GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 15-Oct-04 COURSE: ECE-2025

NAME: ______________________________ GT # ____________________

LAST, FIRST

(e.g. gtg123a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L02:Thurs-9:30 (Anderson)	L03:Tues-Noon (Williams)	L05:Tues-1:30 (Williams)
L06:Thurs-1:30 (Anderson)	L07:Tues-3:00 (Durey)	L08:Thurs-3:00 (Smith)
L09:Tues-4:30 (Durey)	L10:Thurs-4:30 (Smith)	L13:Mon-3:00 (McClellan)
L14:Wed-3:00 (Taylor)	L15:Mon-4:30 (Hayes)	L16:Wed-4:30 (Taylor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

PROBLEM FALL-04-Q.2.1:

A periodic signal $x(t)$ is given by

$$x(t) = 2 + \cos(50\pi t) + \cos(100\pi t + \pi/2).$$

(a) Determine the fundamental period of the signal $x(t)$, i.e., the minimum period.

$T_0 =$ ______ sec. (Give a numerical answer.)

(b) Let the k^{th} Fourier series coefficient of $x(t)$ be a_k. Fill in the table below, giving *numerical values* for each requested $\{a_k\}$ in polar form.

(c) Now define

$$y(t) = x(t - .01).$$

This signal can be expressed in the following Fourier Series with new coefficients $\{b_k\}$

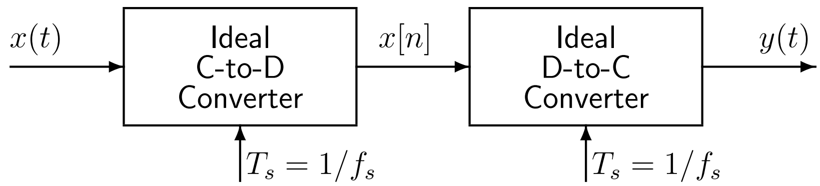
$$y(t) = \sum_{k=-\infty}^{\infty} b_k \, e^{j50\pi kt}.$$

Fill in the second table below, giving *numerical values* for each requested $\{b_k\}$ in polar form.

a_k	Mag	Phase
a_{-4}		
a_{-3}		
a_{-2}		
a_{-1}		
a_0		
a_1		
a_2		
a_3		
a_4		

b_k	Mag	Phase
b_{-4}		
b_{-3}		
b_{-2}		
b_{-1}		
b_0		
b_1		
b_2		
b_3		
b_4		

PROBLEM FALL-04-Q.2.2:



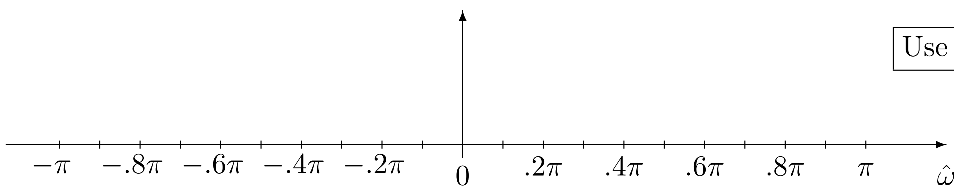
Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(20,000\pi t - \pi/3) + \cos(6000\pi t).$$

(a) Determine a sampling rate above which no aliasing of $x(t)$ occurs. $f_s =$

(b) Given that $f_s = 12,000$ samples/second, plot the frequency spectrum for $x[n]$.

Use $\hat{\omega}$ for digital freq.



(c) Write a simplified expression for the output $y(t)$ in terms of cosine functions.

PROBLEM FALL-04-Q.2.3:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response:

Impulse response:

Difference equation: $y[n] = x[n-1] + x[n-3]$

(b) Frequency response: $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(2j\sin(\hat{\omega}))$

Impulse response:

Difference equation:

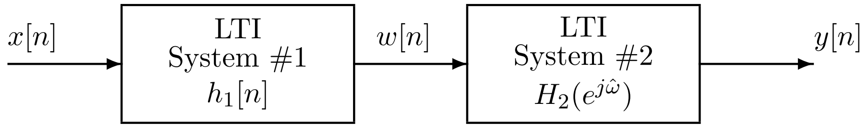
(c) Frequency response:

Impulse response:

MATLAB Implementation: `y = conv([1,-3,1],x)`

PROBLEM FALL-04-Q.2.4:

Consider the following cascade system:



where

$$h_1[n] = \delta[n] + \delta[n-1] - 2\delta[n-2] \quad \text{and} \quad H_2(e^{j\hat{\omega}}) = 2 - e^{-j\hat{\omega}}$$

(a) Find and plot the impulse response $h[n]$ of the overall system.

(b) Find the output signal $y[n]$ if $x[n] = \cos \pi n$.

PROBLEM FALL-04-Q.2.1:

A periodic signal $x(t)$ is given by

$$x(t) = 2 + \cos(50\pi t) + \cos(100\pi t + \pi/2).$$

(a) Determine the fundamental period of the signal $x(t)$, i.e., the minimum period.

$T_0 =$ .04 sec. (Give a numerical answer.)

(b) Let the k^{th} Fourier series coefficient of $x(t)$ be a_k. Fill in the table below, giving *numerical values* for each requested $\{a_k\}$ in polar form.

(c) Now define

$$y(t) = x(t - .01).$$

This signal can be expressed in the following Fourier Series with new coefficients $\{b_k\}$

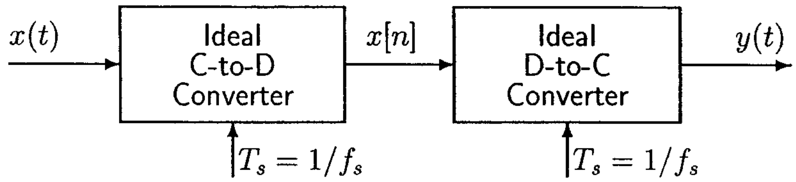
$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j50\pi kt}.$$

Fill in the second table below, giving *numerical values* for each requested $\{b_k\}$ in polar form.

a_k	Mag	Phase
a_{-4}	0	NA
a_{-3}	0	NA
a_{-2}	1/2	$-\pi/2$
a_{-1}	1/2	0
a_0	2	0
a_1	1/2	0
a_2	1/2	$\pi/2$
a_3	0	NA
a_4	0	NA

b_k	Mag	Phase
b_{-4}	0	NA
b_{-3}	0	NA
b_{-2}	1/2	$\pi/2$
b_{-1}	1/2	$\pi/2$
b_0	2	0
b_1	1/2	$-\pi/2$
b_2	1/2	$-\pi/2$
b_3	0	NA
b_4	0	NA

PROBLEM FALL-04-Q.2.2:

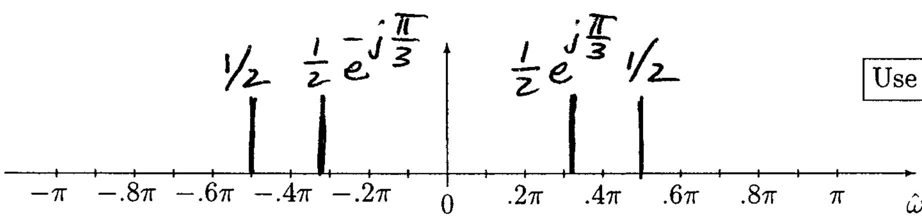


Suppose that the continuous-time input $x(t)$ to the above system is given as

$$x(t) = \cos(20,000\pi t - \pi/3) + \cos(6000\pi t).$$

(a) Determine a sampling rate above which no aliasing of $x(t)$ occurs. $f_s =$ 20,000 Hz.

(b) Given that $f_s = 12,000$ samples/second, plot the frequency spectrum for $x[n]$.



Use $\hat{\omega}$ for digital freq.

(c) Write a simplified expression for the output $y(t)$ in terms of cosine functions.

$$y(t) = \cos(6000\pi t) + \cos\left(4000\pi t + \frac{\pi}{3}\right)$$

PROBLEM FALL-04-Q.2.3:

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response: $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}}$

Impulse response: $h[n] = \delta[n-1] + \delta[n-3]$

Difference equation: $y[n] = x[n-1] + x[n-3]$

(b) Frequency response: $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(2j\sin(\hat{\omega}))$

Impulse response: $h[n] = \delta[n-1] - \delta[n-3]$

Difference equation: $y[n] = x[n-1] - x[n-3]$

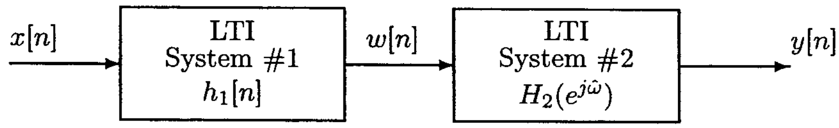
(c) Frequency response: $H(e^{j\hat{\omega}}) = 1 - 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

Impulse response: $h[n] = \delta[n] - 3\delta[n-1] + \delta[n-2]$

MATLAB Implementation: `y = conv([1,-3,1],x)`

PROBLEM FALL-04-Q.2.4:

Consider the following cascade system:



where

$$h_1[n] = \delta[n] + \delta[n-1] - 2\delta[n-2] \quad \text{and} \quad H_2(e^{j\hat{\omega}}) = 2 - e^{-j\hat{\omega}}$$

(a) Find and plot the impulse response $h[n]$ of the overall system.

1	1	-2	
2	-1		
2	2	-4	
	-1	-1	2
2	1	-5	2

$h[n] = 2\delta[n] + \delta[n-1] - 5\delta[n-2] + 2\delta[n-3]$

(b) Find the output signal $y[n]$ if $x[n] = \cos \pi n$.

$H(e^{j\hat{\omega}}) = 2 + e^{-j\hat{\omega}} - 5e^{-j2\hat{\omega}} + 2e^{-j3\hat{\omega}}$

$H(e^{j\pi}) = 2 + e^{-j\pi} - 5e^{-j2\pi} + 2e^{-j3\pi}$

$= 2 - 1 - 5 - 2 = -6$

$y[n] = 6\cos(\pi n - \pi)$ OR

$y[n] = 6\cos(\pi n + \pi)$