GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Spring 1999
Problem Set #5

Assigned: 7 May 99
Due Date: 14 May 99 (FRIDAY)

Quiz #2 will be held on 24-May-99. Closed book, calculators permitted, and one page of hand-written formulas ($8\frac{1}{2}''\times 11''$). It will cover material from Chapters 3, 4, 5, and 6, as represented in Problem Sets #4, #5 and #6.

Reading: In *DSP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response*.

A lab quiz is planned for the labs on 13-May and also 20-May.

$\Longrightarrow$ The five **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 5.1*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

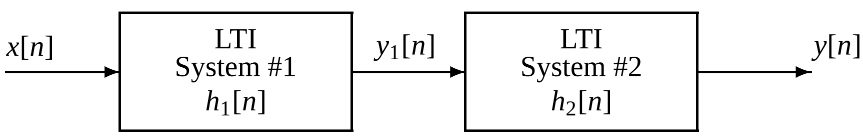


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a blurring filter described by the impulse response:

$$h_1[n] = \begin{cases} 0 & n < 0 \\ \beta^n & n = 0, 1, 2, 3, 4, 5 \\ 0 & n > 5 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - \beta y_1[n-1]$$

Determine the impulse response function of the overall cascade system.

(b) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = \frac{1}{2}$.

PROBLEM 5.2*:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] - \beta x[n-1]$$

(a) When the input to this system is

$$x[n] = \begin{cases} 0 & n < 0 \\ \beta^n & n = 0, 1, 2, 3, 4, 5 \\ 0 & n > 5 \end{cases}$$

Use convolution to compute the values of $y[n]$, over the range $0 \le n \le 6$. Give a general formula in terms of β, and also show that most of the output values are equal to zero.

(b) Use the results from the previous part and plot both $x[n]$ and $y[n]$ for the case where $\beta = \frac{1}{2}$.

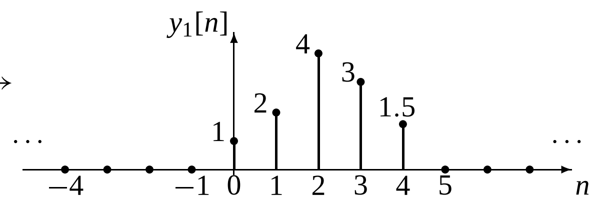
PROBLEM 5.3*:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

(a) When tested with an input signal that is a shifted impulse, $x_1[n] = \delta[n-1]$, the observed output from the filter is the signal $h[n]$ shown below:

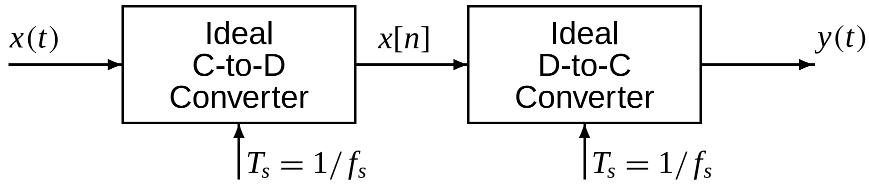
$$\delta[n-1] = \begin{cases} 1 & \text{for } n = 1 \\ 0 & \text{for } n \neq 1 \end{cases} \quad \longrightarrow$$



Use linearity and time-invariance to solve the following problem. Determine the output when the input to the LTI system is $x_2[n] = \delta[n] - \delta[n-2]$. Give your answer as a plot of $y_2[n]$ versus n, or a list of values for $-\infty < n < \infty$.

(b) Define the property of *causality*. Is this system *causal?*

PROBLEM 5.4*:



(a) If the input to the ideal C/D converter is a sinusoid with frequency of 700 Hz, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the frequency of the output.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`.

PROBLEM 5.5*:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] + 3x[n-1] + 3x[n-2] + x[n-3]$$

(a) Find the frequency response $H(\hat{\omega})$, and then express it as a mathematical formula, in polar form (magnitude and phase).

(b) Plot the magnitude and phase of $H(\hat{\omega})$ as a function of $\hat{\omega}$ for $-\pi < \hat{\omega} < \pi$. Do this by hand, but you could check your answer by using the MATLAB function `freqz`.

(c) When the input to the system is $x[n] = \exp(j\pi n/2)$ determine the functional form for the output signal $y[n]$. Find numerical values for the magnitude and phase of $y[n]$.

PROBLEM 5.6:

Consider a system defined by $\quad y[n] = \sum_{k=0}^{13} b_k x[n-k]$

(a) What is the filter length?

(b) Suppose that the input $x[n]$ is non-zero only for $0 \le n \le 33$. Show that $y[n]$ is non-zero at most over a finite interval of the form $0 \le n \le P-1$ and determine P.

(c) Suppose that the input $x[n]$ is non-zero only for $242 \le n \le 942$. Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \le n \le N_4$. Determine N_3 and N_4.

Hint: Draw a sketch similar to Fig. 5.5 to illustrate the zero regions of the output signal.