GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #3C

DATE: 22-November-1999 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ______________
LAST, FIRST

Recitation Section: **Circle the date & time when your Recitation Section meets (not Lab):**

L01:Mon-3pm (Williams) L02:Mon-4:30pm (Williams) L03:Tues-9:30pm (Fan)

L04:Thur-9:30am (Altunbasak) L05:Tues-12pm (McClellan) L06:Thur-12pm (Altunbasak)

L07:Tues-1:30pm (DeWeerth) L08:Thur-1:30pm (Hasler) L09:Tues-3pm (DeWeerth) L10:Thur-3pm (Hasler)

L11:Tues-4:30pm (Zhou) L12:Thur-4:30pm (Yezzi) L13:Weds-3pm (Schafer) L14:Weds-4:30pm (Glytsis)

L15:Tues-8am (Fan) L16:Thur-8am (Smith) L17:Weds-8am (Frazier) L18:Weds-9:30am (Frazier)

L19:Tues-6pm (Zhou) L20:Thur-6pm (Yezzi) L21:Mon-6pm (Smith) L22:Weds-6pm (Glytsis)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY, JUSTIFY, JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

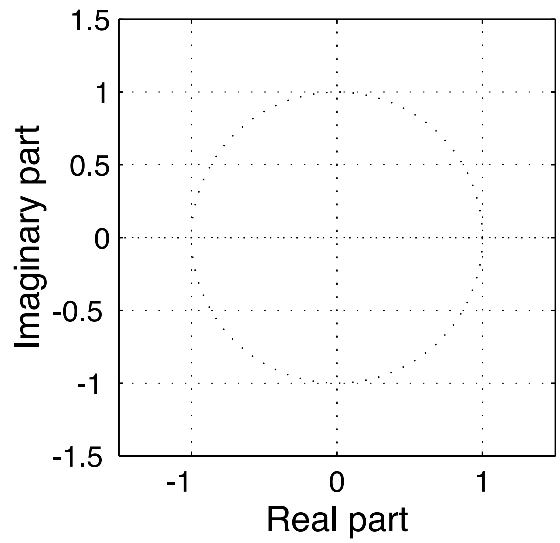
Problem	*Value*	*Score*
1	22	
2	20	
3	18	
4	20	
5	20	

Problem fall-99c-Q.3.1:
A discrete-time system is defined by the following system function:

$$H(z) = \frac{2 - z^{-1}}{1 + 0.49z^{-2}}$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

(b) Determine *all* the poles and zeros of $H(z)$ and plot them in the z-plane.



(c) Fill in numbers for the vectors `bb` and `aa` in the following MATLAB computation of the frequency response of the system:

```
bb=[                    ];     aa=[                    ];

omegahat=-pi:pi/200:pi;
H=freqz(bb,aa,omegahat);
```

Problem fall-99c-Q.3.2:

The system function of a discrete-time LTI system has the following equivalent forms:

$$H(z) = \frac{4 - 4z^{-1}}{1 - 0.25z^{-2}} = \frac{4 - 4z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} = \frac{6}{1 + .5z^{-1}} - \frac{2}{1 - .5z^{-1}}$$

(a) Determine the impulse response of this system; i.e., determine the output $h[n]$ when the input is $\delta[n]$.

(b) Using the form

$$H(z) = \frac{4 - 4z^{-1}}{1 - 0.25z^{-2}},$$

determine an expression for the frequency response as a function of $\hat{\omega}$.

(c) Use the frequency response function to determine the output $y[n]$ when the input is

$$x[n] = e^{j(\pi/2)n} \quad \text{for } -\infty < n < \infty.$$

Problem fall-99c-Q.3.3:
In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal.

(a) $e^{-t}\delta(t - .05) =$

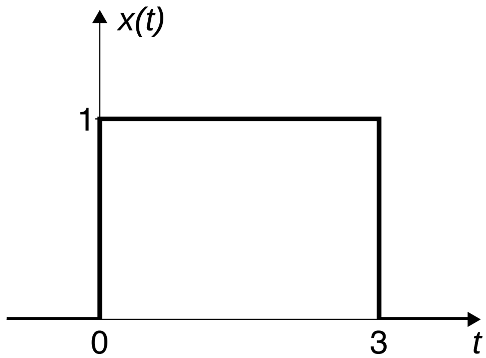
(b) $\delta(t - 2) * \delta(t - 3) =$

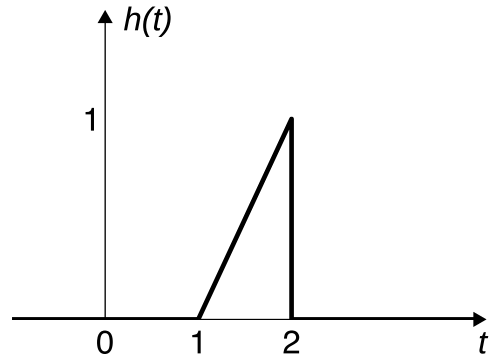
(c) $\int_{-\infty}^{\infty} \delta(\tau - 3)e^{-j\omega\tau} d\tau =$

Problem fall-99c-Q.3.4:
The following figure shows the signal $x(t) = u(t) - u(t-3)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right below) is the triangular function

$$h(t) = \begin{cases} t-1 & 1 < t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

The output of the LTI system is $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.



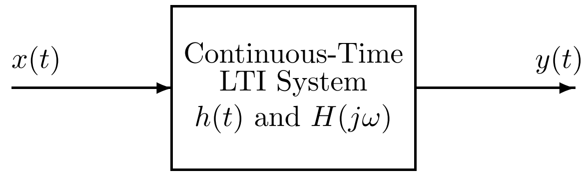


(a) Sketch $h(3-\tau)$ as a function of τ in the space below.

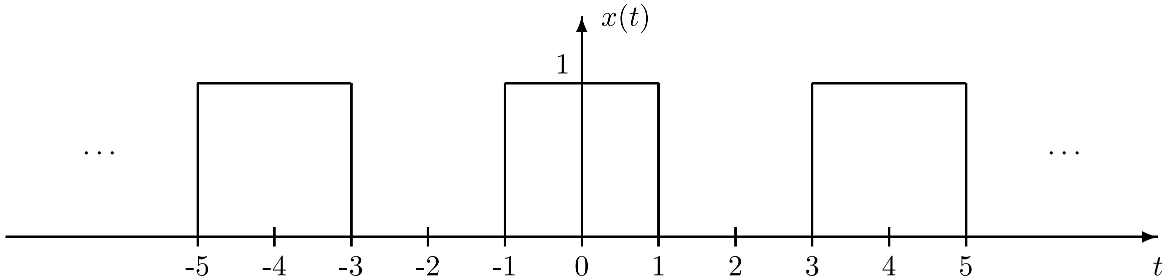
(b) For what values of t can you state with certainty that $y(t) = 0$? **Draw appropriate sketches of $x(\tau)$ and $h(t-\tau)$ to aid your solution.**

(c) Determine the value of $y(t)$ at $t = 3$; that is, determine $y(3)$. **Note carefully: You do not need to evaluate $y(t)$ for all t, only for $t = 3$, and you will not need to "do" any integrals.**

Problem fall-99c-Q.3.5:



The input to the above LTI system is the periodic square wave $x(t)$ depicted below:

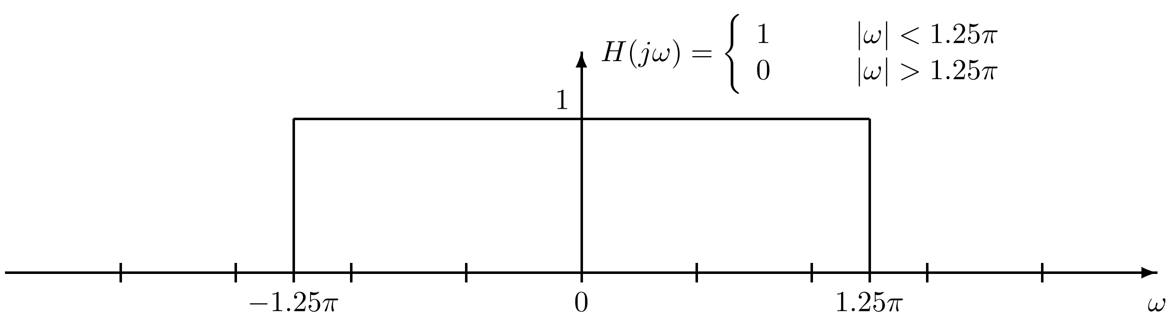


The Fourier series for this input is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where $a_k = \begin{cases} 1 & k = 0 \\ 2\dfrac{\sin(\pi k/2)}{\pi k} & k \neq 0. \end{cases}$

(a) Determine the fundamental frequency ω_0 of the input signal $x(t)$. $\omega_0 =$ __________ rad/sec

(b) Write the general expression for the Fourier series of the corresponding output $y(t)$.*Knowing that $y(t)$ has Fourier Series coefficients b_k, give an explicit formula for b_k in terms of a_k and the system's frequency response $H(j\omega)$.*

(c) Now, assume the frequency response of the system is the ideal lowpass filter plotted below. Plot the spectrum of the input signal on the same graph; i.e., make a **(carefully labeled)** plot showing the Fourier coefficients a_k plotted at the frequencies $k\omega_0$ for $-3\omega_0 \leq \omega \leq 3\omega_0$.



(d) Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. **Your answer should be expressed in terms of only real quantities.**

Problem fall-99-Q.3.1:

A discrete-time system is defined by the following system function:

$$H(z) = \frac{2 - z^{-1}}{1 + 0.49z^{-2}}$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

"By inspection"

$$y[n] = -0.49\, y[n-2] + 2x[n] - x[n-1]$$

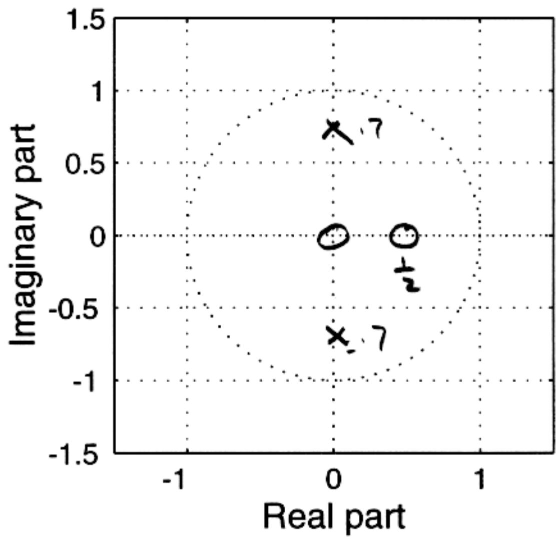
(b) Determine *all* the poles and zeros of $H(z)$ and plot them in the z-plane.

$$H(z) = \frac{2z(z - 1/2)}{z^2 + .49}$$

$$= \frac{2z(z - 1/2)}{(z - j.7)(z + j.7)}$$

poles: $z = \pm j.7$

zeros: $z = 0, 1/2$



(c) Fill in numbers for the vectors bb and aa in the following MATLAB computation of the frequency response of the system:

```
bb=[ 2 , -1 ];     aa=[ 1, 0 , 0.49 ];

omegahat=-pi:pi/200:pi;
H=freqz(bb,aa,omegahat);
```

Problem fall-99-Q.3.2:

The system function of a discrete-time LTI system has the following equivalent forms:

$$H(z) = \frac{4 - 4z^{-1}}{1 - 0.25z^{-2}} = \frac{4 - 4z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} = \frac{6}{1 + .5z^{-1}} - \frac{2}{1 - .5z^{-1}}$$

(a) Determine the impulse response of this system; i.e., determine the output $h[n]$ when the input is $\delta[n]$.

Use the partial fraction form and

$$a^n u[n] \Longleftrightarrow \frac{1}{1 - az^{-1}}$$

$$h[n] = 6(-0.5)^n u[n] - 2(0.5)^n u[n]$$

(b) Using the form

$$H(z) = \frac{4 - 4z^{-1}}{1 - 0.25z^{-2}},$$

determine an expression for the frequency response as a function of $\hat{\omega}$.

Substitute $z = e^{j\hat{\omega}} \Rightarrow H(e^{j\hat{\omega}}) = \dfrac{4 - 4e^{-j\hat{\omega}}}{1 - 0.25e^{-j\hat{\omega}2}}$

(c) Use the frequency response function to determine the output $y[n]$ when the input is

$$x[n] = e^{j(\pi/2)n} \quad \text{for } -\infty < n < \infty.$$

$$y[n] = H(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{2}n}$$

$$= \frac{4 - 4e^{-j\pi/2}}{1 - 0.25e^{-j\pi}} e^{j\frac{\pi}{2}n} = \frac{4\sqrt{2}e^{j\frac{\pi}{4}}}{1 + 0.25} e^{j\frac{\pi}{2}n}$$

$$= \frac{4\sqrt{2}}{1.25} e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$$

Problem fall-99-Q.3.3:
In each of the following cases, simplify the expression using the properties of the continuous-time unit impulse signal.

(a) $e^{-t}\delta(t - .05) =$ $e^{-.05}\delta(t-.05)$

uses the property $f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$

(b) $\delta(t-2) * \delta(t-3) =$ $\delta(t-5)$

uses the property $f(t)*\delta(t-t_0) = f(t-t_0)$

(c) $\int_{-\infty}^{\infty} \delta(\tau - 3)e^{-j\omega\tau}d\tau =$ $e^{-j\omega 3}$

uses the properties

$$\int_{-\infty}^{\infty} \delta(\tau - t_0)d\tau = 1$$ and

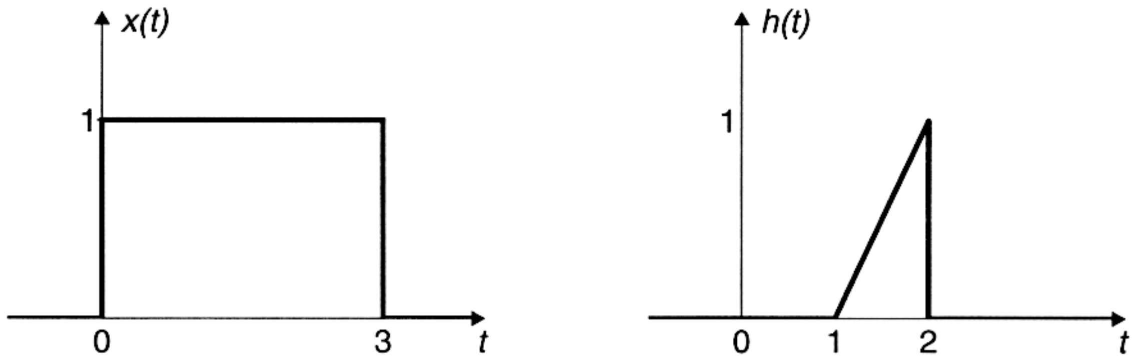
$$f(\tau)\delta(\tau-t_0) = f(t_0)\delta(\tau-t_0)$$

Problem fall-99-Q.3.4:

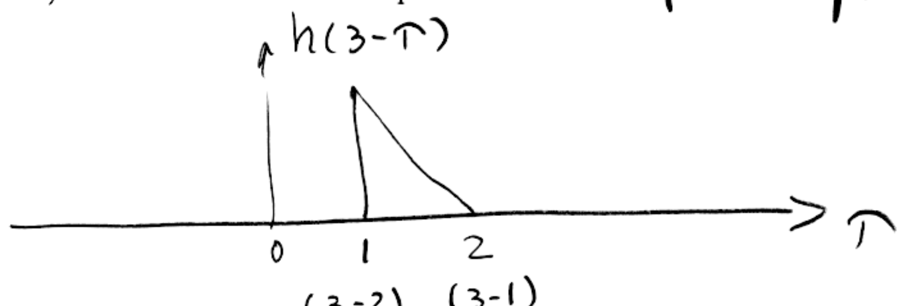
The following figure shows the signal $x(t) = u(t) - u(t-3)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right below) is the triangular function

$$h(t) = \begin{cases} t-1 & 1 < t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

The output of the LTI system is $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.



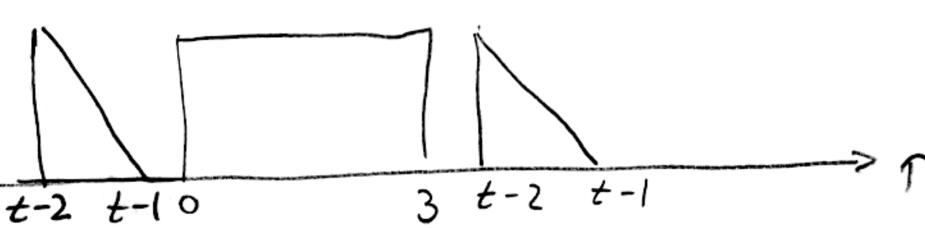
(a) Sketch $h(3-\tau)$ as a function of τ in the space below. "flip & shift



(b) For what values of t can you state with certainty that $y(t) = 0$? **Draw appropriate sketches of $x(\tau)$ and $h(t-\tau)$ to aid your solution.**

$t-1 < 0 \Rightarrow t < 1$ $\qquad$ $t-2 > 3 \Rightarrow t > 5$

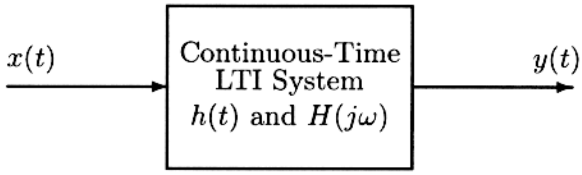
y(t) = 0 for $t < 1$ & $t > 5$



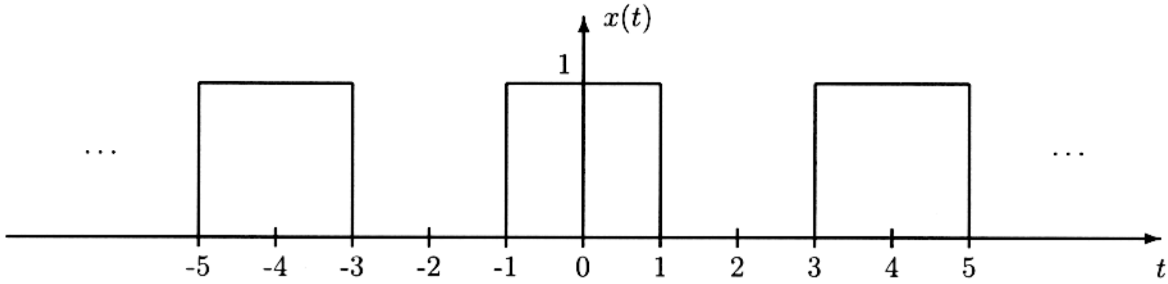
(c) Determine the value of $y(t)$ at $t = 3$; that is, determine $y(3)$. **Note carefully: You do not need to evaluate $y(t)$ for all t, only for $t = 3$, and you will not need to "do" any integrals.** To determine y(3) plot X(T) & h(3-T)

Since h(3-T) overlaps completely y(3) is the area of the triangle: ∴ $y(3) = 1/2$

Problem fall-99-Q.3.5:



The input to the above LTI system is the periodic square wave $x(t)$ depicted below:



The Fourier series for this input is $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where $a_k = \begin{cases} 1 & k = 0 \\ 2\frac{\sin(\pi k/2)}{\pi k} & k \neq 0. \end{cases}$

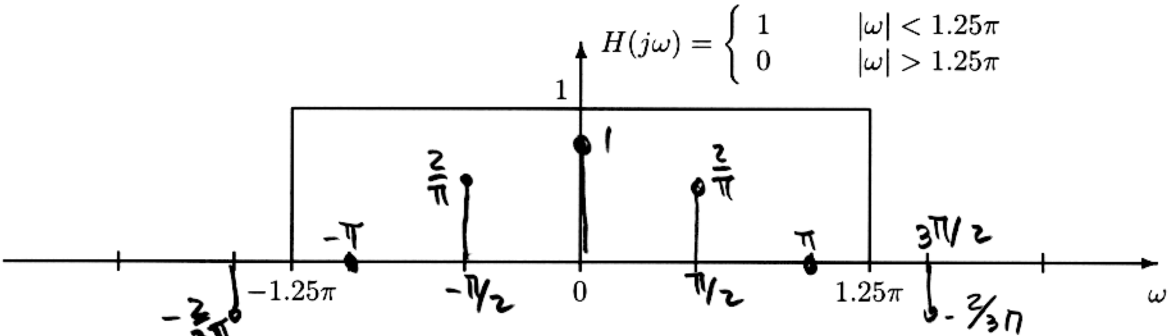
(a) Determine the fundamental frequency ω_0 of the input signal $x(t)$. $\omega_0 =$ $\pi/2$ rad/sec

$T_0 = 4 \Rightarrow \omega_0 = 2\pi/4 = \pi/2$

(b) Write the general expression for the Fourier series of the corresponding output $y(t)$.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t} \qquad b_k = a_k H(jk\omega_0)$$

(c) Now, assume the frequency response of the system is the ideal lowpass filter plotted below. Plot the spectrum of the input signal on the same graph; i.e., make a **(carefully labeled)** plot showing the Fourier coefficients a_k plotted at the frequencies $k\omega_0$ for $-3\omega_0 \leq \omega \leq 3\omega_0$.



(d) Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. **Your answer should be expressed in terms of only real quantities.**

Since $y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$

$$y(t) = 1 + \frac{2}{\pi} e^{j\frac{\pi}{2}t} + \frac{2}{\pi} e^{-j\frac{\pi}{2}t}$$

$$= 1 + \frac{4}{\pi} \cos\left(\frac{\pi}{2}t\right)$$