**Problem fall-99-F.1:**
In each of the following cases, simplify the expression as much as possible.

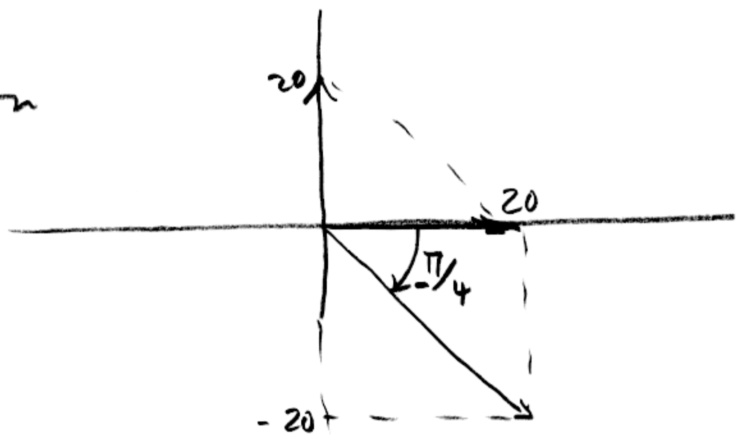
(a) $e^{-t}\delta(t - .05) = \boxed{e^{-.05}\,\delta(t-.05)}$

(b) $e^{-t} * \delta(t - 2) = \boxed{e^{-(t-2)}}$

(c) $\int_{-\infty}^{\infty} e^{-\tau}\delta(\tau - .05)d\tau = \boxed{e^{-.05}}$

(d) $x[n] = 20\cos(0.2\pi n + \pi/2) + 20\sqrt{2}\cos(0.2\pi n - \pi/4) = \boxed{20\cos(0.2\pi n)}$

Phasor Addition




$20e^{j\pi/2} + 20\sqrt{2}e^{-j\pi/4} = 20j + 20\sqrt{2}\cos(-\pi/4) + j20\sqrt{2}\sin(-\pi/4)$

$= 20j + 20 - 20j = 20$

**Problem fall-99-F.2:**

A discrete-time system is defined by the following system function:

$$H(z) = \frac{0.81 + z^{-2}}{1 + 0.81 z^{-2}}.$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

$$y[n] = -0.81\,y[n-2] + 0.81\,x[n] + x[n-2]$$

(b) Fill in numbers for the vectors `bb` and `aa` in the following MATLAB computation of the frequency response of the system:

```
bb=[0.81, 0, 1 ];     aa=[ 1, 0, .81 ];

yy=filter(bb,aa,xx)
```

where `xx` is the input signal to be filtered.

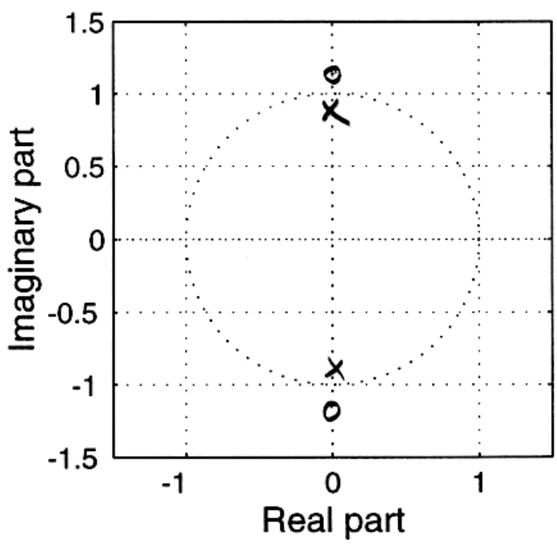
(c) Determine *all* the poles and zeros of $H(z)$ and plot them in the $z$-plane.

$$H(z) = \frac{0.81 z^2 + 1}{z^2 + .81}$$

$$= \frac{.81\left(z - j\frac{1}{.9}\right)\left(z + j\frac{1}{.9}\right)}{(z - j.9)(z + j.9)}$$

Zeros at $z = \pm \frac{10}{9} j$

Poles at $z = \pm .9 j$

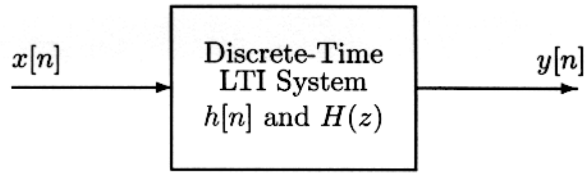



(d) Compute $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$, the magnitude-squared of the frequency response.

$$|H(e^{j\hat{\omega}})|^2 = \left(\frac{.81 + e^{-j2\hat{\omega}}}{1 + .81 e^{-j2\hat{\omega}}}\right)\left(\frac{.81 + e^{j2\hat{\omega}}}{1 + .81 e^{j2\hat{\omega}}}\right)$$

$$= \frac{(.81)^2 + 1.62 \cos 2\hat{\omega} + 1}{1 + 1.62 \cos 2\hat{\omega} + (.81)^2} = 1$$

**Problem fall-99-F.3:**




An LTI discrete-time system is depicted above. The system function of the system is

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1})}.$$

(a) It is desired that the *output* of the system be $y[n] = (\frac{1}{2})^n u[n]$. Find the $z$-transform $Y(z)$ of this output signal.

$$Y(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

(b) Use the $z$-transform method to determine the $z$-transform $X(z)$ of the input to the system such that the output of the system will be $y[n] = (\frac{1}{2})^n u[n]$.

$$Y(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} = H(z)\,X(z)$$

$$\therefore X(z) = \frac{\frac{1}{1 - \frac{1}{2}z^{-1}}}{\frac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1})}} = 1 + \frac{3}{4}z^{-1}$$

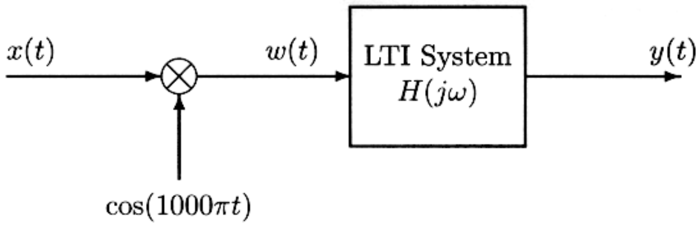
(c) Use the partial fraction expansion method to determine the impulse response $h[n]$ of the system with system function $H(z)$ given above.

$$H(z) = \frac{\frac{1}{1 + \frac{1}{2}\cdot\frac{4}{3}}}{1 + \frac{3}{4}z^{-1}} + \frac{\frac{1}{1 + \frac{3}{4}\cdot 2}}{1 - \frac{1}{2}z^{-1}}$$

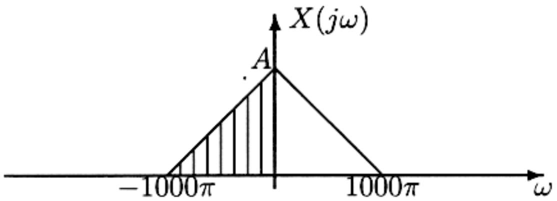
$$= \frac{\frac{3}{5}}{1 + \frac{3}{4}z^{-1}} + \frac{\frac{2}{5}}{1 - \frac{1}{2}z^{-1}}$$

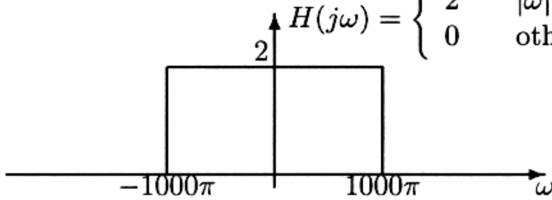
$$h[n] = \frac{3}{5}\left(-\frac{3}{4}\right)^n u[n] + \frac{2}{5}\left(\frac{1}{2}\right)^n u[n]$$

**Problem fall-99-F.4:**




In the above modulation system, assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted on the left below, and assume that the frequency response of the LTI system is $H(j\omega)$ as plotted on the right below.

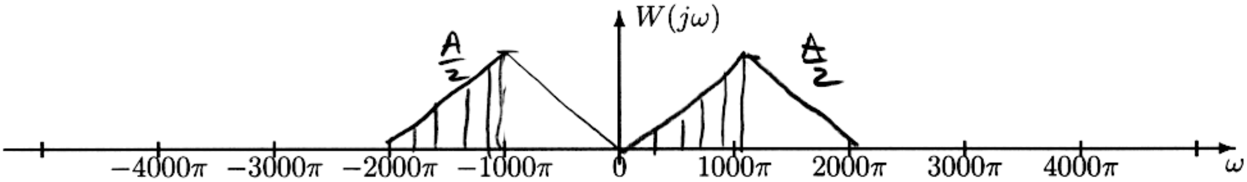


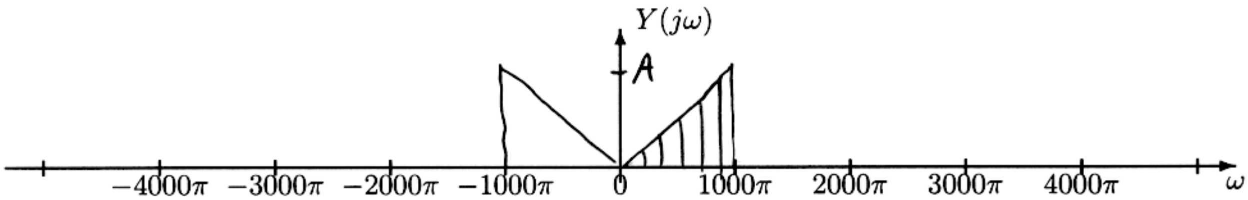

$$H(j\omega) = \begin{cases} 2 & |\omega| < 1000\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)\cos(1000\pi t)$, in terms of $X(j\omega)$ and plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above. *Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*

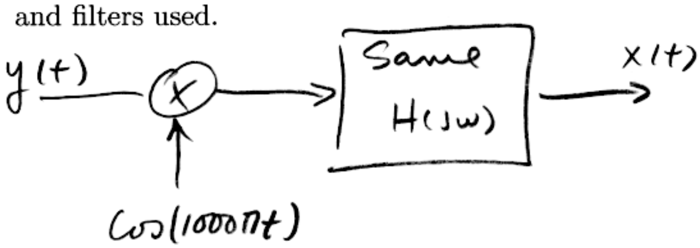
$W(j\omega) = \frac{1}{2}X(j(\omega - 1000\pi)) + \frac{1}{2}X(j(\omega + 1000\pi))$




(b) Plot the Fourier transform $Y(j\omega)$ of the output of the LTI system when its input is $W(j\omega)$ as plotted in part (a).



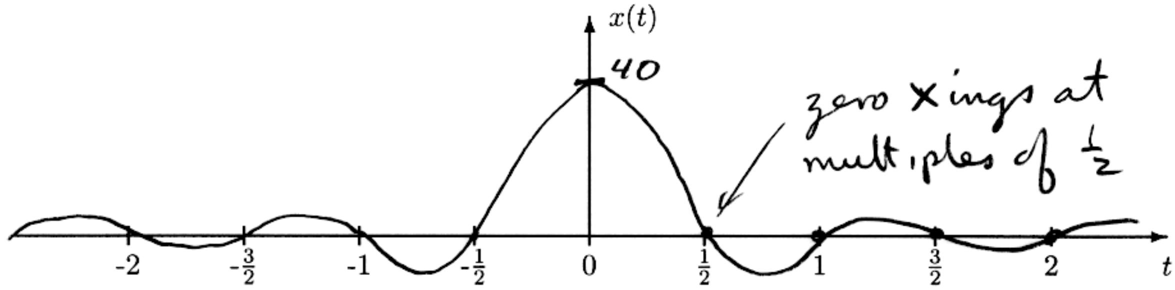

(c) Draw a block diagram of a system to recover $x(t)$ from the signal $y(t)$. Specify all modulators and filters used.




i.e. just run $y(t)$ thru the same system

**Problem fall-99-F.5:**

(a) Consider the signal $x(t) = \dfrac{20\sin(2\pi t)}{\pi t}$. Make a carefully labeled sketch of $x(t)$ in the space below.

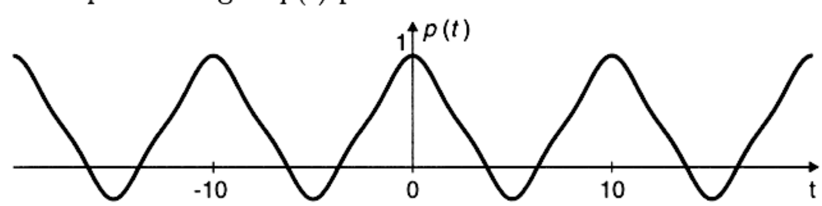



(b) Determine the Fourier transform of $y(t) = x(t-2)$.

$$Y(j\omega) = X(j\omega)e^{-j\omega 2}$$

$$= \begin{cases} 20e^{-j\omega 2} & |\omega| < 2\pi \\ 0 & |\omega| > 2\pi \end{cases}$$

(c) Now consider the periodic signal $p(t)$ plotted below:



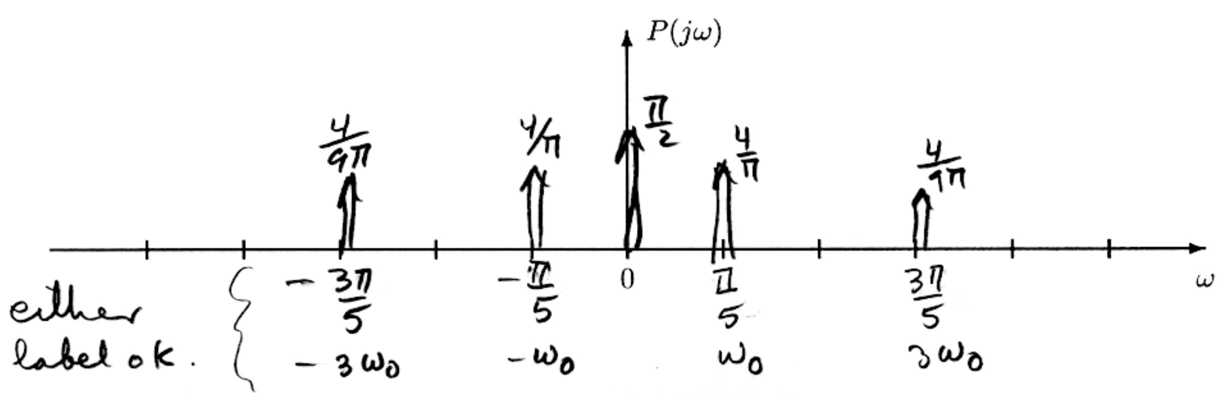

The Fourier series for this input can be simplified to the following form:

$$p(t) = \frac{1}{4} + \frac{4}{\pi^2}\cos(\omega_0 t) + \frac{4}{9\pi^2}\cos(3\omega_0 t)$$

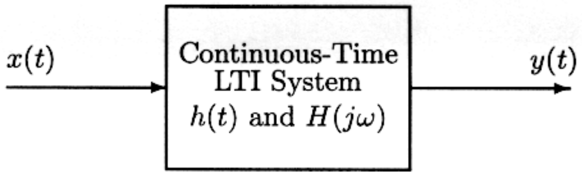
$\boxed{\omega_0 = \underline{2\pi/10 = \pi/5} \text{ rad/sec??}}$

**Either** write an equation for $P(j\omega)$, the Fourier transform of $p(t)$, in the space below, **or** plot it on the axes below. *You must label your plot carefully to receive full credit.*

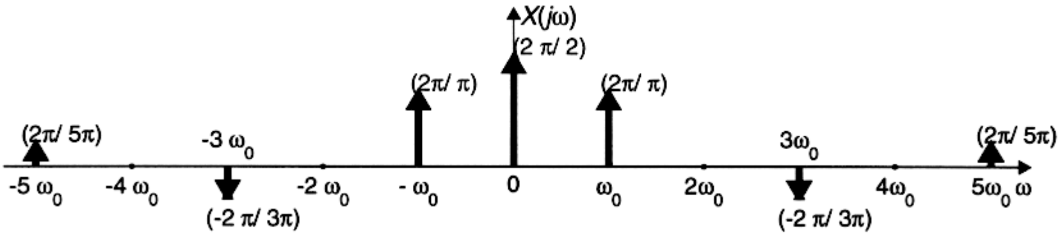
$$P(j\omega) = \frac{\pi}{2}\delta(\omega) + \frac{4}{\pi}\delta(\omega - \omega_0) + \frac{4}{\pi}\delta(\omega + \omega_0) + \frac{4}{9\pi}\delta(\omega - 3\omega_0) + \frac{4}{9\pi}\delta(\omega + 3\omega_0)$$



**Problem fall-99-F.6:**




The periodic input to the above LTI system has Fourier transform $X(j\omega)$ as below:




where the dark arrows denote impulses.

For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform. [*Circle the correct answer. There is only one correct answer in each case.*]

(a) $y(t) = x(t - \frac{1}{2})$: (1) (2) (3) (4) (5) (6) (7)

(b) $y(t) = x(t) - \frac{1}{2}$: (1) (2) (3) (4) (5) (6) (7)

(c) $y(t) = \frac{2}{\pi}\cos(\omega_0 t)$: (1) (2) (3) (4) (5) (6) (7)

(d) $y(t) = \frac{1}{2} + \frac{1}{\pi}\cos(\omega_0 t)$: (1) (2) (3) (4) (5) (6) (7)

(e) $y(t) = \frac{1}{2}$: (1) (2) (3) (4) (5) (6) (7)

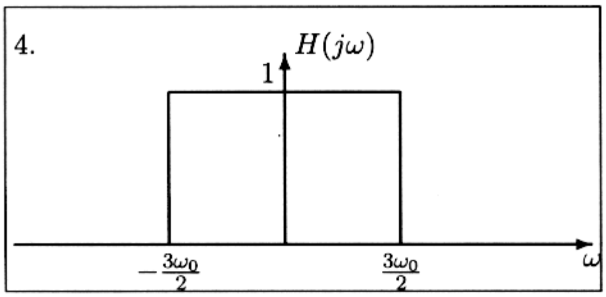
The possible filters are described by the following equations and graphs. *Some of these may not be used.*

1. $H(j\omega) = \begin{cases} 1 & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

2. $H(j\omega) = e^{-j\omega/2}$

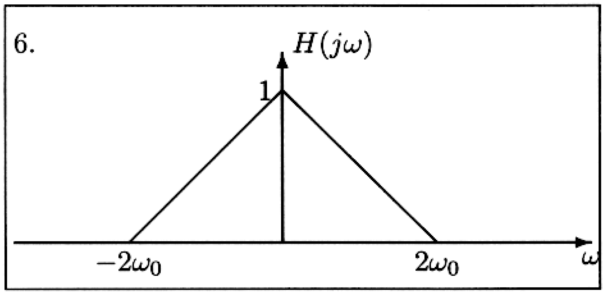
3. $H(j\omega) = \begin{cases} e^{-j\omega/2} & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

4.

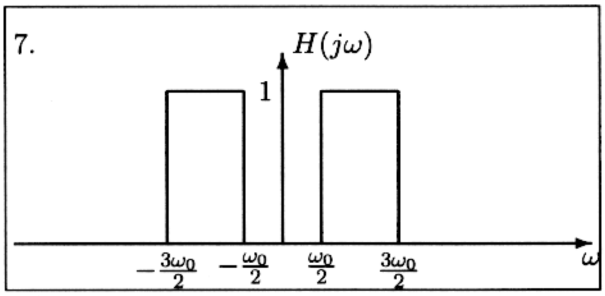



5. $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ 1 & |\omega| > \omega_0/2 \end{cases}$

6.

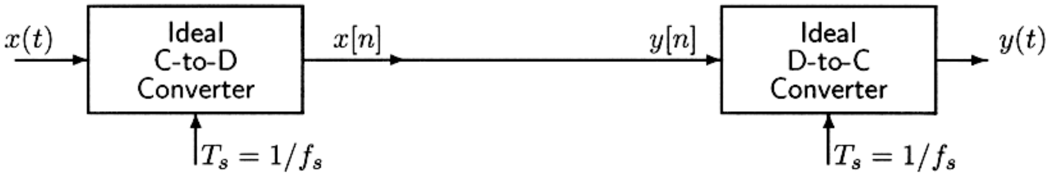



7.



## Problem fall-99-F.7:

Consider the following system for sampling and reconstruction of a continuous-time signal:




(a) In this part, assume that the input to the system is a bandlimited signal whose Fourier transform satisfies the condition $X(j\omega) = 0$ for $|\omega| \geq 200\pi$. For this input signal, what is the *smallest* value of the sampling frequency $f_s$ such that $y(t) = x(t)$?

smallest $f_s =$ 200 samples/sec

(b) Now suppose that the sampling rate is $f_s = 200$ samples/sec and assume that the $\boxed{\textit{output}}$ signal is

$$y(t) = \cos(20\pi t + \pi/4).$$

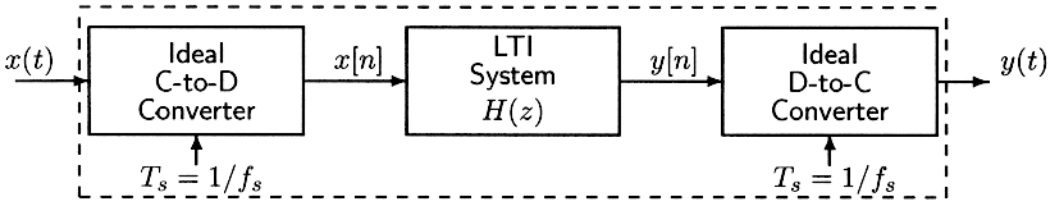
Determine *two different* signals $x_1(t)$ and $x_2(t)$ that could have been the input $x(t)$ that produced this output. *Be sure that the frequencies of both inputs are positive and less than 200 Hz.*

$x_1(t) = \cos(20\pi t + \pi/4)$ – no aliasing

$x_2(t) = \cos(380\pi t - \pi/4)$ – folding

**Problem fall-99-F.8:**

Consider the following system for discrete-time filtering of a continuous-time signal:




(a) In this part, assume that the input to the system is a bandlimited signal whose Fourier transform satisfies the condition $X(j\omega) = 0$ for $|\omega| \geq 200\pi$. Furthermore, assume that the system function of the discrete-time system is

$$H(z) = z^{-1} + z^{-3}.$$

For a sampling rate of $f_s = 200$ samples/sec, we have shown that the overall system (dashed box) behaves as a continuous-time LTI system; that is, $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$. Give an expression for the overall effective frequency response $H_{\text{eff}}(j\omega)$. *Substitute the appropriate value of $T_s$ into your answer so that you obtain an expression that only depends on $\omega$.*

$$\boxed{H_{\text{eff}}(j\omega) = e^{-j\omega/200}\left(1+e^{-j\omega/100}\right)}$$

$T_s = \frac{1}{200}$

$$H_{\text{eff}}(j\omega) = H(e^{j\omega T_s}) \qquad |\omega| < \frac{\pi}{T_s}$$

$$= e^{-j\omega T_s} + e^{-j3\omega T_s} = \left(1+e^{-j2\omega T_s}\right)e^{-j\omega T_s}$$

$$= 2\cos(\omega T_s)\, e^{-j\omega 2T_s}$$

(b) In this part, assume that the discrete-time system is as defined by the system function $H(z)$ in part (a), and the input is

$$x(t) = 10 + 20\cos(100\pi t) \qquad \text{for } -\infty < t < \infty.$$

Use the effective frequency response $H_{\text{eff}}(j\omega)$ determined in part (a) to determine the output $y(t)$ for $-\infty < t < \infty$.

$$y(t) = 10H_{\text{eff}}(0) + 20\left|H_{\text{eff}}(j100\pi)\right|\cos\left(100\pi t + \angle H_{\text{eff}}(j100\pi)\right)$$

$$= 20 + 20\,\underbrace{\left|1+e^{-j\pi}\right|}_{0}\cos(100\pi t - 2\cdot 100\pi/200)$$

$$= 20$$