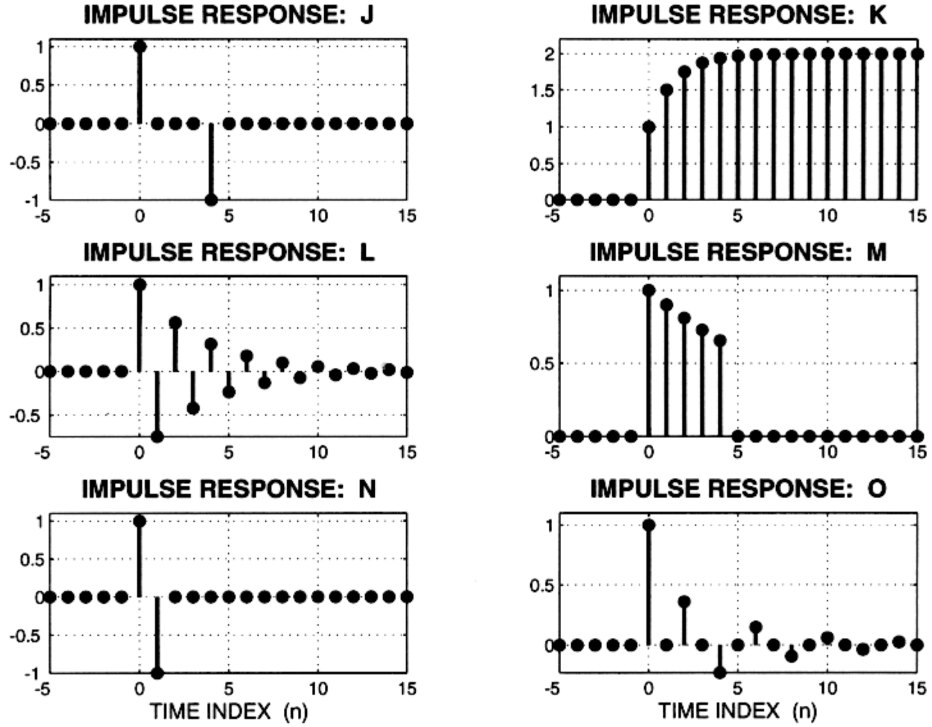
## Problem fall-99-F.1:




For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the impulse response.

$\mathcal{S}_1 :\quad y[n] = -.75y[n-1] + x[n]$

$\mathcal{S}_2 :\quad H(z) = \dfrac{1+z^{-2}}{1+0.64z^{-2}}$

$\mathcal{S}_3 :\quad H(z) = \displaystyle\sum_{k=0}^{4} z^{-k}$

$\mathcal{S}_4 :\quad H(z) = \dfrac{1+z^{-2}}{1-0.75z^{-1}}$

$\mathcal{S}_5 :\quad H(z) = \dfrac{2}{1-z^{-1}} + \dfrac{-1}{1-.5z^{-1}}$

$\mathcal{S}_6 :\quad H(z) = \displaystyle\sum_{k=0}^{4} (0.9)^k z^{-k}$

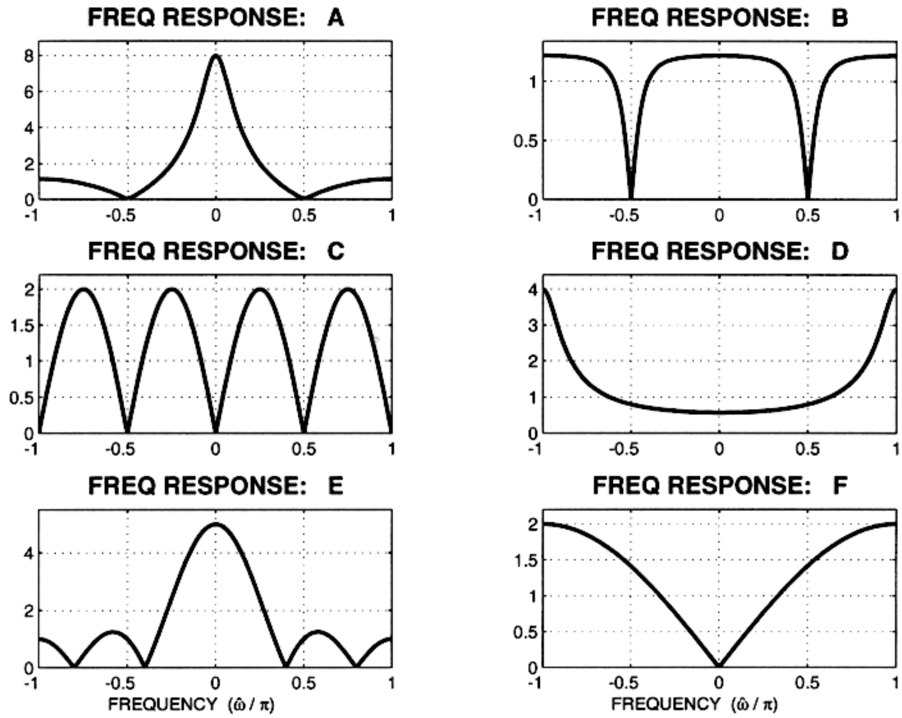
$\mathcal{S}_7 :\quad y[n] = x[n] - x[n-1]$

$\mathcal{S}_8 :\quad H(z) = 1 - z^{-4}$

Mark your answer in the following table:

| IMPULSE RESPONSE | SYSTEM ($\mathcal{S}_\#$) | IMPULSE RESPONSE | SYSTEM ($\mathcal{S}_\#$) |
|---|---|---|---|
| J | 8 | K | 5 |
| L | 1 | M | 6 |
| N | 7 | O | 2 |

**Problem fall-99-F.2:**




For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1: \quad y[n] = -.75y[n-1] + x[n]$

$\mathcal{S}_2: \quad H(z) = \dfrac{1+z^{-2}}{1+0.64z^{-2}}$

$\mathcal{S}_3: \quad H(z) = \displaystyle\sum_{k=0}^{4} z^{-k}$

$\mathcal{S}_4: \quad H(z) = \dfrac{1+z^{-2}}{1-0.75z^{-1}}$

$\mathcal{S}_5: \quad H(z) = \dfrac{2}{1-z^{-1}} + \dfrac{-1}{1-.5z^{-1}}$

$\mathcal{S}_6: \quad H(z) = \displaystyle\sum_{k=0}^{4} (0.9)^k z^{-k}$

$\mathcal{S}_7: \quad y[n] = x[n] - x[n-1]$

$\mathcal{S}_8: \quad H(z) = 1 - z^{-4}$

Mark your answer in the following table:

| FREQUENCY RESPONSE | SYSTEM ($\mathcal{S}_\#$) | FREQUENCY RESPONSE | SYSTEM ($\mathcal{S}_\#$) |
|---|---|---|---|
| A | 4 | B | 2 |
| C | 8 | D | 1 |
| E | 3 | F | 7 |

**Problem fall-99-F.3:**
A discrete-time system is defined by the following system function:

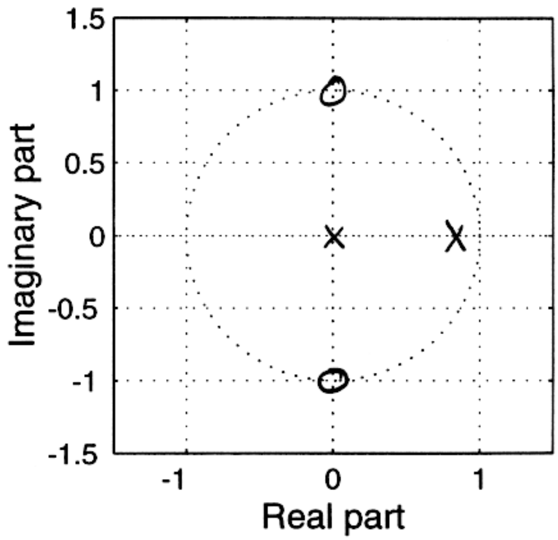
$$H(z) = \frac{1+z^{-2}}{1-0.75z^{-1}} = \frac{1}{1-0.75z^{-1}} + \frac{z^{-2}}{1-0.75z^{-1}}.$$

(a) Use the first form of $H(z)$ to determine *all* the poles and zeros of $H(z)$ and plot them in the $z$-plane.

$$H(z) = \frac{z^2+1}{z(z-.75)} = \frac{(z-j)(z+j)}{z(z-.75)}$$

zeros at $z = \pm j$

Poles at $z = 0, .75$




(b) Use the second form of $H(z)$ above to find the corresponding impulse response $h[n]$.

$$h[n] = (.75)^n u[n] + (.75)^{n-2} u[n-2]$$

(c) Use the first form of $H(z)$ to obtain an expression for the magnitude-squared of the frequency response $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$. Your answer should involve only real quantities.

$$|H(e^{j\hat{\omega}})|^2 = \left(\frac{1+e^{-j2\hat{\omega}}}{1-.75e^{-j\hat{\omega}}}\right)\left(\frac{1+e^{j2\hat{\omega}}}{1-.75e^{j\hat{\omega}}}\right)$$

$$= \frac{1+2\cos(2\hat{\omega})+1}{1-1.5\cos\hat{\omega}+(.75)^2} = \frac{2(1+\cos(2\hat{\omega}))}{1.5625-1.5\cos\hat{\omega}}$$

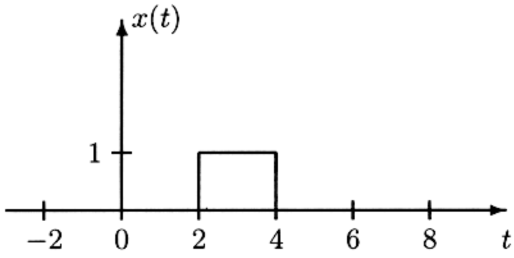
(d) For what value of $\hat{\omega}$ will it be true that $y[n] = 0$ for $-\infty < n < \infty$ when the input to the system is $x[n] = e^{j\hat{\omega}n}$ for $-\infty < n < \infty$?

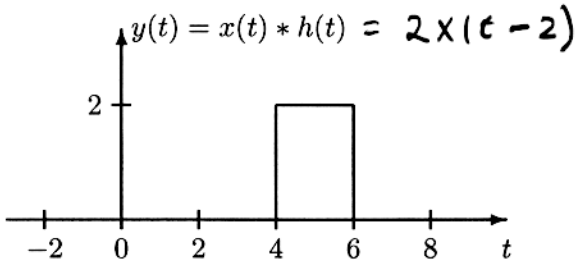
Since we have a zero at $z = \pm j = e^{j\pm\pi/2}$

$$\hat{\omega} = \pm \pi/2$$

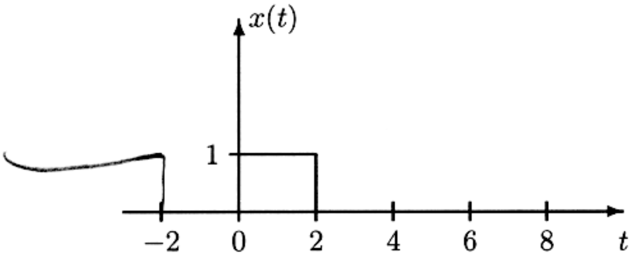
**Problem fall-99-F.4:**

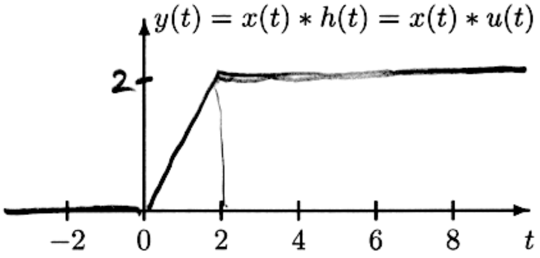
(a) Given that $y(t) = x(t) * h(t)$, find $h(t)$. $h(t) =$ $2\delta(t-2)$



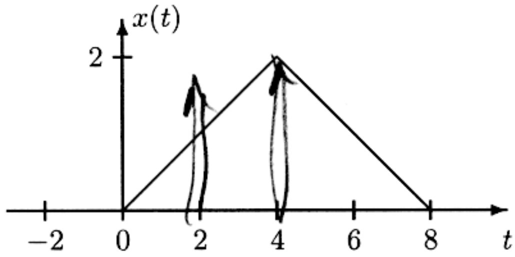



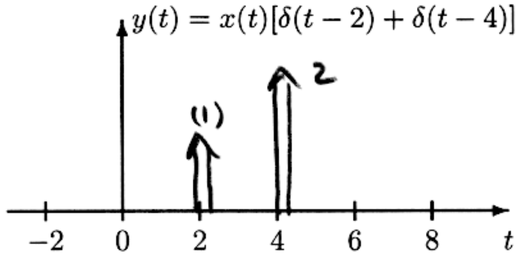

(b) If $h(t) = u(t)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis.*



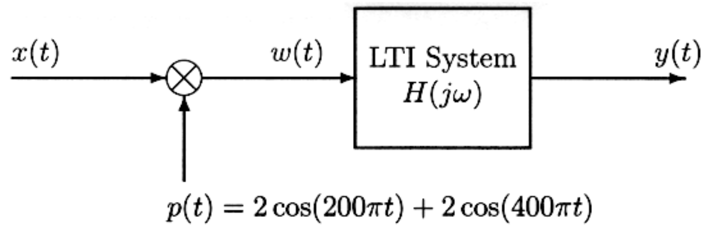

(c) Plot $y(t) = x(t)[\delta(t-2) + \delta(t-4)]$ on the graph on the right.



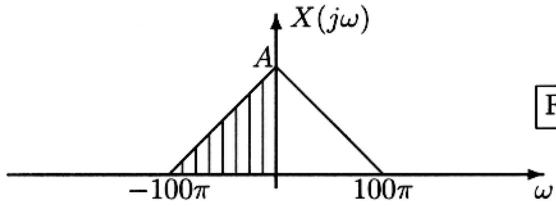




$$
\begin{aligned}
y(t) &= x(t)[\delta(t-2) + \delta(t-4)] \\
&= x(t)\,\delta(t-2) + x(t)\,\delta(t-4) \\
&= x(2)\,\delta(t-2) + x(4)\,\delta(t-4) \\
&= \delta(t-2) + 2\,\delta(t-4)
\end{aligned}
$$

**Problem fall-99-F.5:**




In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted in $\boxed{\text{Figure F.5}}$ below.

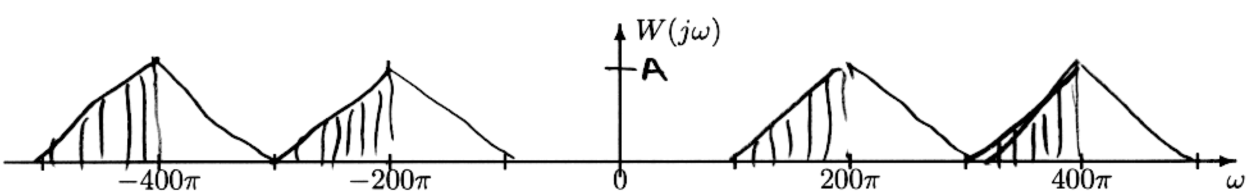



$\boxed{\text{Figure F.5}}$

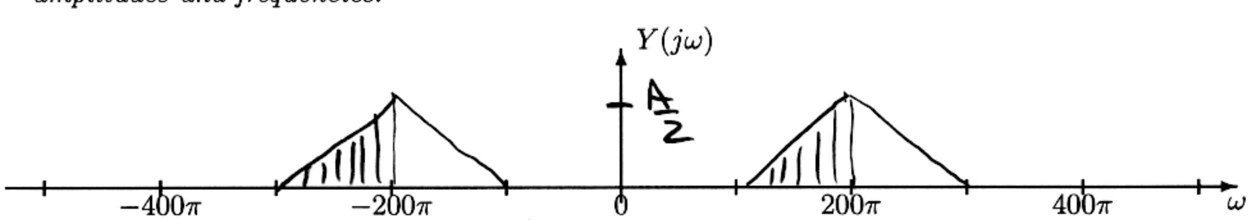
(a) Give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)[2\cos(200\pi t) + 2\cos(400\pi t)]$, in terms of $X(j\omega)$.

$$\boxed{W(j\omega) = X(j(\omega-200\pi)) + X(j(\omega+200\pi)) + X(j(\omega-400\pi)) + X(j(\omega+400\pi))}$$

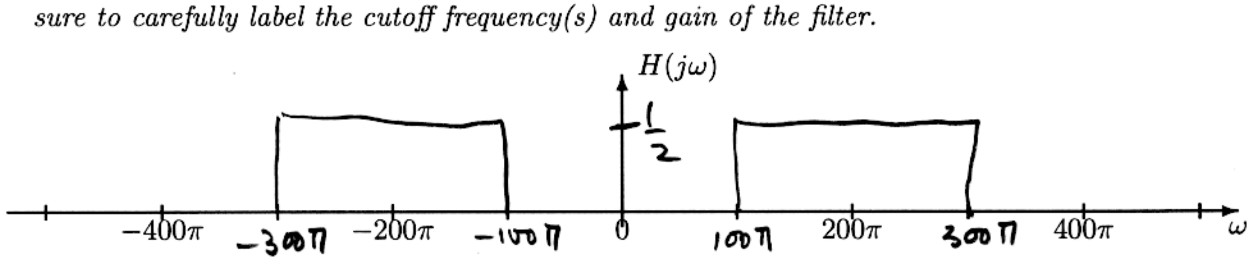
Also, plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above in $\boxed{\text{Figure F.5}}$. *Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*




(b) It is desired that the output of the filter be $y(t) = x(t)\cos(200\pi t)$. Plot the Fourier transform $Y(j\omega)$ below for the $X(j\omega)$ given in $\boxed{\text{Figure F.5}}$ above. *Be sure to carefully label both amplitudes and frequencies.*



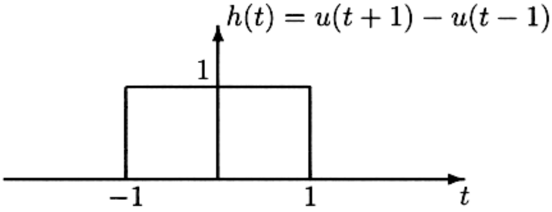

(c) Plot the frequency response $H(j\omega)$ of a filter that is required to obtain $y(t)$ from $w(t)$. *Be sure to carefully label the cutoff frequency(s) and gain of the filter.*




Lowpass also works.

**Problem fall-99-F.6:**

The impulse response of an LTI system is

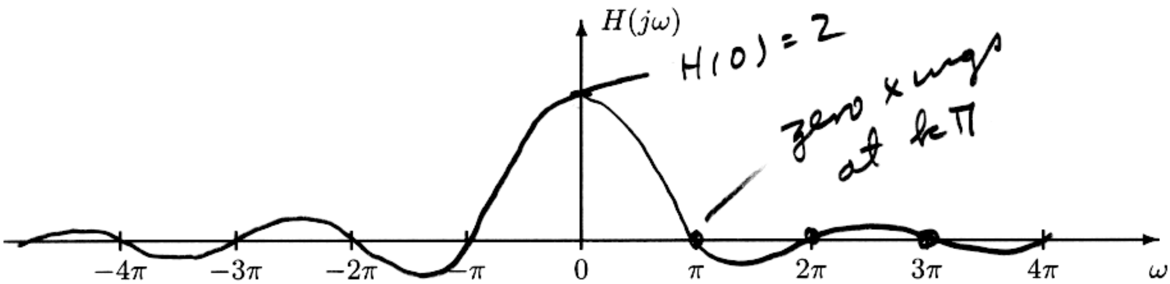



(a) Determine the frequency response $H(j\omega)$ of the system.

$$H(j\omega) = \int_{-1}^{1} e^{-j\omega t}\,dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} = \underline{\underline{\frac{2\sin\omega}{\omega}}}$$

or look up on sheet

(b) Make a carefully labeled sketch of $H(j\omega)$ on the axes below.




(c) Is the LTI system stable? Explain your answer to receive full credit.

Yes it is stable since

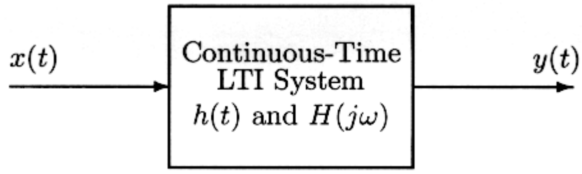
$$\int_{-1}^{1} 1\,dt = 2 < \infty$$

(d) Is the LTI system causal? Explain your answer to receive full credit.

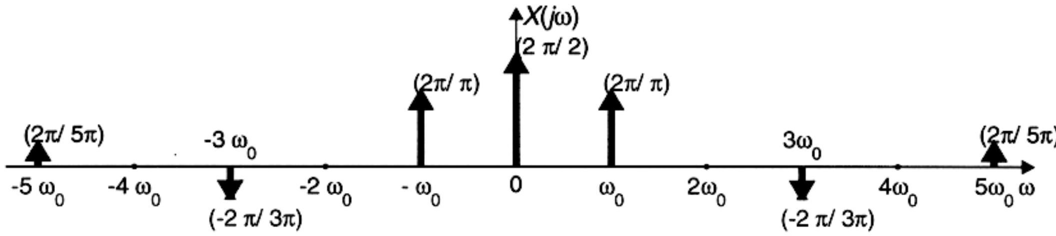
No it is <u>not</u> causal since

$$h(t) \neq 0 \quad t < 0$$

**Problem fall-99-F.7:**




The periodic input to the above LTI system has Fourier transform $X(j\omega)$ as below:




where the dark arrows denote impulses.

For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform. [*Circle the correct answer. There is only one correct answer in each case.*]

(a) $y(t) = x(t - \frac{1}{2})$: (1) (2) (3) (4) (5) (6) (7)

(b) $y(t) = x(t) - \frac{1}{2}$: (1) (2) (3) (4) (5) (6) (7)

(c) $y(t) = \frac{1}{2} + \frac{2}{\pi}\cos[\omega_0(t - \frac{1}{2})]$: (1) (2) (3) (4) (5) (6) (7)

(d) $y(t) = \frac{2}{\pi}\cos(\omega_0 t)$: (1) (2) (3) (4) (5) (6) (7)

(e) $y(t) = \frac{1}{2}$: (1) (2) (3) (4) (5) (6) (7)

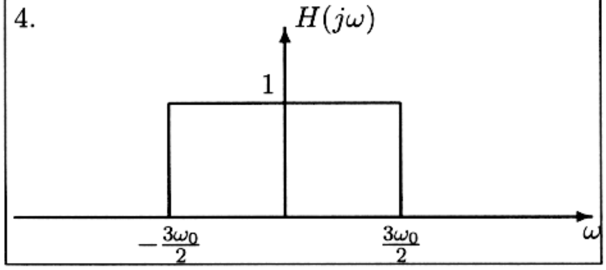
The possible filters are described by the following equations and graphs. *Some of these may not be used.*

1. $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ 1 & |\omega| > \omega_0/2 \end{cases}$

2. $H(j\omega) = e^{-j\omega/2}$

3. $H(j\omega) = \frac{1}{2}[1 + \cos(\omega T_0)]$

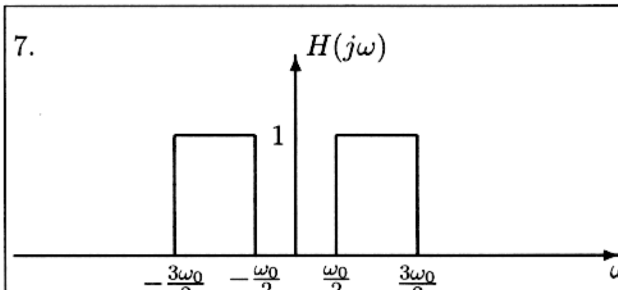
4.




5. $H(j\omega) = \begin{cases} 1 & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

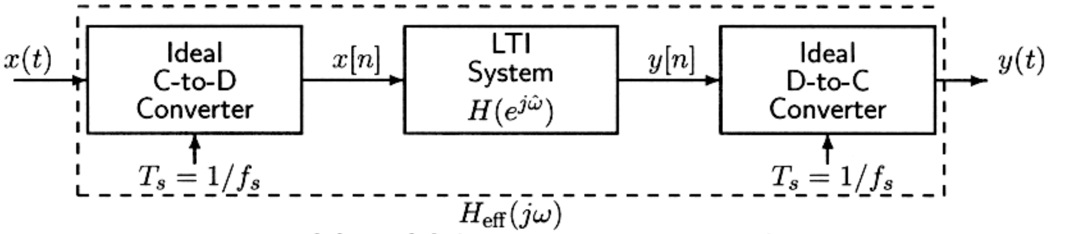
6. $H(j\omega) = \begin{cases} e^{-j\omega/2} & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

7.

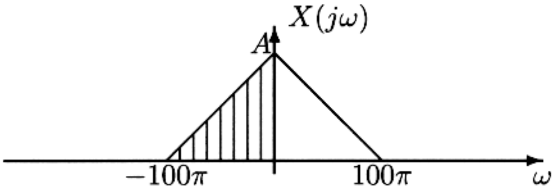


**Problem fall-99-F.8:**

Consider the following system for discrete-time filtering of a continuous-time signal:




(a) In this part, assume that $y[n] = x[n]$ (i.e., the identity system) and assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency $f_s$ such that $y(t) = x(t)$?




smallest $f_s =$ 100 samples/sec

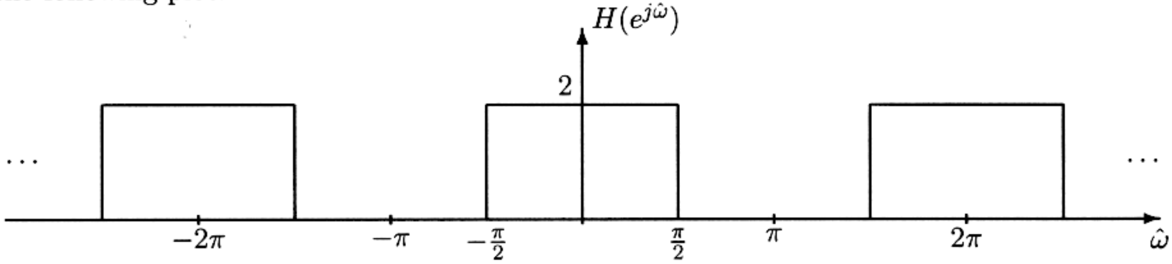
(b) In this part again assume that $y[n] = x[n]$ (i.e., the identity system) but now assume that the input signal is $x(t) = 10\cos(150\pi t + \pi/3)$. If the sampling rate is $\boxed{f_s = 100 \text{ samples/sec,}}$ what is the corresponding output $y(t)$?

$$x[n] = 10\cos(150\pi\, n/100 + \pi/3) = 10\cos(1.5\pi n + \pi/3)$$

$$= 10\cos(2\pi n - .5\pi n + \pi/3) = 10\cos(.5\pi n - \pi/3)$$

$$\therefore\ y(t) = 10\cos(.5\pi \cdot 100t - \pi/3) = 10\cos(50\pi t - \pi/3)$$

(c) In this part, assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot:




Now, if $\boxed{f_s = 200 \text{ samples/sec}}$, make a carefully labeled plot below of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system. Also plot $Y(j\omega)$, the Fourier transform of the output $y(t)$, when the input has Fourier transform $X(j\omega)$ as depicted in the graph of part (a).

$$H_{\text{eff}}(j\omega) = H(e^{j\omega/200})$$