GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: May 4, 2001 COURSE: ECE 2025

NAME: ______________________________ STUDENT #: ____________________

LAST, FIRST

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

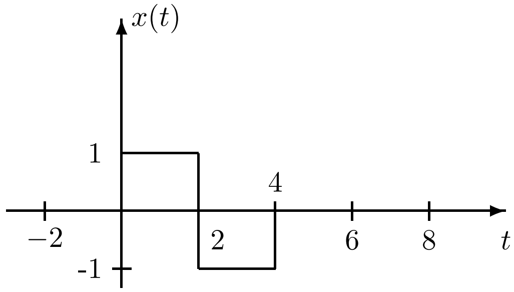
Mon-3p (L11:McClellan)	M-4:30p (L13:Frazier)			
Tues-9:30a (L01:Casinovi)	T-Noon (L03:Casinovi)	T-1:30p (L05:Bordelon)	T-3p (L07:Bordelon)	
Thur-9:30a (L02:Bordelon)	Th-Noon (L04:Bordelon)	Th-1:30p (L06:Smith)	Th-3p (L08:Smith)	Th-4:30p (L09:Casinovi)
				Th-6p (L10:Casinovi)

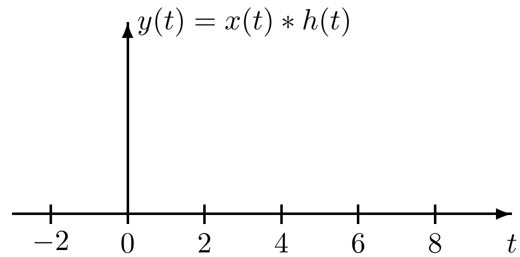
- Write your name on the front page ONLY.
- Closed book, but one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (original only) permitted. OK to write on both sides. A calculator is permitted. **TEAR OFF LAST PAGE OF FOURIER TRANSFORM TABLES.**
- JUSTIFY your reasoning CLEARLY to receive any partial credit. Explanations are also REQUIRED to receive FULL credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
TOTAL	180	

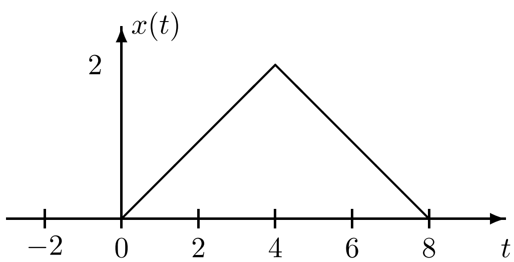
Problem s-01-F.1:

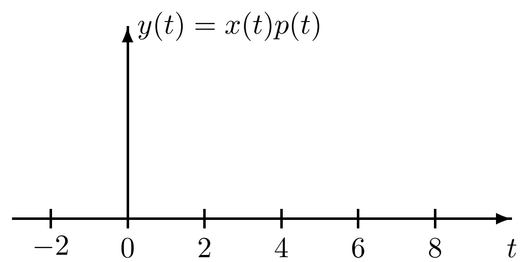
(a) If $h(t) = \delta(t) + \delta(t-2)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis. First write an equation for $y(t)$ in terms of $x(t)$ for the given $h(t)$ in the space below the graph. Then plot the graph.*





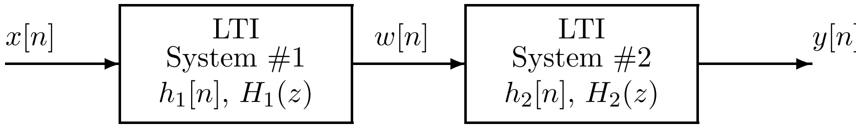
(b) Determine the simplest form for $w(t) = x(t)p(t)$ when $p(t) = \delta(t+2) + \delta(t-2)$ and plot $w(t)$ on the graph on the right. *First write the simplified equation for $w(t)$ in the space below the graph. Then plot $w(t)$.*





Problem s-01-F.2:

A cascade of two discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 + z^{-2}) \quad \text{and} \quad h_2[n] = (0.5)^{n-1}u[n-1].$$

(a) If the input to the first system is

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2],$$

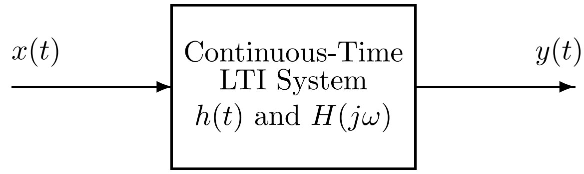
determine the output, $w[n]$, of the **first** system.

$w[n] =$

(b) Determine the system function $H(z)$ of the overall system.

$H(z) =$

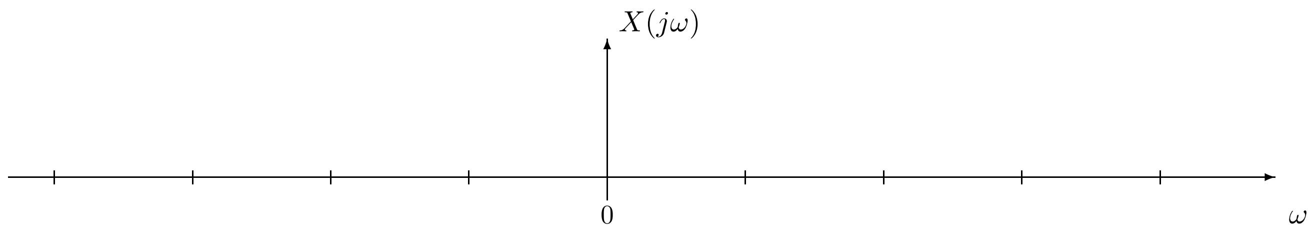
Problem s-01-F.3:



The periodic input to the above system is defined by the equation:

$$x(t) = \sum_{k=-4}^{4} a_k e^{j200\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1}{\pi|k|^2} & k \neq 0 \\ 1 & k = 0 \end{cases}.$$

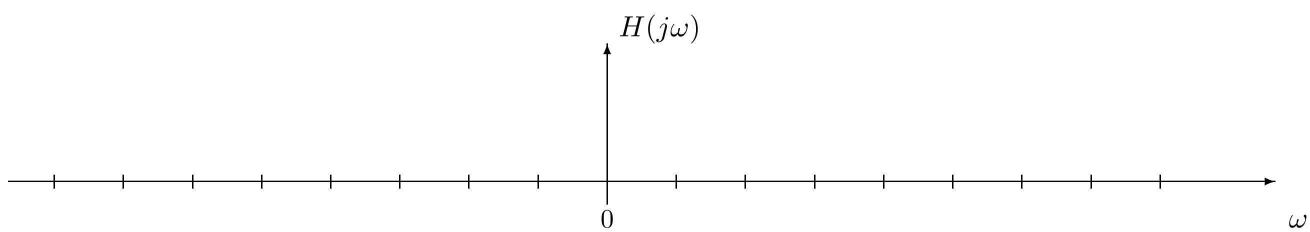
(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot carefully to receive full credit.



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \begin{cases} 10 & |\omega| \leq 300\pi \\ 0 & |\omega| > 300\pi \end{cases}.$$

Plot this function on the graph below using the same frequency scale as the plot in part (a). *Note carefully what type of filter (i.e., lowpass, bandpass, highpass) this is.*



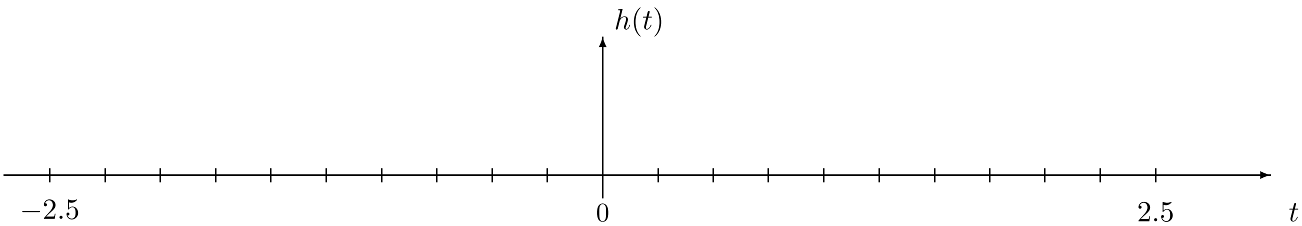
(c) Write an equation for $y(t)$. In order to get full credit you must simplify it to include only cosine functions.

Problem s-01-F.4:

A linear time-invariant system has impulse response

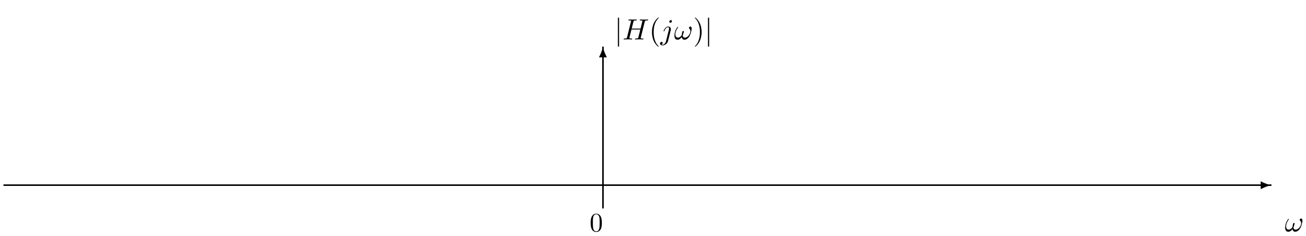
$$h(t) = 10\frac{\sin(2\pi(t-1))}{\pi(t-1)}.$$

(a) Sketch $h(t)$ very carefully on the graph below for the indicated range of values of t. Label the maximum value and the points where $h(t) = 0$.

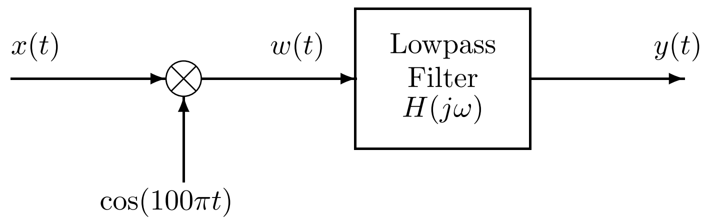


(b) Determine the frequency response $H(j\omega)$ of this system.

(c) Then plot $|H(j\omega)|$ for this system on the graph below. Label your plot carefully to receive full credit.



Problem s-01-F.5:



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform, $X(j\omega)$, as depicted in Figure A below.

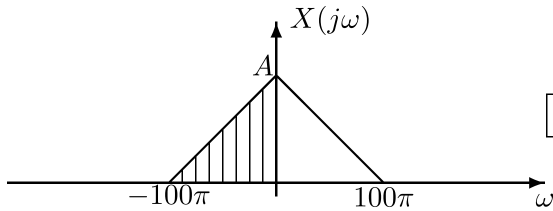
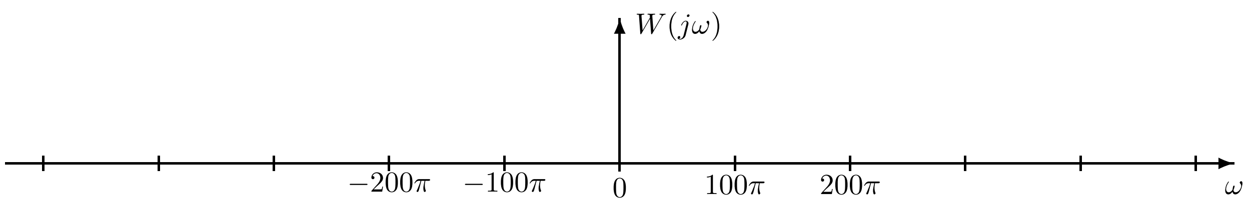


Figure A

(a) First give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)\cos(100\pi t)$, in terms of $X(j\omega)$.

$W(j\omega) =$ ______________________________

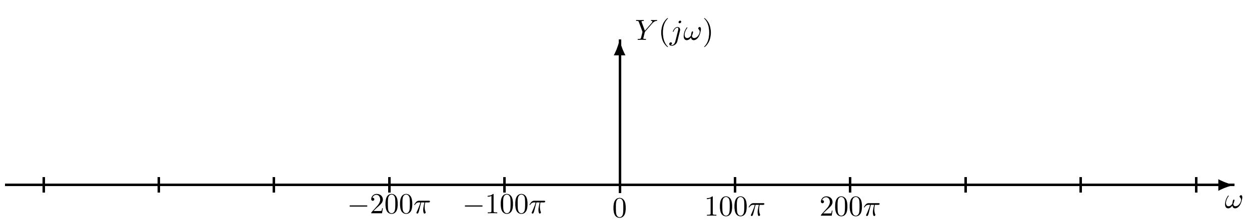
(b) Now **carefully** plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above in Figure A. *Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*



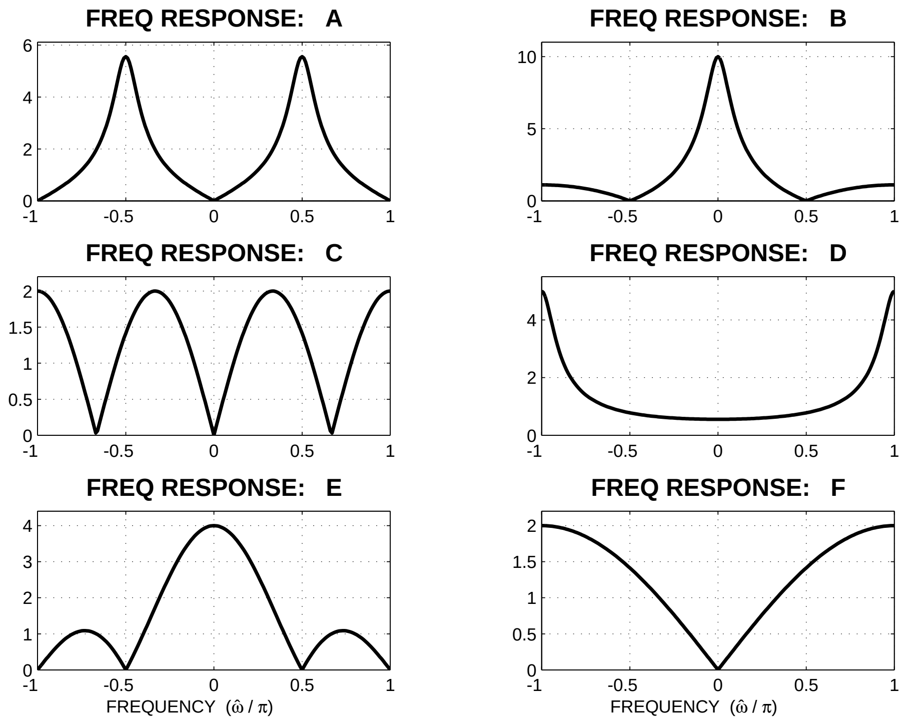
(c) The frequency response of the lowpass filter is

$$H(j\omega) = \begin{cases} 2 & |\omega| \le 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$

Plot the Fourier transform $Y(j\omega)$ below for the $X(j\omega)$ given in Figure F.5 above. *Be sure to carefully label both amplitudes and frequencies and be sure to shade the region corresponding to the negative frequencies of the input.*



Problem s-01-F.6:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$, a difference equation, or a MATLAB statement) matches the frequency response (magnitude only). *There is only ONE correct match per graph.* NOTE: The discrete-time frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $y[n] = -0.8y[n-1] + x[n]$

$\mathcal{S}_2$: `H=freqz([1,0,1],[1,0,0.64],omega)`

$\mathcal{S}_3$: $H(z) = \sum_{k=0}^{3} z^{-k}$

$\mathcal{S}_4$: $H(z) = \dfrac{1+z^{-2}}{1-0.8z^{-1}}$

$\mathcal{S}_5$: $H(z) = 1 + 0.64z^{-2}$

$\mathcal{S}_6$: $H(z) = \dfrac{1-z^{-2}}{1+0.64z^{-2}}$

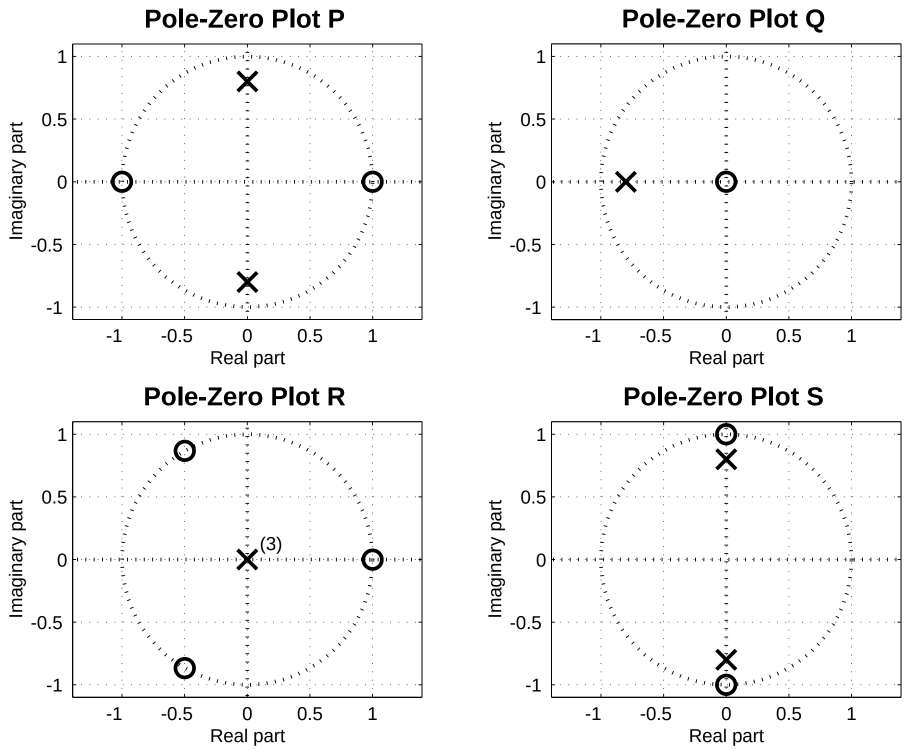
$\mathcal{S}_7$: $y[n] = x[n] - x[n-1]$

$\mathcal{S}_8$: $H(z) = 1 - z^{-3}$

Mark your answer in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_\#$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_\#$)
A		B	
C		D	
E		F	

Problem s-01-F.7:



For each of the pole-zero plots (P, Q, R, S), determine which one of the following systems (specified by either an impulse response $h[n]$, a difference equation, or a MATLAB statement) matches the pole-zero plot. *There is only ONE correct match per plot.*

$\mathcal{S}_1$: $h[n] = \delta[n] + 0.8\delta[n-1]$

$\mathcal{S}_2$: $h[n] = (-0.8)^n u[n]$

$\mathcal{S}_3$: $y[n] = -0.64y[n-2] + x[n] + x[n-2]$

$\mathcal{S}_4$: $y[n] = x[n] - x[n-3]$

$\mathcal{S}_5$: y=filter([1,1,1,1],1,x)

$\mathcal{S}_6$: $y[n] = -0.64y[n-2] + x[n] - x[n-2]$

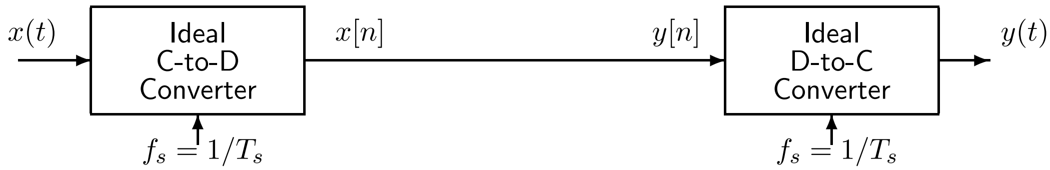
$\mathcal{S}_7$: $h[n] = (-0.8)^n u[n] - (0.8)^n u[n]$

Mark your answer in the following table:

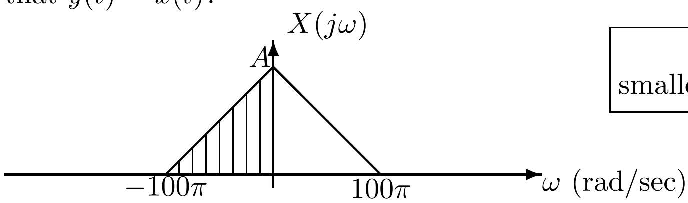
POLE-ZERO PLOT	SYSTEM ($\mathcal{S}_{\#}$)	POLE-ZERO PLOT	SYSTEM ($\mathcal{S}_{\#}$)
P		Q	
R		S	

Problem s-01-F.8:

Consider the following system for sampling and reconstruction of a continuous-time signal:



(a) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that $y(t) = x(t)$?



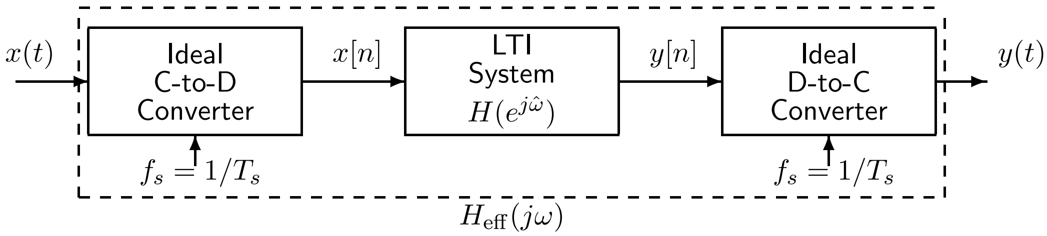
smallest $f_s =$ ______________ samples/sec

(b) In this part, the input signal is $x(t) = 10 + 10\cos(100\pi t + \pi/3)$. If the sampling rate is $f_s = 80$ samples/sec, what is the corresponding output $y(t)$?

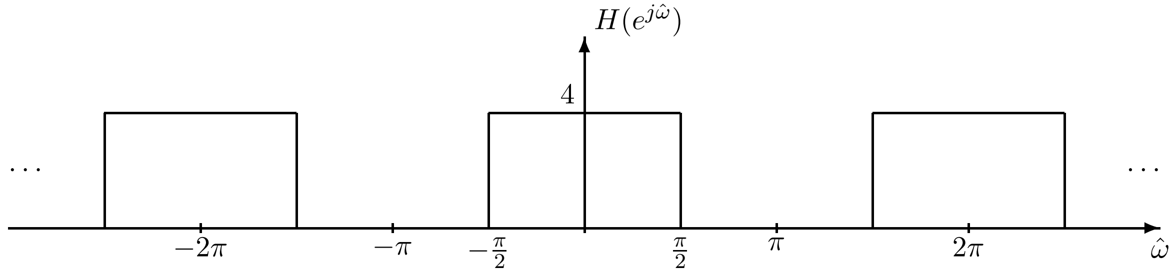
$y(t) =$

Problem s-01-F.9:

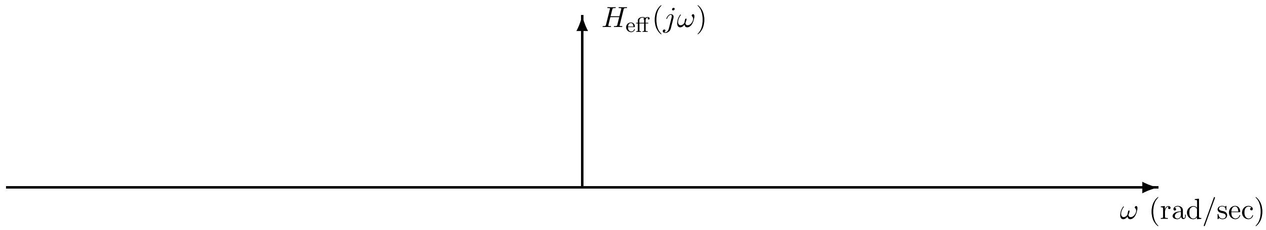
Consider the following system for discrete-time filtering of a continuous-time signal:



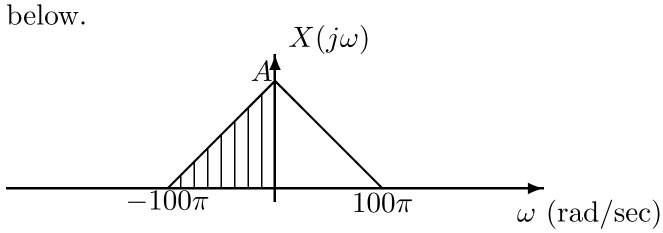
(a) Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot of $H(e^{j\hat{\omega}})$ versus normalized frequency $\hat{\omega}$:



If $f_s = 100$ samples/sec, make a carefully labeled plot below of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system, versus frequency in rads/sec.



(b) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below.



For the system in part (a), what is the smallest sampling rate such that the input signal passes through the lowpass filter unaltered; i.e., what is the minimum f_s such that $Y(j\omega) = X(j\omega)$?

smallest $f_s =$ ____________ samples/sec

Table of Fourier Transform Pairs		
Signal Name	*Time-Domain:* $x(t)$	*Frequency-Domain:* $X(j\omega)$
Right-sided exponential	$e^{-at}u(t) \quad (a>0)$	$\dfrac{1}{a+j\omega}$
Left-sided exponential	$e^{bt}u(-t) \quad (b>0)$	$\dfrac{1}{b-j\omega}$
Square pulse	$[u(t+T/2)-u(t-T/2)]$	$\dfrac{\sin(\omega T/2)}{\omega/2}$
"sinc" function	$\dfrac{\sin(\omega_0 t)}{\pi t}$	$[u(\omega+\omega_0)-u(\omega-\omega_0)]$
Impulse	$\delta(t)$	1
Shifted impulse	$\delta(t-t_0)$	$e^{-j\omega t_0}$
Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
General cosine	$A\cos(\omega_0 t+\phi)$	$\pi A e^{j\phi}\delta(\omega-\omega_0)+\pi A e^{-j\phi}\delta(\omega+\omega_0)$
Cosine	$\cos(\omega_0 t)$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
Sine	$\sin(\omega_0 t)$	$-j\pi\delta(\omega-\omega_0)+j\pi\delta(\omega+\omega_0)$
General periodic signal	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega-k\omega_0)$

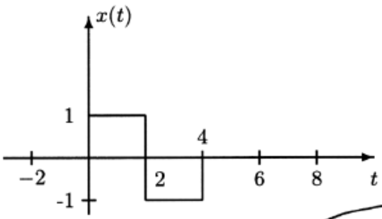
Table of Fourier Transform Properties		
Property Name	*Time-Domain* $x(t)$	*Frequency-Domain* $X(j\omega)$
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(j\omega)+bX_2(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X(j(\omega/a))$
Delay	$x(t-t_d)$	$e^{-j\omega t_d}X(j\omega)$
Modulation	$x(t)e^{j\omega_0 t}$	$X(j(\omega-\omega_0))$
Modulation	$x(t)\cos(\omega_0 t)$	$\frac{1}{2}X(j(\omega-\omega_0))+\frac{1}{2}X(j(\omega+\omega_0))$
Differentiation	$\dfrac{dx^k(t)}{dt^k}$	$(j\omega)^k X(j\omega)$
Convolution	$x(t)*h(t)$	$X(j\omega)H(j\omega)$
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega)*P(j\omega)$

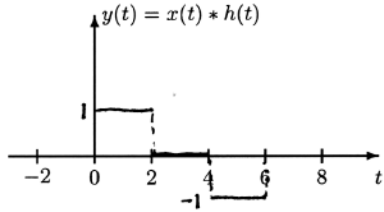
Table of z-Transform Pairs		
Signal Name	*Time-Domain:* $x[n]$	z*-Domain:* $X(z)$
Impulse	$\delta[n]$	1
Shifted impulse	$\delta[n-n_0]$	z^{-n_0}
Right-sided exponential	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$
General cosine	$r^n \cos(\hat{\omega}_0 n)$	$\dfrac{1-r\cos(\hat{\omega}_0)z^{-1}}{1-2r\cos(\hat{\omega}_0)z^{-1}+r^2z^{-2}}$

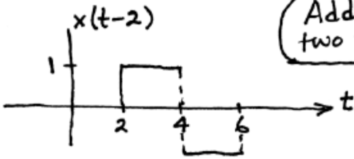
Table of z-Transform Properties		
Property Name	*Time-Domain* $x[n]$	z*-Domain* $X(z)$
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$
Delay	$x[n-n_d]$	$z^{-n_d}X(z)$
Convolution	$x[n]*h[n]$	$X(z)H(z)$

Problem s-01-F.1:

(a) If $h(t) = \delta(t) + \delta(t-2)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis. First write an equation for $y(t)$ in terms of $x(t)$ for the given $h(t)$ in the space below the graph. Then plot the graph.*







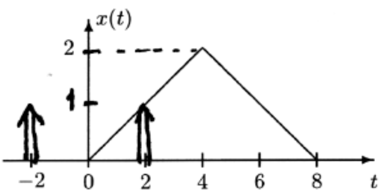
Add these two signals

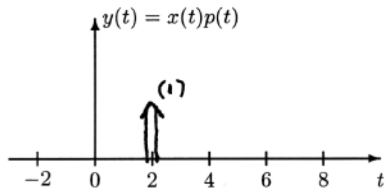
$$y(t) = x(t) * \{\delta(t) + \delta(t-2)\}$$

$$y(t) = x(t) + x(t-2)$$

$$y(t) = \begin{cases} 1 & 0 \le t \le 2 \\ -1 & 4 \le t \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

(b) Determine the simplest form for $w(t) = x(t)p(t)$ when $p(t) = \delta(t+2) + \delta(t-2)$ and plot $w(t)$ on the graph on the right. *First write the simplified equation for $w(t)$ in the space below the graph. Then plot $w(t)$.*

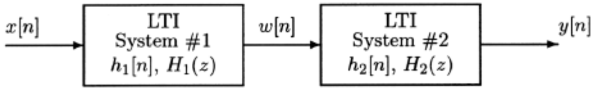




$$\begin{aligned} w(t) &= x(t)[\delta(t+2) + \delta(t-2)] \\ &= x(t)\delta(t+2) + x(t)\delta(t-2) \\ &= x(-2)\delta(t+2) + x(2)\delta(t-2) \\ &= 0 + 1\delta(t-2) \end{aligned}$$

Problem s-01-F.2:

A cascade of two discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 + z^{-2}) \quad \text{and} \quad h_2[n] = (0.5)^{n-1}u[n-1].$$

(a) If the input to the first system is

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2],$$

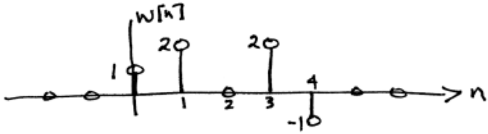
determine the output, $w[n]$, of the **first** system.

Use Z-transforms:

$$X(z) = 1 + 2z^{-1} - z^{-2}$$

$$W(z) = H_1(z)X(z) = (1+z^{-2})(1+2z^{-1}-z^{-2})$$

$$= 1 + 2z^{-1} + 0z^{-2} + 2z^{-3} - z^{-3}$$



$w[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-3] - \delta[n-4]$

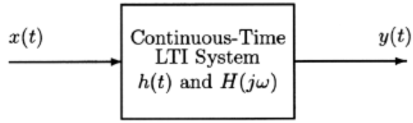
(b) Determine the system function $H(z)$ of the overall system.

$$H(z) = H_1(z)H_2(z) = (1+z^{-2})\left(\frac{z^{-1}}{1-0.5z^{-1}}\right)$$

$$H_2(z) = z^{-1}\left(\frac{1}{1-0.5z^{-1}}\right)$$

$$H(z) = \frac{z^{-1} + z^{-3}}{1 - 0.5z^{-1}}$$

Problem s-01-F.3:



The periodic input to the above system is defined by the equation:

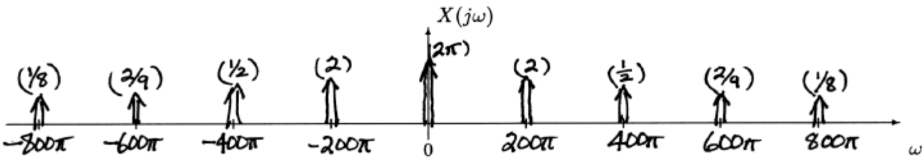
$$x(t) = \sum_{k=-4}^{4} a_k e^{j200\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1}{\pi |k|^2} & k \neq 0 \\ 1 & k = 0 \end{cases}.$$

$a_1 = 1/\pi$
$a_2 = 1/4\pi$
$a_3 = 1/9\pi$
$a_4 = 1/16\pi$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below. Label your plot carefully to receive full credit.

$$X(j\omega) = \sum_{k=-4}^{4} 2\pi a_k \delta(\omega - 200\pi k)$$

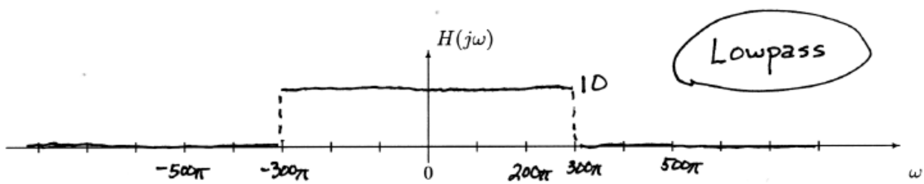
$\omega_0 = 200\pi$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \begin{cases} 10 & |\omega| \leq 300\pi \\ 0 & |\omega| > 300\pi \end{cases}.$$

Plot this function on the graph below using the same frequency scale as the plot in part (a). *Note carefully what type of filter (i.e., lowpass, bandpass, highpass) this is.*



(c) Write an equation for $y(t)$. In order to get full credit you must simplify it to include only cosine functions.

$$Y(j\omega) = H(j\omega)X(j\omega) = 20\pi\delta(\omega) + 2\delta(\omega - 200\pi) + 2\delta(\omega + 200\pi)$$

Invert: $y(t) = 10 + \frac{10}{\pi} e^{j200\pi t} + \frac{10}{\pi} e^{-j200\pi t}$

use Euler's inverse formula

$$y(t) = 10 + \frac{20}{\pi}\cos(200\pi t)$$

Problem s-01-F.4:

A linear time-invariant system has impulse response

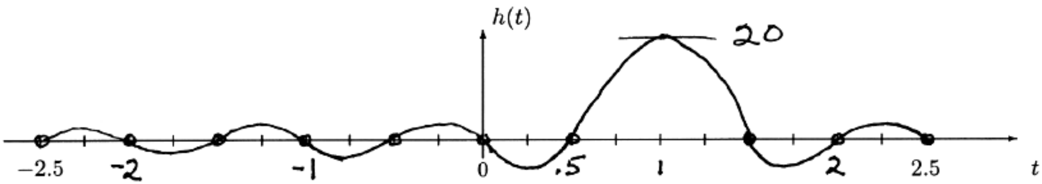
$$h(t) = 10\frac{\sin(2\pi(t-1))}{\pi(t-1)}.$$

shifted "sinc" $10\frac{\sin(2\pi t)}{\pi t}$ ← shift by 1

(a) Sketch $h(t)$ very carefully on the graph below for the indicated range of values of t. Label the maximum value and the points where $h(t) = 0$.

For $10\frac{\sin 2\pi t}{\pi t}$, the max is at $t=0$. Max $= \frac{10 \cdot 2\pi}{\pi} = 20$

↳ zero crossings spaced by ½ because sin( ) is zero when $2\pi t = \pi k$ $\Rightarrow t = k/2$ sec.



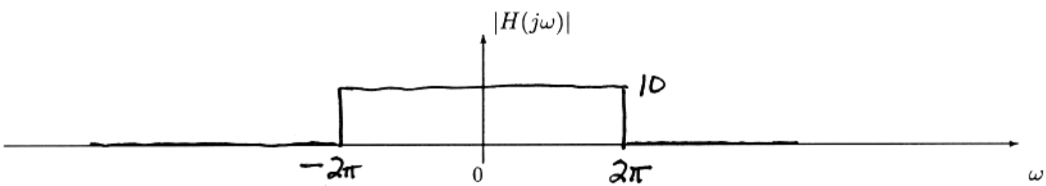
(b) Determine the frequency response $H(j\omega)$ of this system.

Take the Fourier Transform. "Sinc" ⇒ rectangle. Shifting in time becomes multiply by $e^{-j\omega t_d}$ in frequency

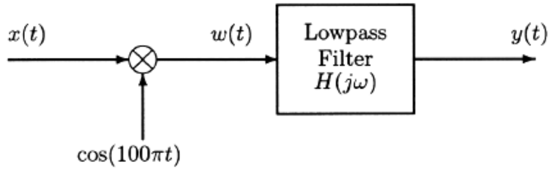
$$H(j\omega) = e^{-j\omega}\left[10u(\omega+2\pi) - 10u(\omega-2\pi)\right]$$

(c) Then plot $|H(j\omega)|$ for this system on the graph below. Label your plot carefully to receive full credit. The term $e^{-j\omega}$ does NOT contribute to the magnitude.

$$|H(j\omega)| = 10\left[u(\omega+2\pi) - u(\omega-2\pi)\right] = \begin{cases} 10 & \text{for } |\omega| \le 2\pi \\ 0 & \text{for } |\omega| > 2\pi \end{cases}$$



Problem s-01-F.5:



In the above modulation/filtering system, assume that the input signal $x(t)$ has a bandlimited Fourier transform, $X(j\omega)$, as depicted in Figure A below.

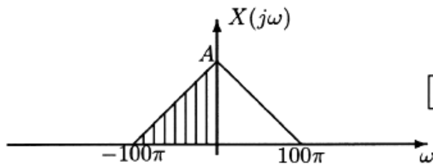


Figure A

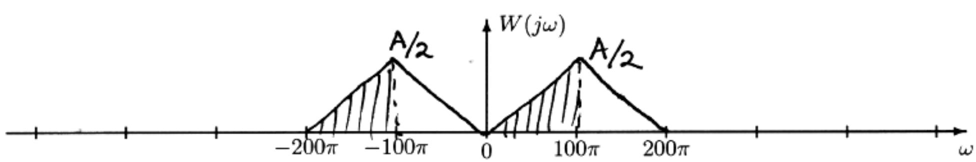
(a) First give the general equation that expresses $W(j\omega)$, the Fourier transform of $w(t) = x(t)\cos(100\pi t)$, in terms of $X(j\omega)$.

$$W(j\omega) = \tfrac{1}{2}X(j(\omega-100\pi)) + \tfrac{1}{2}X(j(\omega+100\pi))$$

(Frequency shifting)

(b) Now **carefully** plot the Fourier transform $W(j\omega)$ for the specific input $x(t)$ whose Fourier transform $X(j\omega)$ is given above in Figure A. *Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plot of $W(j\omega)$, and be sure to carefully label both amplitudes and frequencies.*

Shift up by 100π rad/sec & shift down by 100π rad/s



(c) The frequency response of the lowpass filter is

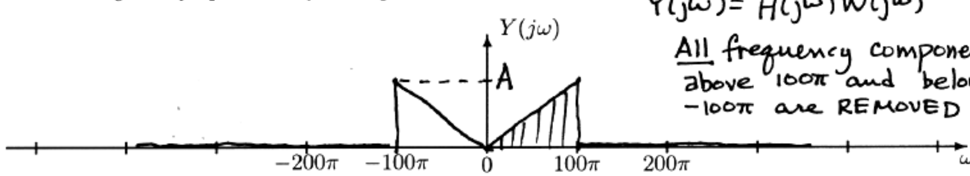
$$H(j\omega) = \begin{cases} 2 & |\omega| \le 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$

Lowpass Filter Gain = 2

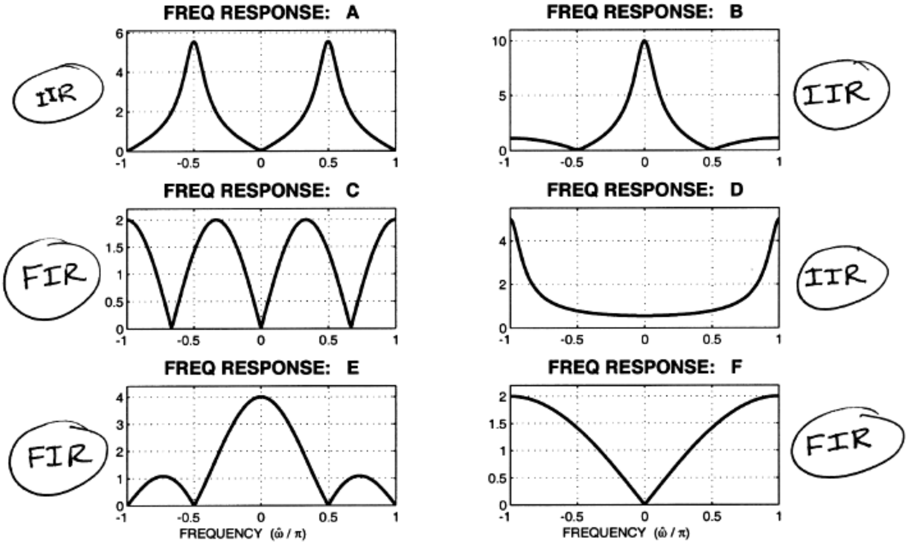
Plot the Fourier transform $Y(j\omega)$ below for the $X(j\omega)$ given in Figure F.5 above. *Be sure to carefully label both amplitudes and frequencies and be sure to shade the region corresponding to the negative frequencies of the input.*

$Y(j\omega) = H(j\omega)W(j\omega)$

All frequency components above 100π and below -100π are REMOVED



Problem s-01-F.6:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$, a difference equation, or a MATLAB statement) matches the frequency response (magnitude only). *There is only ONE correct match per graph.* NOTE: The discrete-time frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $y[n] = -0.8y[n-1] + x[n]$ $\quad H_1(z) = \frac{1}{1+0.8z^{-1}}$

$\mathcal{S}_2$: H=freqz([1,0,1],[1,0,0.64],omega) $\quad \rightarrow H_2(z) = \frac{1+z^{-2}}{1+0.64z^{-2}}$ ← zeros at $z=\pm j \Rightarrow \hat{\omega} = \pm\pi/2$.

$\mathcal{S}_3$: $H(z) = \sum_{k=0}^{3} z^{-k} = \frac{1-z^{-4}}{1-z^{-1}}$ zeros at $z=-1, \pm j \Rightarrow \hat{\omega} = \pi, \pm\pi/2$

$\mathcal{S}_4$: $H(z) = \dfrac{1+z^{-2}}{1-0.8z^{-1}}$

$\mathcal{S}_5$: $H(z) = 1 + 0.64z^{-2}$ ← NO zeros on U.C.

$\mathcal{S}_6$: $H(z) = \dfrac{1-z^{-2}}{1+0.64z^{-2}}$ ← zeros at $z=\pm 1 \Rightarrow \hat{\omega} = 0, \pi$

$\mathcal{S}_7$: $y[n] = x[n] - x[n-1]$ $\quad H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}}$ ← zero @ $\hat{\omega}=0$, $=2$ @ $\hat{\omega}=\pi$

$\mathcal{S}_8$: $H(z) = 1 - z^{-3}$ ← zeros at $z=1, e^{\pm j2\pi/3} \Rightarrow \hat{\omega} = 0, \pm 2\pi/3$

Mark your answer in the following table:

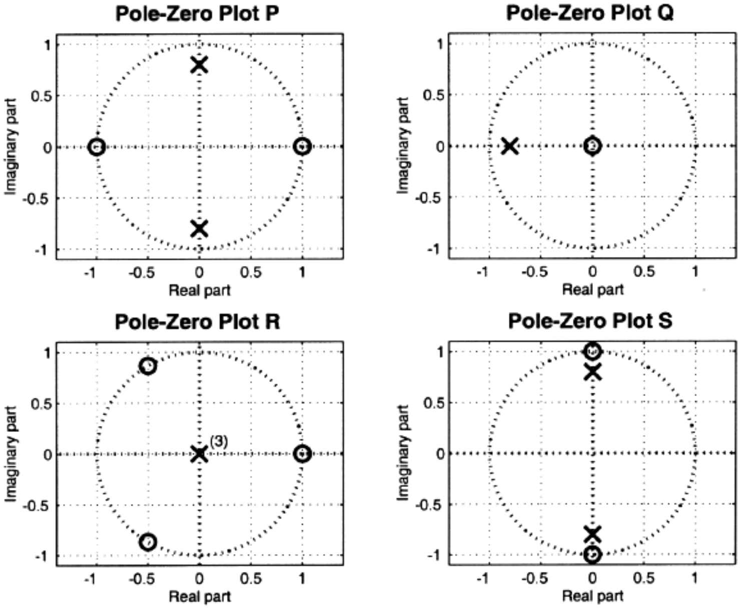
FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_\#$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_\#$)
A	S_6 6	B	S_4 4
C	S_8 8	D	S_1 1
E	S_3 3	F	S_7 7

DC values: S_1 is $\frac{1}{1.8} \approx .6$ $\quad S_3$ is 4 $\quad S_5$ is 1.64

S_2 is $\frac{2}{1.64} \approx 1.25$ $\quad S_4$ is $\frac{2}{.2} = 10$ $\quad S_6, S_7, S_8$ are 0 at $\hat{\omega}=0$ or $z=1$

at $\hat{\omega}=\pi$, S_1 is $\frac{1}{1-.8} = \frac{1}{.2} = 5$

Problem s-01-F.7:



For each of the pole-zero plots (P, Q, R, S), determine which one of the following systems (specified by either an impulse response $h[n]$, a difference equation, or a MATLAB statement) matches the pole-zero plot. *There is only ONE correct match per plot.*

$\mathcal{S}_1$: $h[n] = \delta[n] + 0.8\delta[n-1]$

$\mathcal{S}_2$: $h[n] = (-0.8)^n u[n]$ → $H(z) = \frac{1}{1+.8z^{-1}}$ pole @ -0.8

$\mathcal{S}_3$: $y[n] = -0.64y[n-2] + x[n] + x[n-2]$

$\mathcal{S}_4$: $y[n] = x[n] - x[n-3]$ $H(z) = 1 - z^{-3}$ has zeros at $z = 1, e^{\pm j2\pi/3}$

$\mathcal{S}_5$: y=filter([1,1,1,1],1,x)

$\mathcal{S}_6$: $y[n] = -0.64y[n-2] + x[n] - x[n-2]$

$\mathcal{S}_7$: $h[n] = (-0.8)^n u[n] - (0.8)^n u[n]$

Mark your answer in the following table:

POLE-ZERO PLOT	SYSTEM ($\mathcal{S}_{\#}$)	POLE-ZERO PLOT	SYSTEM ($\mathcal{S}_{\#}$)
P	S_6 6	Q	S_2 2
R	S_4 4	S	S_3 3

$\mathcal{S}_3$: $H(z) = \frac{1+z^{-2}}{1+0.64z^{-2}}$

zeros @ $z = \pm j$

poles @ $z = \pm j0.8$

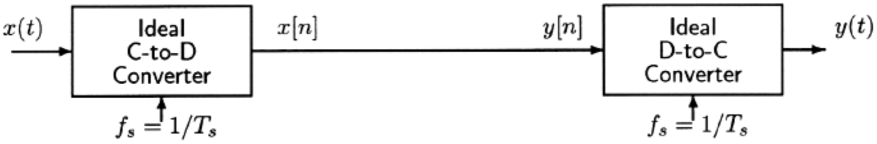
$\mathcal{S}_6$: $H(z) = \frac{1-z^{-2}}{1+0.64z^{-2}}$

zeros @ $z = \pm 1$

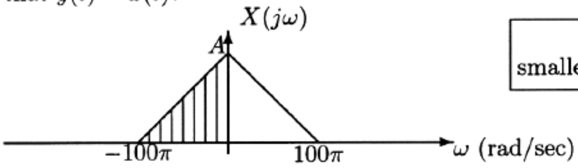
poles @ $z = \pm j0.8$

Problem s-01-F.8:

Consider the following system for sampling and reconstruction of a continuous-time signal:



(a) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that $y(t) = x(t)$?



smallest f_s = 100 samples/sec

Sampling Thm $\Rightarrow$ $f_s \geq 2 f_{MAX} = 100$ samples/sec

$f_{MAX} = \frac{100\pi}{2\pi} = 50\,Hz$

(b) In this part, the input signal is $x(t) = 10 + 10\cos(100\pi t + \pi/3)$. If the sampling rate is $f_s = 80$ samples/sec, what is the corresponding output $y(t)$?

$f_{MAX} = \frac{100\pi}{2\pi} = 50\,Hz$

Will we have aliasing? Yes!

$$x[n] = 10 + 10\cos(100\pi \tfrac{n}{80} + \pi/3)$$

$$= 10 + 10\cos(1.25\pi n + \pi/3)$$

$1.25\pi - 2\pi = -0.75\pi$

$$= 10 + 10\cos(-0.75\pi n + \pi/3)$$

$$= 10 + 10\cos(0.75\pi n - \pi/3)$$

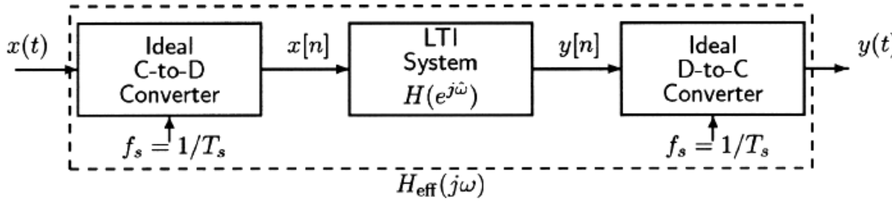
run this $\hat{\omega}$ thru the C/D converter

$(0.75\pi) f_s = \frac{3\pi}{4} \cdot 80 = 60\pi$

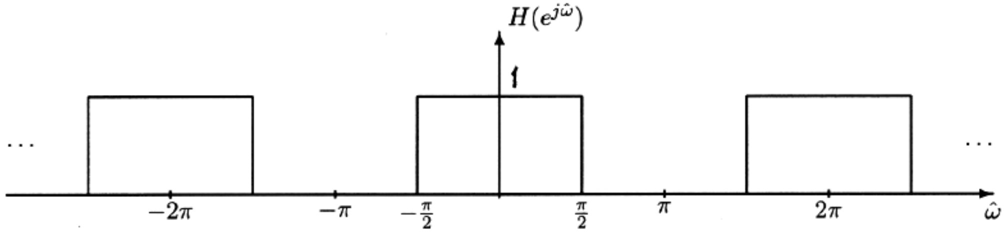
$y(t) = 10 + 10\cos(60\pi t - \pi/3)$

Problem s-01-F.9:

Consider the following system for discrete-time filtering of a continuous-time signal:



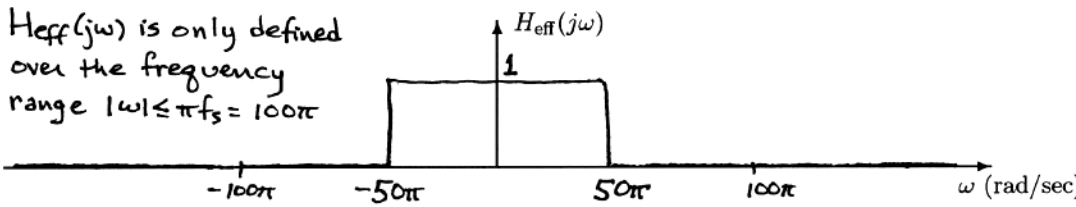
(a) Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot of $H(e^{j\hat{\omega}})$ versus normalized frequency $\hat{\omega}$:



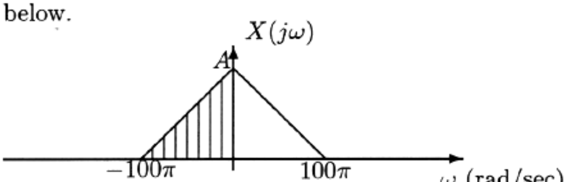
If $f_s = 100$ samples/sec, make a carefully labeled plot below of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system, versus frequency in rads/sec.

$H_{eff}(j\omega) = H(e^{j\hat{\omega}})\big|_{\hat{\omega}=\omega/f_s = \omega/100}$ $\Rightarrow$ $\hat{\omega} = \pi/2$ maps to $\omega = 100(\pi/2)$

$H_{eff}(j\omega)$ is only defined over the frequency range $|\omega| \le \pi f_s = 100\pi$



(b) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below.



Through the C/D converter $\omega = 100\pi$ maps to $\hat{\omega} = 100\pi/f_s$

We need $100\pi/f_s$ inside the passband, $\Rightarrow \frac{100\pi}{f_s} \le \frac{\pi}{2}$

For the system in part (a), what is the smallest sampling rate such that the input signal passes through the lowpass filter unaltered; i.e., what is the minimum f_s such that $Y(j\omega) = X(j\omega)$?

$f_s \ge \frac{100\pi}{\pi/2} = 200$ Hz

smallest f_s = 200 samples/sec