9.1*:

A linear time-invariant system has impulse response:

$$h(t) = e^{0.5t}\{u(t+2) - u(t-2)\} = \begin{cases} e^{0.5t} & -2 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine whether or not this system is *causal.* Give a reason to support your answer.

(b) Plot $h(t-\tau)$ as a function of τ for $t = 0$, 2, and 10.

(c) Find the output $y(t)$ when the input is $x(t) = 3\delta(t+2)$, and make a sketch of $y(t)$.

(d) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = 6e^{-0.5t}\{u(t) - u(t-4)\} = \begin{cases} 6e^{-0.5t} & 0 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$$

PROBLEM 9.2*:

A linear time-invariant system has impulse response: $h(t) = t^{-1}u(t - \frac{1}{2})$

(a) Plot $h(t-\tau)$ versus τ, for $t = -3$ and $t = 2$. Label your plot.

(b) Is the LTI system causal? Give a reason to support your answer.

(c) Is the system stable? Explain with a proof or counter-example.

(d) If the input is $x(t) = u(t-1)$, then it will be true that the output $y(t)$ is zero for $t \le t_1$. Find t_1.

(e) The rest of the output signal (for $t > t_1$) is non-zero, when the input is $x(t) = u(t-1)$. Use the convolution integral to find the non-zero portion of the output, i.e., find $y(t)$ for $t > t_1$.

PROBLEM 9.3*:

A linear time-invariant system has impulse response: $h(t) = 10\,\{u(t-2) - u(t)\}$

(a) When two finite-duration signals are convolved, the result is a finite-duration signal, $y(t) = x(t)*h(t)$. Suppose that the input signal is:

$$x(t) = e^{t-3}\,\{u(t-3) - u(t-7)\}$$

Determine the duration (in secs.) of the output signal $y(t) = x(t) * h(t)$.

(b) Using the same input signal as in the previous part, determine the minimum value of the signal $y(t)$, and also the time where the minimum occurs, i.e., $y(t_{\min}) = \min_n\{y(t)\} = y_{\min}$.

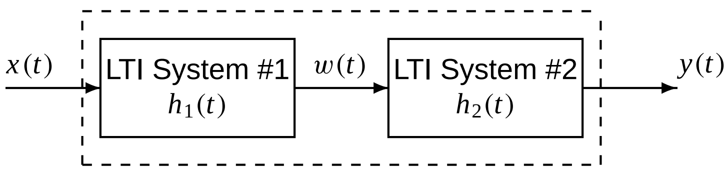
Hint: It is not necessary to perform the entire convolution to answer this question. Consider the Use the **cconvdemo** GUI in MATLAB to visualize the "flip and slide" nature of this convolution.

(c) If the input is changed to $x(t) = 7u(t-3)$, then it will be true that the output $y(t)$ from the convolution can be written as

$$y(t) = B(t-T_{12})\,\{u(t-T_{12}) - u(t-T_{23})\} + Cu(t-T_{23})$$

where the constants B and C and the times T_{12} and T_{23} can be determined from the flip and slide view of convolution. Determine the values of these four parameters.

PROBLEM 9.4*:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

$$h_1(t) = e^{-t}u(t - \tfrac{1}{2})$$

and the second system is described by the input/output relation

$$y(t) = \frac{d}{dt}w(t-2) - \int_{-\infty}^{t-2} w(\tau)d\tau$$

(a) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Find the output when the input signal is $x(t) = e^{t}u(t)$.

PROBLEM 9.5*:

A continuous-time system is defined by the impulse response:

$$h(t) = \delta(t) - \tfrac{1}{2}be^{-bt}u(t)$$

(a) Determine a simple expression for the frequency response of this system.

(b) Make a plot of the frequency response (magnitude only) when $b = 10\pi$.

(c) Describe the type of filter in the plot of part (b), e.g., LPF, HPF, BPF, or something else.

(d) Find the output $y(t)$ when the input signal is $x(t) = 10 + 40\cos(90\pi t)$, and the parameter b is $b = 10\pi$.