GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2A

DATE: 25-October-1999 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ____________________
LAST, FIRST

Recitation Section: **Circle the date & time when your Recitation Section meets (not Lab):**

L01:Mon-3pm (Williams) L02:Mon-4:30pm (Williams) L03:Tues-9:30pm (Fan)

L04:Thur-9:30am (Altunbasak) L05:Tues-12pm (McClellan) L06:Thur-12pm (Altunbasak)

L07:Tues-1:30pm (DeWeerth) L08:Thur-1:30pm (Hasler) L09:Tues-3pm (DeWeerth) L10:Thur-3pm (Hasler)

L11:Tues-4:30pm (Zhou) L12:Thur-4:30pm (Yezzi) L13:Weds-3pm (Schafer) L14:Weds-4:30pm (Glytsis)

L15:Tues-8am (Fan) L16:Thur-8am (Smith) L17:Weds-8am (Frazier) L18:Weds-9:30am (Frazier)

L19:Tues-6pm (Zhou) L20:Thur-6pm (Yezzi) L21:Mon-6pm (Smith) L22:Weds-6pm (Glytsis)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY, JUSTIFY, JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem fall-99-Q.2.1:
A linear time-invariant system has impulse response

$$h[n] = \delta[n] + \delta[n-1] - \delta[n-3].$$

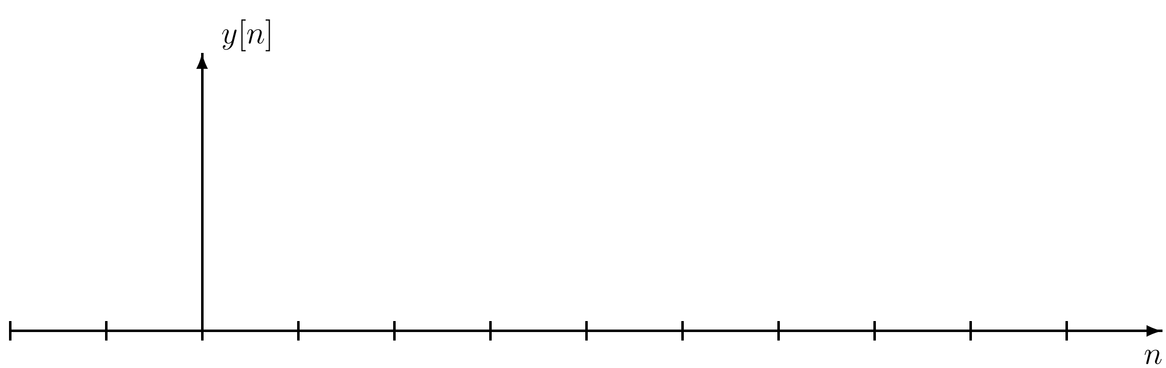
(a) Determinine the difference equation that relates the output $y[n]$ to an input $x[n]$.

(b) Determine the system function $H(z)$ for the system.

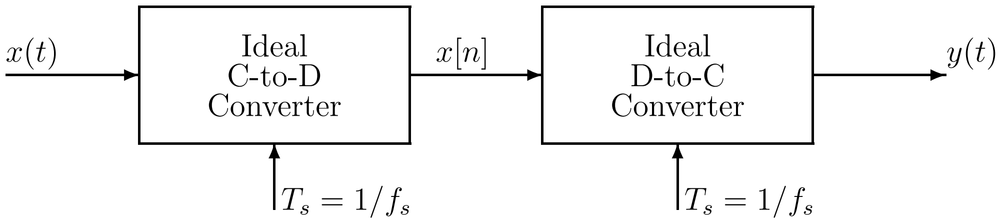
(c) Determine the output $y[n]$ of the system when the input is

$$x[n] = \sum_{k=0}^{3} \delta[n-k].$$

Plot the values of $y[n]$ for $-2 \leq n \leq 9$ on the axis below. *Be sure to label your plot carefully to receive full credit.*



Problem fall-99-Q.2.2:



Suppose that the output of the D-to-C converter in the above system is found to be

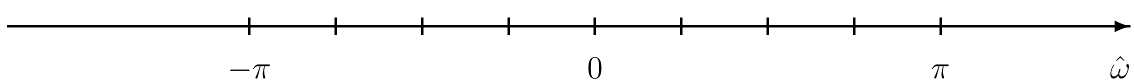
$$y(t) = 5 + 20\cos(2\pi(100)t + \pi/4)$$

when the sampling rate is $f_s = 1/T_s = 400$ samples/second.

(a) Give an equation for $x[n]$ in terms of cosine functions. **Write your answer on the line below.**

Answer: $x[n] =$ ____________________

Plot the spectrum of $x[n]$ for normalized frequencies $-\pi \le \hat{\omega} \le \pi$. **Carefully label and dimension your plot to receive full credit.**



(b) Determine two *different* input signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have produced the given output of the D-to-C converter. **All of the frequencies in your answers must be positive and less than 400 Hz. Give equations for both inputs on the lines below.**

Answer: $x_1(t) =$ ____________________

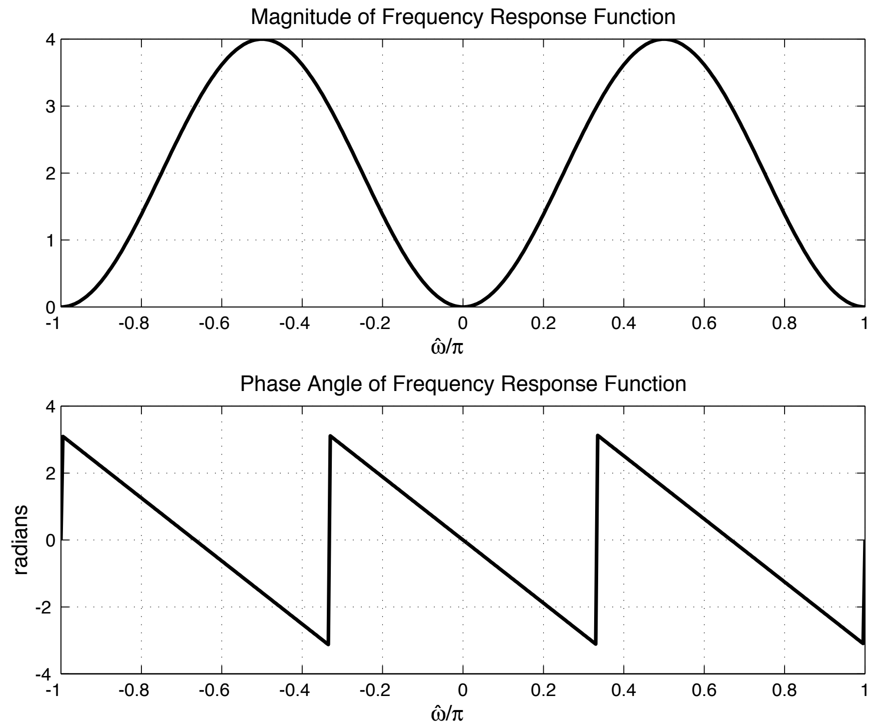
Answer: $x_2(t) =$ ____________________

Problem fall-99-Q.2.3:

A linear time-invariant system is defined by the system function

$$H(z) = -z^{-1} + 2z^{-3} - z^{-5}$$

The magnitude and phase of the frequency response of this system are plotted in the following figure. **Note that the frequency scale is $\hat{\omega}/\pi$.**



(a) This filter is a *lowpass* *bandpass* *highpass* filter. (Circle one.)

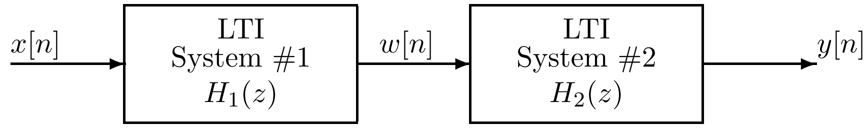
(b) Use the above graph to determine (as accurately as you can) the output $y[n]$ of this system when the input is

$$x[n] = 10 + 10\cos(0.5\pi n).$$

Mark the points on the graph that you used in your solution.

(c) Determine an expression for the frequency response, $H(e^{j\hat{\omega}})$. To receive full credit write your answer in the form $H(e^{j\hat{\omega}}) = A(\hat{\omega})e^{-j\hat{\omega}n_0}$, where $A(\hat{\omega})$ is real and n_0 is an integer.

Problem fall-99-Q.2.4:
Consider the following cascade system:

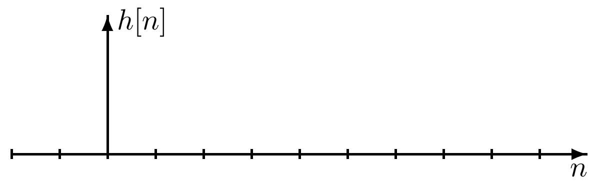


where

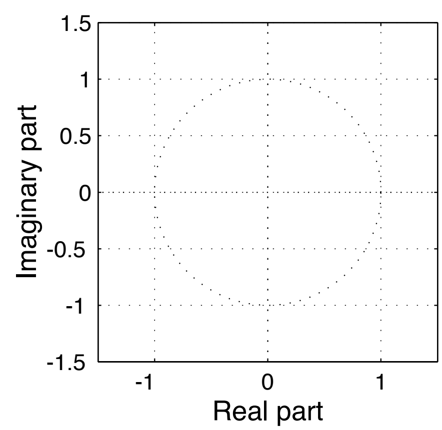
$$H_1(z) = 2 + 2z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}.$$

(a) Determine the system function $H(z)$ of the overall system. Express your answer as a polynomial in z^{-1}.

(b) Determine and plot the impulse response $h[n]$ of the overall system.



(c) Return to your result in part (a). Express $H(z)$ as the product of a constant and three first-order factors each written in the form $(1 - az^{-1})$. From this, determine the zeros and poles of $H(z)$ and plot them in the z-plane plot below.



(d) If the input is $x[n] = Ae^{j\phi}e^{j\hat{\omega}_0 n}$ for $-\infty < n < \infty$, for what values of $-\pi \le \hat{\omega}_0 \le \pi$ will the output be $y[n] = 0$ for $-\infty < n < \infty$?

Problem fall-99-Q.2.1:

A linear time-invariant system has impulse response

$$h[n] = \delta[n] + \delta[n-1] - \delta[n-3].$$

(a) Determinine the difference equation that relates the output $y[n]$ to an input $x[n]$.

$$y[n] = x[n] + x[n-1] - x[n-3]$$

(b) Determine the system function $H(z)$ for the system.

$$H(z) = 1 + z^{-1} - z^{-3}$$

(c) Determine the output $y[n]$ of the system when the input is

$$x[n] = \sum_{k=0}^{3} \delta[n-k].$$

Plot the values of $y[n]$ for $-2 \leq n \leq 9$ on the axis below. *Be sure to label your plot carefully to receive full credit.*

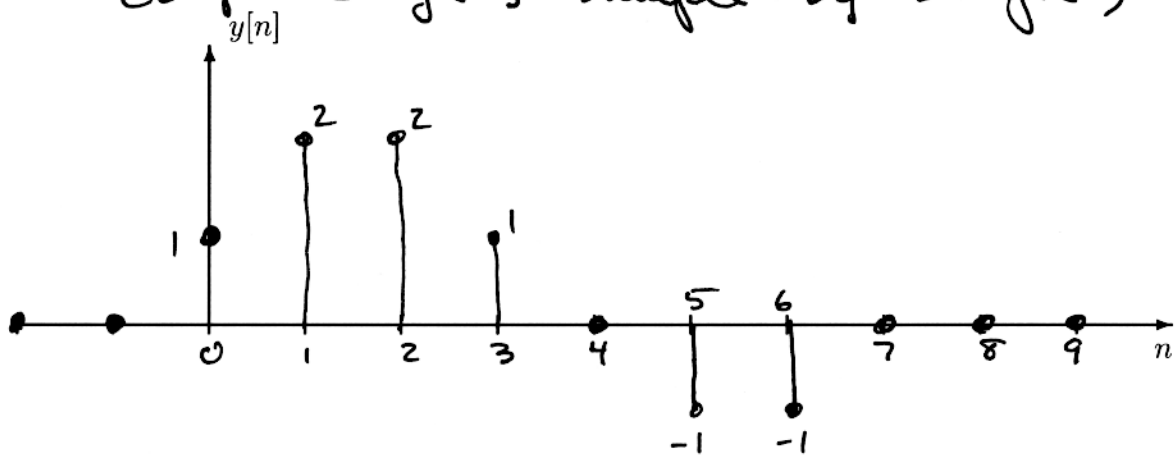
Using z-transforms: $X(z) = 1 + z^{-1} + z^{-2} + z^{-3}$

$H(z) = 1 + z^{-1} - z^{-3}$

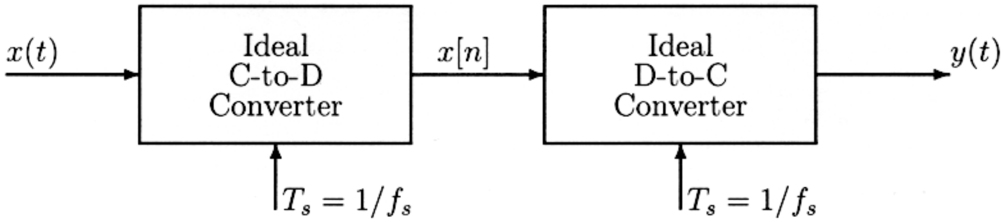
$Y(z) = (1 + z^{-1} - z^{-3})(1 + z^{-1} + z^{-2} + z^{-3})$

$\quad = 1 + 2z^{-1} + 2z^{-2} + z^{-3} - z^{-5} - z^{-6}$

(you could also use the difference equ. to compute $y[n]$ sample-by-sample)



Problem fall-99-Q.2.2:



Suppose that the output of the D-to-C converter in the above system is found to be

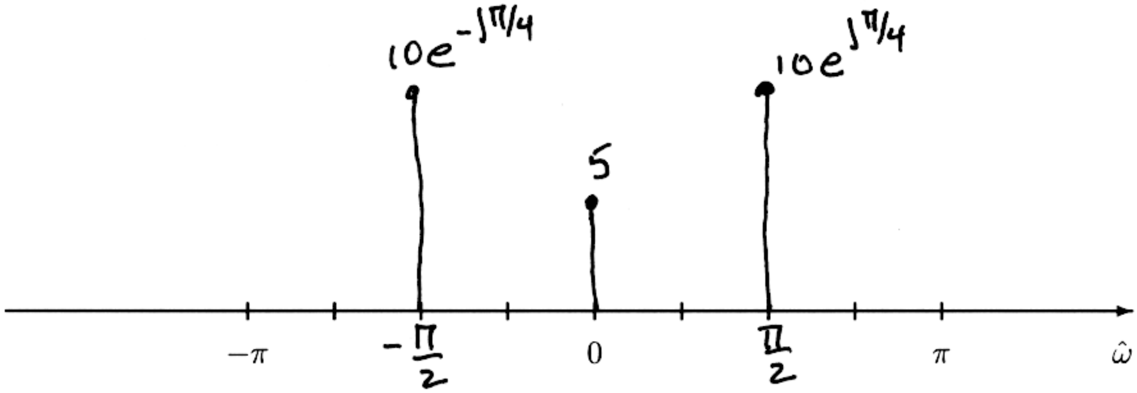
$$y(t) = 5 + 20\cos(2\pi(100)t + \pi/4)$$

when the sampling rate is $f_s = 1/T_s = 400$ samples/second.

(a) Give an equation for $x[n]$ in terms of cosine functions. **Write your answer on the line below.** $x[n] = y(nT_s) = 5 + 20\cos(2\pi(100)\frac{n}{400} + \pi/4)$

Answer: $x[n] =$ $5 + 20\cos(\frac{\pi}{2}n + \pi/4)$

Plot the spectrum of $x[n]$ for normalized frequencies $-\pi \leq \hat{\omega} \leq \pi$. **Carefully label and dimension your plot to receive full credit.**



(b) Determine two *different* input signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have produced the given output of the D-to-C converter. **All of the frequencies in your answers must be positive and less than 400 Hz. Give equations for both inputs on the lines below.**

If no aliasing occurs $y(t) = x(t)$

Answer: $x_1(t) =$ $5 + 20\cos(2\pi(100)t + \pi/4)$

If folding occurs the frequency will be $400 - 100 = 300$

Answer: $x_2(t) =$ $5 + 20\cos(2\pi(300)t - \pi/4)$

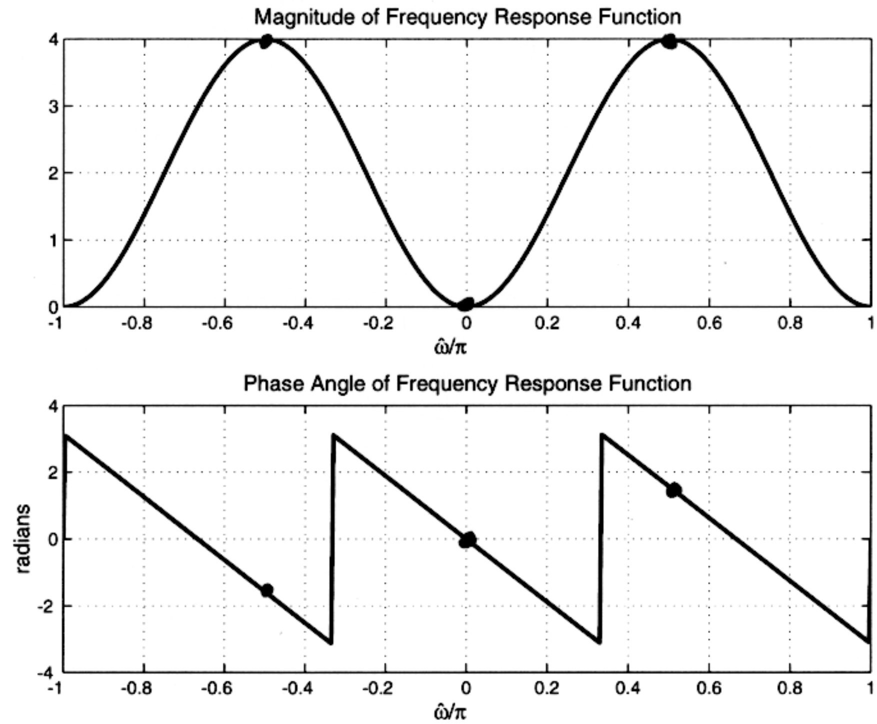
↑ note − sign

Problem fall-99-Q.2.3:

A linear time-invariant system is defined by the system function

$$H(z) = -z^{-1} + 2z^{-3} - z^{-5}$$

The magnitude and phase of the frequency response of this system are plotted in the following figure. **Note that the frequency scale is $\hat{\omega}/\pi$.**



(a) This filter is a *lowpass* *bandpass* *highpass* filter. (Circle one.)

(b) Use the above graph to determine (as accurately as you can) the output $y[n]$ of this system when the input is

$$x[n] = 10 + 10\cos(0.5\pi n). = 10 + 5e^{j.5\pi n} + 5e^{-j.5\pi n}$$

Mark the points on the graph that you used in your solution.

$$y[n] = 10H(e^{j0}) + 5H(e^{j.5\pi})e^{j.5\pi n} + 5H(e^{-j.5\pi})e^{-j.5\pi n}$$

$$= 0 + 5.4e^{j1.6}e^{j.5\pi n} + 5.4e^{-j1.6}e^{-j.5\pi n}$$

$$= 40\cos(.5\pi n + 1.6) \qquad (1.6 \text{ actually} = \pi/2) \text{ see (b)}$$

(c) Determine an expression for the frequency response, $H(e^{j\hat{\omega}})$. To receive full credit write your answer in the form $H(e^{j\hat{\omega}}) = A(\hat{\omega})e^{-j\hat{\omega}n_0}$, where $A(\hat{\omega})$ is real and n_0 is an integer.

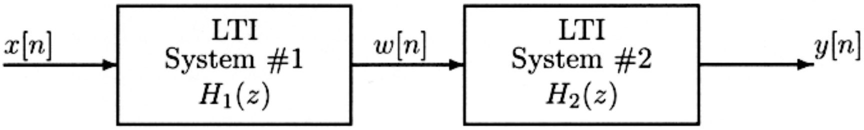
$$H(e^{j\hat{\omega}}) = -e^{-j\hat{\omega}} + 2e^{-j\hat{\omega}3} - e^{-j\hat{\omega}5}$$

$$= (2 - e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})e^{-j\hat{\omega}3}$$

$$= (2 - 2\cos 2\hat{\omega})e^{-j\hat{\omega}3}$$

Problem fall-99-Q.2.4:

Consider the following cascade system:



where

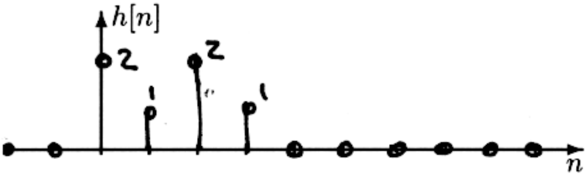
$$H_1(z) = 2 + 2z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}.$$

(a) Determine the system function $H(z)$ of the overall system. Express your answer as a polynomial in z^{-1}.

$$H(z) = H_1(z)\,H_2(z) = (2 + 2z^{-2})(1 + \tfrac{1}{2}z^{-1})$$
$$= 2 + z^{-1} + 2z^{-2} + z^{-3}$$

(b) Determine and plot the impulse response $h[n]$ of the overall system.

$$h[n] = 2\delta[n] + \delta[n-1]$$
$$+ 2\delta[n-2] + \delta[n-3]$$



(c) Return to your result in part (a). Express $H(z)$ as the product of a constant and three first-order factors each written in the form $(1 - az^{-1})$. From this, determine the zeros and poles of $H(z)$ and plot them in the z-plane plot below.

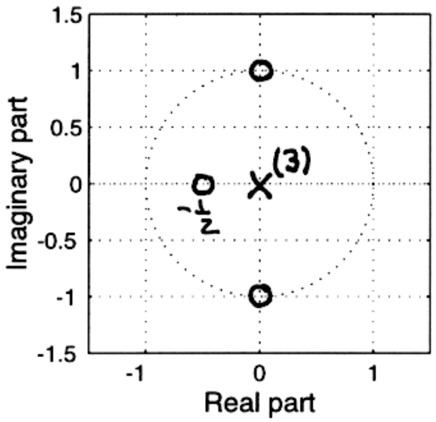
Zeros satisfy $2 + 2z^{-2} = 0$

$2(z^2 + 1)/z^2 = 0 \Rightarrow z = \pm j$

and $1 + \frac{1}{2}z^{-1} = \frac{z + 1/2}{z} \Rightarrow z = -\frac{1}{2}$

There are 3 poles at $z = 0$

$$H(z) = 2(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})$$
$$\cdot (1 + \tfrac{1}{2}z^{-1})$$



(d) If the input is $x[n] = Ae^{j\phi}e^{j\hat{\omega}_0 n}$ for $-\infty < n < \infty$, for what values of $-\pi \le \hat{\omega}_0 \le \pi$ will the output be $y[n] = 0$ for $-\infty < n < \infty$?

$y[n] = 0$ for all n if $\omega_0 = \pm\frac{\pi}{2}$

since $H(e^{\pm j\pi/2}) = 0$