GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Spring 1999
Problem Set #3

Assigned: 16 April 1999
Due Date: 23 April 1999 (FRIDAY)

Quiz #1 will be held in lecture on Monday 26-April-99. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation*, especially pp. 48–73.

The web site: `http://webct.ece.gatech.edu/SCRIPT/SPR99EE2200/scripts/serve_home`
You should change your password; look under `COURSE TOOLS`. Please check the "Bulletin Board" often.

$\Longrightarrow$ **Look for another on-line HW this week.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

The Homework must be turned in at the Friday lecture. After NOON on Friday, the homework is considered late and will be given a zero.

PROBLEM 3.1*:

In AM radio, the transmitted signal is voice (or music) mixed with a *carrier signal.* The carrier is a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta is 750 kHz. If we use the notation $v(t)$ to denote the voice/music signal, then the actual transmitted signal for WSB might be:

$$x(t) = (v(t) + A)\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $v(t)$.)

(a) Voice-band signals tend to contain frequencies less than 4000 Hz. Suppose that $v(t)$ is a sinusoid, $v(t) = \sin(2\pi(1000)t)$. Draw the spectrum for $v(t)$.

(b) Now draw the spectrum for $x(t)$, assuming a carrier at 750 kHz. Use $v(t)$ from part (a).

(c) Music signals might contain frequencies as high as 20 kHz. Suppose that $v(t)$ is a higher frequency sinusoid, $v(t) = \sin(2\pi(13000)t)$. Draw the spectrum for $x(t)$, assuming a carrier at 750 kHz.

(d) If the carrier frequency is changed to 680 kHz (WCNN), describe in words how the spectrum will change for both of the cases above. In particular, describe the relationship between the spectrum for $v(t)$ and the carrier frequency. Use the spectra that you sketched in the previous parts as examples to support your explanation.

PROBLEM 3.2*:

A signal composed of sinusoids is defined by an equation that uses the Fourier Series *synthesis* notation:

$$x(t) = \sum_{k=-3}^{3} jke^{j\pi kt}$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. Label the complex amplitude value at the appropriate frequencies.

(b) Is $x(t)$ periodic? If so, what is the fundamental period?

(c) Determine the DC value of $x(t)$, i.e., the average value over one period: $\frac{1}{T_0}\int_0^{T_0} x(t)dt$.

PROBLEM 3.3*:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \le t \le T_0/2$ by the equation

$$x(t) = \begin{cases} 10 & |t| < t_c \\ -2 & t_c < |t| \le T_0/2 \end{cases}$$

where $t_c < T_0/2$. In this problem assume that $T_0 = 5$ and $t_c = 1$.

(a) Sketch the periodic function $x(t)$ for t in the range $-T_0 < t < 2T_0$.

(b) Determine the D.C. coefficient X_0 using the parameters $T_0 = 5$ and $t_c = 1$.

(c) Determine the *fundamental frequency* ω_0 in the Fourier Series representation (rad/sec).

(d) Use the Fourier *analysis* integral[1] (for $k \neq 0$)

$$X_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-jk\omega_0 t}dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e\left\{\sum_{k=1}^{\infty} X_k e^{jk\omega_0 t}\right\}$$

Your final result could depend on t_c and T_0, but use $t_c = 1$ and $T_0 = 5$.

(e) Sketch the spectrum of $x(t)$ for the case $t_c = 1$ and $T_0 = 5$. Include the DC component and also the first 2 non-zero frequency components in both positive and negative frequency. Label each component with its complex amplitude (magnitude and phase). Check your work by verifying that the conjugate property, $\frac{1}{2}X_{-k} = \frac{1}{2}X_k^*$, holds.[2]

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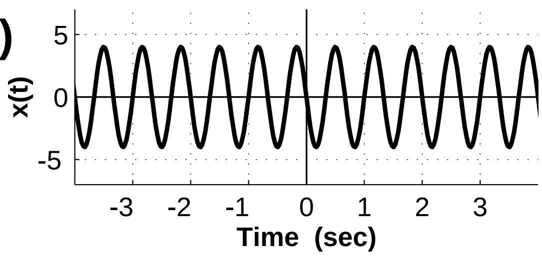
[1] The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

[2] When converting from $\Re e\{X\}$ to the spectrum, remember that $\Re e\{X\} = \frac{1}{2}X + \frac{1}{2}X^*$

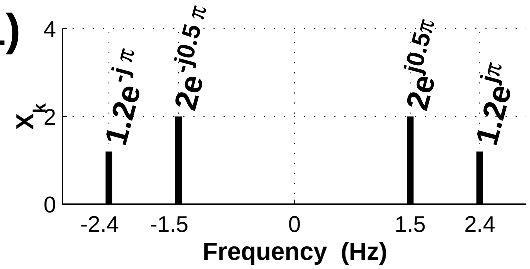
PROBLEM 3.4:

Several signals are plotted below along with their corresponding spectra. However, they are in a random order. For each of the signals (a)–(e), determine the correct spectrum (1)–(5). Explain your answers by deriving the formula for a time signal from each of the spectrum plots.

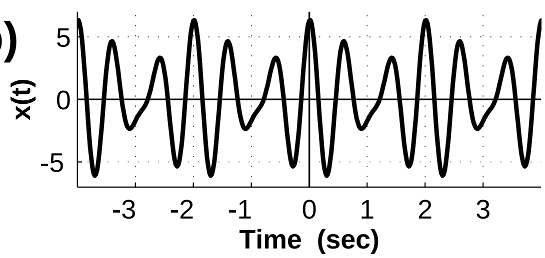
(a)



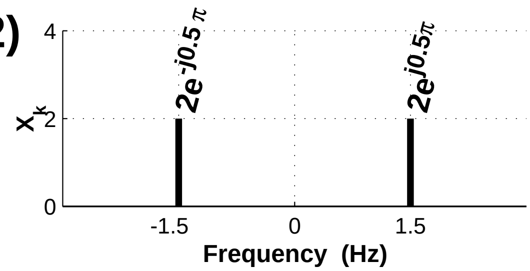
(1)



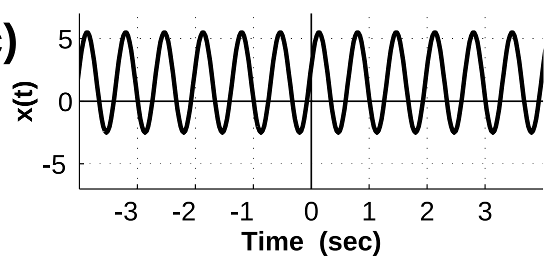
(b)



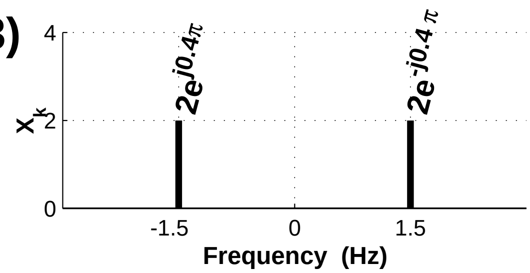
(2)



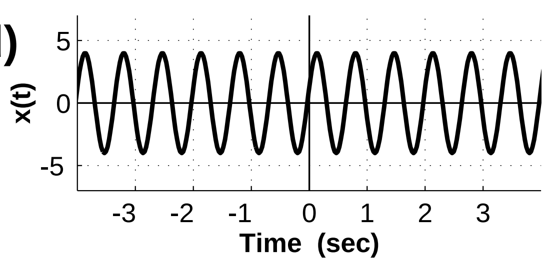
(c)



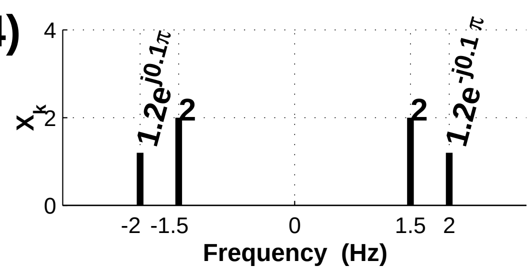
(3)



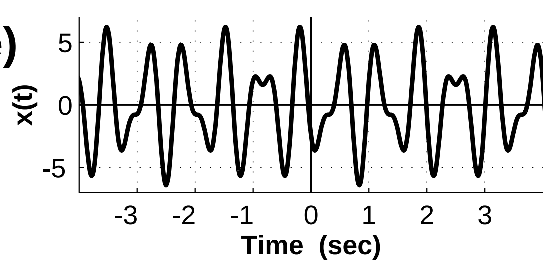
(d)



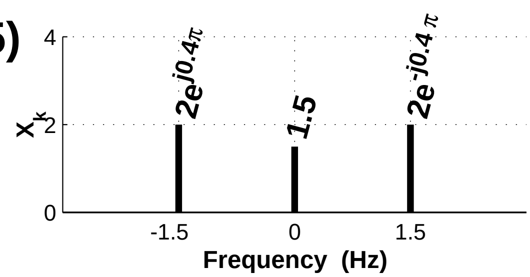
(4)



(e)



(5)



PROBLEM 3.5*:

DSP First, Chapter 3, Problem 8, page 80.

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^{\#}$	D	$E^{\flat}$	E	F	$F^{\#}$	G	$G^{\#}$	A	$B^{\flat}$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C (note #49) is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The E Minor chord is composed of the tones of E, G, B sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the E-Minor chord assuming that each note is realized by a pure sinusoidal tone and that each note is equally loud. (You do not have to specify the complex amplitudes precisely.)