

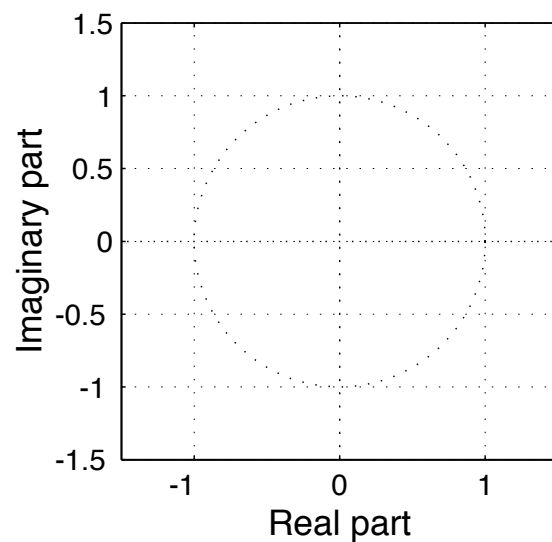
Problem fall-99b-Q.3.1:

A discrete-time system is defined by the following system function:

$$H(z) = \frac{10 + 10z^{-1}}{1 + 0.81z^{-2}}$$

- (a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

- (b) Determine *all* the poles and zeros of $H(z)$ and plot them in the z -plane.



- (c) Fill in numbers for the vectors **bb** and **aa** in the following MATLAB computation of the frequency response of the system:

```
bb=[           ];    aa=[           ];
```

```
omegahat=-pi:pi/200:pi;  
H=freqz(bb,aa,omegahat);
```

Problem fall-99b-Q.3.2:

The system function of a discrete-time LTI system has the following equivalent forms:

$$H(z) = \frac{10z^{-1}}{1 - 0.25z^{-2}} = \frac{10z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} = \frac{10}{1 - 0.5z^{-1}} - \frac{10}{1 + 0.5z^{-1}}$$

- (a) Determine the impulse response of this system; i.e., determine the output $h[n]$ when the input is $\delta[n]$.

- (b) Using the form

$$H(z) = \frac{10z^{-1}}{1 - 0.25z^{-2}},$$

determine an expression for the frequency response as a function of $\hat{\omega}$.

- (c) Use the frequency response function to determine the output $y[n]$ when the input is

$$x[n] = e^{j(\pi/2)n} \quad \text{for } -\infty < n < \infty.$$

Problem fall-99b-Q.3.3:

In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal.

(a) $\cos(20\pi t)\delta(t - .05) =$

(b) $\cos(20\pi t) * \delta(t - 5) =$

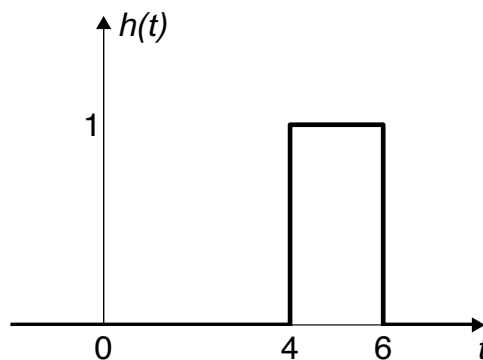
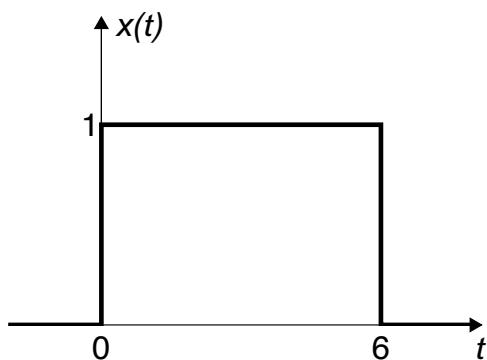
(c) $\int_{-\infty}^{\infty} \delta(\tau - 4)e^{-j\omega\tau}d\tau =$

Problem fall-99b-Q.3.4:

The following figure shows the signal $x(t) = u(t) - u(t - 6)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is the pulse

$$h(t) = \begin{cases} 1 & 4 < t < 6 \\ 0 & \text{otherwise.} \end{cases}$$

The output of the LTI system is $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$.

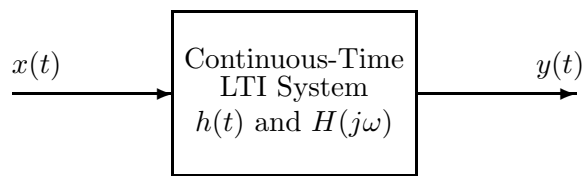


(a) Sketch $h(5 - \tau)$ as a function of τ in the space below.

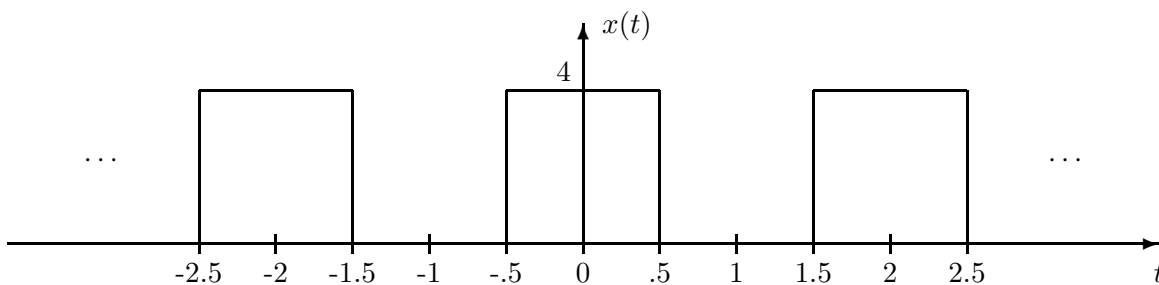
(b) For what values of t can you state with certainty that $y(t) = 0$? **Draw appropriate sketches of $x(\tau)$ and $h(t - \tau)$ to aid your solution.**

(c) Determine the value of $y(t)$ at $t = 5$; that is, determine $y(5)$. **Note carefully: You do not need to evaluate $y(t)$ for all t , only for $t = 5$, and you will not need to “do” any integrals.**

Problem fall-99b-Q.3.5:



The input to the above LTI system is the periodic square wave $x(t)$ depicted below:

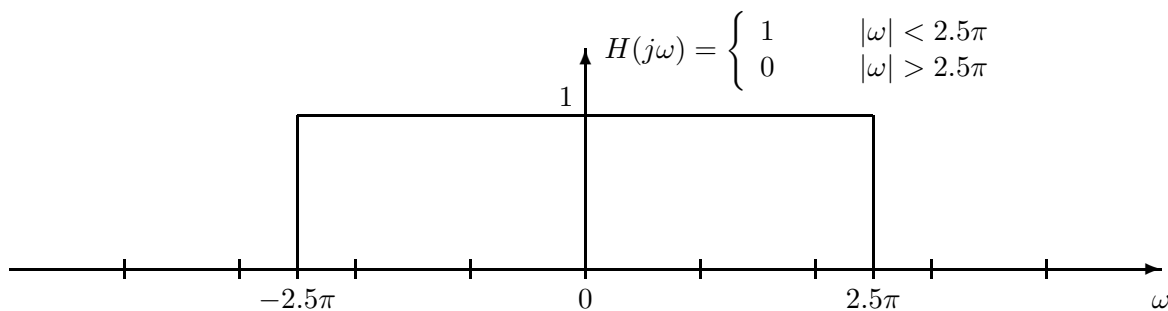


The Fourier series for this input is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where $a_k = \begin{cases} 2 & k = 0 \\ 4 \frac{\sin(\pi k/2)}{\pi k} & k \neq 0. \end{cases}$

(a) Determine the fundamental frequency ω_0 of the input signal $x(t)$. $\omega_0 = \underline{\hspace{2cm}}$ rad/sec

(b) Write the general expression for the Fourier series of the corresponding output $y(t)$. *Knowing that $y(t)$ has Fourier Series coefficients b_k , give an explicit formula for b_k in terms of a_k and the system's frequency response $H(j\omega)$.*

(c) Now, assume the frequency response of the system is the ideal lowpass filter plotted below. Plot the spectrum of the input signal on the same graph; i.e., make a (**carefully labeled**) plot showing the Fourier coefficients a_k plotted at the frequencies $k\omega_0$ for $-3\omega_0 \leq \omega \leq 3\omega_0$.



(d) Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. **Your answer should be expressed in terms of only real quantities.**

Problem fall-99-Q.3.1:

A discrete-time system is defined by the following system function:

$$H(z) = \frac{10 + 10z^{-1}}{1 + 0.81z^{-2}}$$

- (a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

"By inspection"

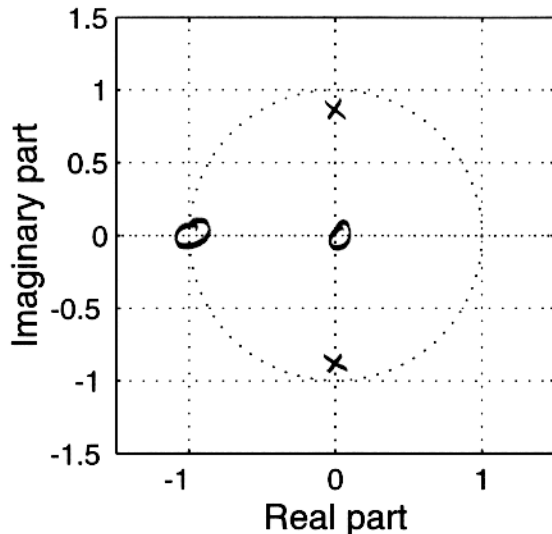
$$y[n] = -0.81 y[n-2] + 10 x[n] + 10 x[n-1]$$

- (b) Determine *all* the poles and zeros of $H(z)$ and plot them in the z -plane.

$$\begin{aligned} H(z) &= \frac{10z(z+1)}{z^2 + 0.81} \\ &= \frac{10z(z+1)}{(z - j.9)(z + j.9)} \end{aligned}$$

poles: $z = \pm j.9$

zeros: $z = 0, -1$



- (c) Fill in numbers for the vectors `bb` and `aa` in the following MATLAB computation of the frequency response of the system:

```
bb=[ 10, 10    ];    aa=[ 1, 0, 0.81 ];
```

```
omegahat=-pi:pi/200:pi;
H=freqz(bb,aa,omegahat);
```

Problem fall-99-Q.3.2:

The system function of a discrete-time LTI system has the following equivalent forms:

$$H(z) = \frac{10z^{-1}}{1 - 0.25z^{-2}} = \frac{10z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} = \frac{10}{1 - 0.5z^{-1}} - \frac{10}{1 + 0.5z^{-1}}$$

- (a) Determine the impulse response of this system; i.e., determine the output $h[n]$ when the input is $\delta[n]$.

Use the partial fraction form and
 $a^n u[n] \iff \frac{1}{1 - az^{-1}}$

$$h[n] = 10(.5)^n u[n] - 10(-.5)^n u[n]$$

- (b) Using the form

$$H(z) = \frac{10z^{-1}}{1 - 0.25z^{-2}},$$

determine an expression for the frequency response as a function of $\hat{\omega}$.

Substitute
 $z = e^{j\hat{\omega}} \Rightarrow H(e^{j\hat{\omega}}) = \frac{10e^{-j\hat{\omega}}}{1 - 0.25e^{-j\hat{\omega}2}}$

- (c) Use the frequency response function to determine the output $y[n]$ when the input is

$$x[n] = e^{j(\pi/2)n} \text{ for } -\infty < n < \infty.$$

$$\begin{aligned} y[n] &= H(e^{j\frac{\pi}{2}}) e^{j\frac{\pi}{2}n} \\ &= \frac{10e^{-j\frac{\pi}{2}}}{1 - 0.25e^{-j\pi}} e^{j\frac{\pi}{2}n} = \frac{10e^{-j\frac{\pi}{2}}}{1 + 0.25} e^{j\frac{\pi}{2}n} \\ &= \frac{10}{1.25} e^{j\frac{\pi}{2}(n-1)} \end{aligned}$$

Problem fall-99-Q.3.3:

In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal.

$$(a) \cos(20\pi t)\delta(t - .05) = \overset{-1}{\cancel{\cos(20\pi(.05))}} \delta(t - .05) = -\delta(t - .05)$$

Uses the property $f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$

$$(b) \cos(20\pi t) * \delta(t - 5) = \cos(20\pi(t - 5))$$

Uses the property $f(t) * \delta(t - t_0) = f(t - t_0)$

$$(c) \int_{-\infty}^{\infty} \delta(\tau - 4)e^{-j\omega\tau} d\tau = e^{-j\omega 4}$$

Uses the properties

$$\int_{-\infty}^{\infty} \delta(\tau - t_0) d\tau = 1 \quad \text{and}$$

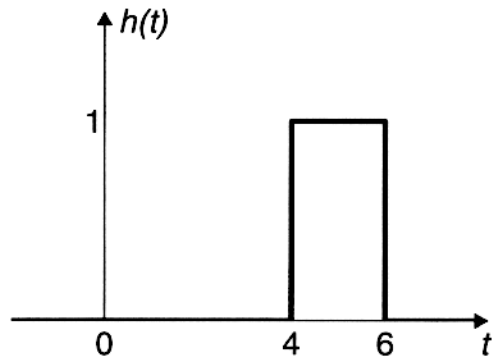
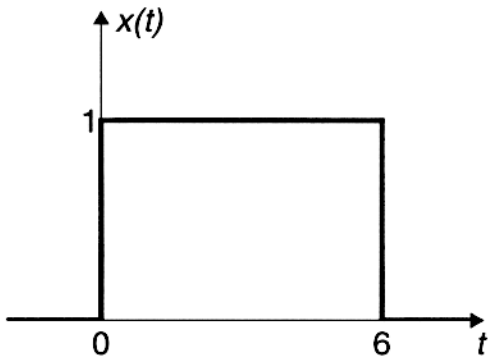
$$f(\tau)\delta(\tau - t_0) = f(t_0)\delta(\tau - t_0)$$

Problem fall-99-Q.3.4:

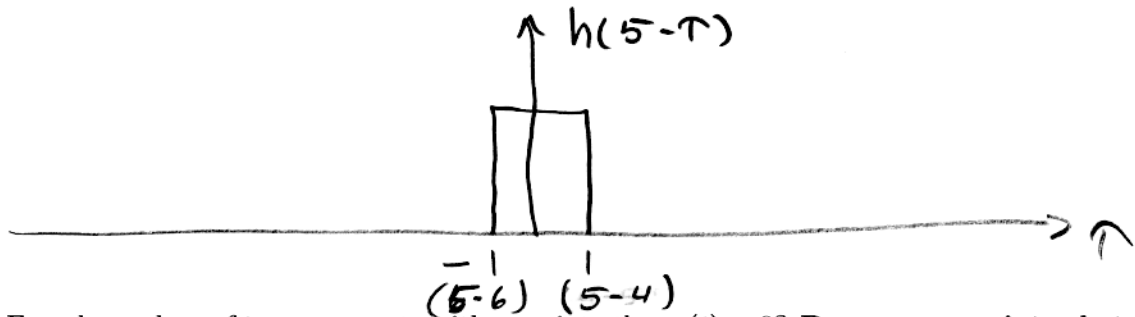
The following figure shows the signal $x(t) = u(t) - u(t-6)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is the pulse

$$h(t) = \begin{cases} 1 & 4 < t < 6 \\ 0 & \text{otherwise.} \end{cases}$$

The output of the LTI system is $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.



(a) Sketch $h(5-\tau)$ as a function of τ in the space below.



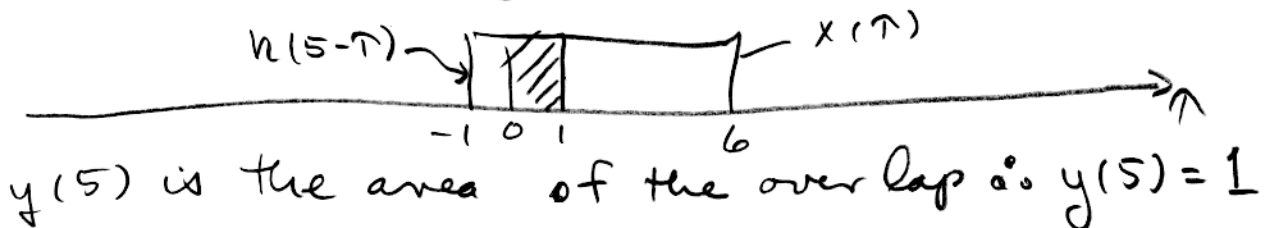
(b) For what values of t can you state with certainty that $y(t) = 0$? Draw appropriate sketches of $x(\tau)$ and $h(t-\tau)$ to aid your solution.

$$t-4 < 0 \Rightarrow t < 4$$

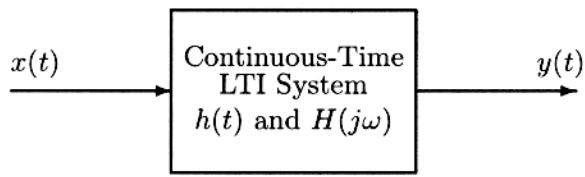
$$t-6 > 6 \Rightarrow t > 12$$



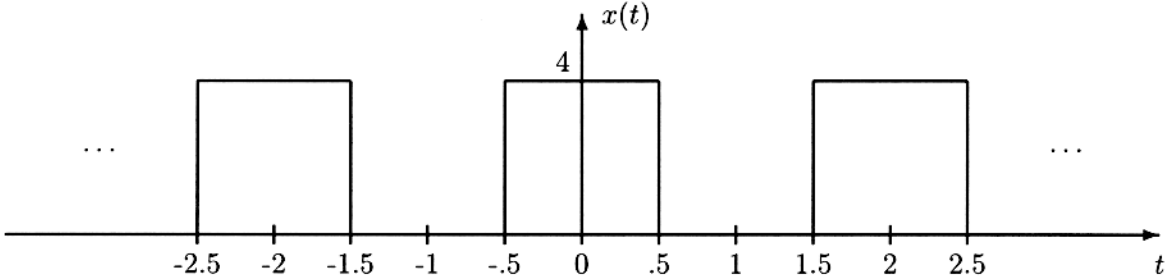
(c) Determine the value of $y(t)$ at $t = 5$; that is, determine $y(5)$. Note carefully: You do not need to evaluate $y(t)$ for all t , only for $t = 5$, and you will not need to “do” any integrals. To determine $y(5)$, plot $x(\tau)$ & $h(5-\tau)$



Problem fall-99-Q.3.5:



The input to the above LTI system is the periodic square wave $x(t)$ depicted below:



The Fourier series for this input is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where $a_k = \begin{cases} 2 & k = 0 \\ 4 \frac{\sin(\pi k/2)}{\pi k} & k \neq 0 \end{cases}$

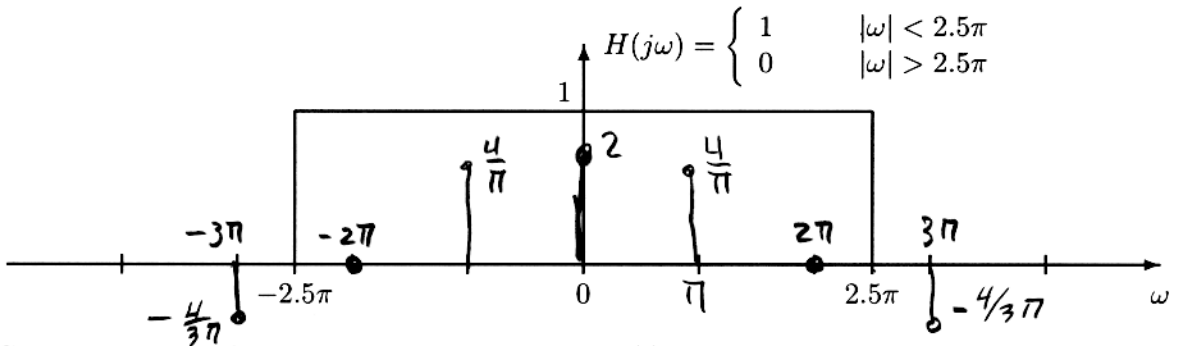
(a) Determine the fundamental frequency ω_0 of the input signal $x(t)$. $\omega_0 = \underline{\underline{\pi}}$ rad/sec

$T_0 = 2 \Rightarrow \omega_0 = 2\pi/2$

(b) Write the general expression for the Fourier series of the corresponding output $y(t)$.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t} \quad b_k = a_k H(jk\omega_0)$$

(c) Now, assume the frequency response of the system is the ideal lowpass filter plotted below. Plot the spectrum of the input signal on the same graph; i.e., make a (**carefully labeled**) plot showing the Fourier coefficients a_k plotted at the frequencies $k\omega_0$ for $-3\omega_0 \leq \omega \leq 3\omega_0$.



(d) Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. Your answer should be expressed in terms of only real quantities.

Since $y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$,

$$y(t) = 2 + \frac{4}{\pi} e^{j\pi t} + \frac{4}{\pi} e^{-j\pi t}$$

$$= 2 + \frac{8}{\pi} \cos(\pi t)$$