GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #10

Assigned: 27-Mar-04
Due Date: Week of 5-April-04

Quiz #3 will be given on 9-April. One page ($8\frac{1}{2} \times 11$ in.) of **handwritten** notes allowed.

Reading: In *SP First*, all Chapter 11: *Continuous-Time Fourier Transform*.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 10.1*:

Signal Processing First, Chapter **11**, Problem **3**, page 342. (Inverse Fourier Transform)

PROBLEM 10.2*:

Signal Processing First, Chapter **11**, Problem **4**, page 342. (Forward and Inverse Fourier Transforms)

PROBLEM 10.3*:

The impulse response of an LTI system is given by

$$h(t) = \frac{\sin(8\pi t - 4\pi)}{2t - 1}$$

(a) First, make a detailed and accurately labeled sketch of $h(t)$ over the time interval $-\frac{1}{2} \leq t \leq \frac{1}{2}$. Mark the important amplitudes and time locations of peaks and zero crossings.

(b) Now determine the Fourier transform $H(j\omega)$ of this impulse response, which is equivalent to finding the frequency response of the system.

(c) Make detailed plots of $|H(j\omega)|$ and $\angle H(j\omega)$ versus ω. Label your plots carefully. Mark the important amplitudes and time locations of peaks and zero crossings.

PROBLEM 10.4*:

The frequency responses of two ideal lowpass LTI systems are

$$H_1(j\omega) = \{u(\omega + 16\pi) - u(\omega - 16\pi)\}\, e^{-j3\omega}$$
$$H_2(j\omega) = \{u(\omega + 32\pi) - u(\omega - 32\pi)\}\, e^{-j3\omega}$$

(a) Create a bandpass filter by subtracting the two LPFs defined above.

$$H_b(j\omega) = H_2(j\omega) - H_1(j\omega)$$

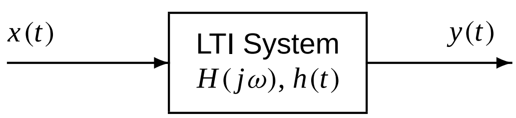
Make a sketch of the magnitude and phase of $H_b(j\omega)$.

(b) Using the BPF from part (a), determine the output of the system[1] when the input signal is

$$x(t) = \sin(14\pi t) + \frac{\sin(35\pi t)}{\pi t}$$

(c) Determine the the impulse response of the system. Express your result in a *simple form.*

PROBLEM 10.5*:



The impulse response of the system (above) is

$$h(t) = \frac{\sin(\omega_{co} t)}{\pi t} \cos(5\omega_{co} t)$$

and the input to this system is a periodic signal (with period $T_0 = 1/6$ sec.) given by a Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 k t}$$

where the Fourier coefficients are $a_k = \dfrac{7k^2}{6 + k^2}, \quad k = 0, \pm 1, \pm 2, \ldots$

(a) Recall how the spectrum is related to the Fourier series, and then plot the spectrum of the input signal, $x(t)$, over the frequency range $-40\pi < \omega < 40\pi$ in rad/s.

(b) Determine the frequency response $H(j\omega)$ of the system as a general formula. Exploit the fact that $h(t)$ and $H(j\omega)$ are a "Fourier Transform pair." Then, for the case $\omega_{co} = 5\pi$ rad/s, plot the magnitude $|H(j\omega)|$ vs. ω, and the phase $\angle H(j\omega)$ vs. ω. Use the frequency range $-40\pi < \omega < 40\pi$ rad/s.

(c) Determine the spectrum of the output signal, $y(t)$. Make a plot versus ω. This will be easy to do if you overlay the plots from parts (a) and (b) on the same frequency axis.

(d) Use your spectrum plot in (c) to determine an equation for $y(t)$, the output of the LTI system for the given input $x(t)$ when the cutoff frequency is $\omega_{co} = 5\pi$ rad/s.

[1] Use frequency-domain methods: Determine the Fourier transform of the input signal and then apply the filter in the frequency-domain to determine the corresponding output signal.