GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Spring 1999
Problem Set #6

Assigned: 14 May 99
Due Date: 21 May 99 (FRIDAY)

Quiz #2 will be held on 24-May-99. Closed book, calculators permitted, and one page of hand-written formulas ($8\frac{1}{2}''\times 11''$). It will cover material from Chapters 3, 4, 5, and 6, as represented in Problem Sets #4, #5 and #6.

Reading: In *DSP First*, Chapter 6 on *Frequency Response* and Chapter 7 on *z-Transforms.*

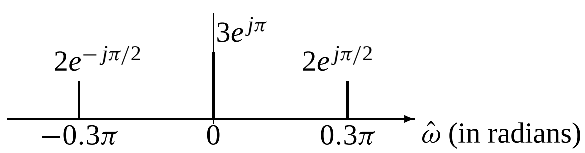
The last lab quiz is planned for the labs on 20-May.

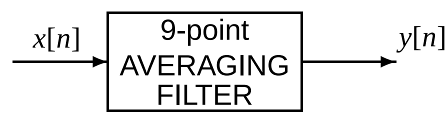
$\Longrightarrow$ The six **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 6.1*:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Use the MATLAB GUI called `ltidemo.m` to determine the output $y[n]$ when the FIR filter is a 9-point averaging filter. Include a screen shot of the result from `ltidemo`.

(c) Determine the formula for the output signal $y[n]$. Do this calculation by hand.

PROBLEM 6.2*:

For the *Dirichlet* function:

$$D(\hat{\omega}, 9) = \frac{\sin(4.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

(a) Make a plot of $D(\hat{\omega}, 9)$ over the range $-2\pi \le \hat{\omega} \le +2\pi$. Label all the zero crossings.

(b) Determine the period of $D(\hat{\omega}, 9)$. Is it equal to 2π; why, or why not?

(c) Find the maximum value of the function.

NOTE: the *Dirichlet* function is defined via: $D(\hat{\omega}, L) = \dfrac{\sin(L\hat{\omega}/2)}{\sin(\frac{1}{2}\hat{\omega})}$
In MATLAB consult help on `diric` for more information.

PROBLEM 6.3*:

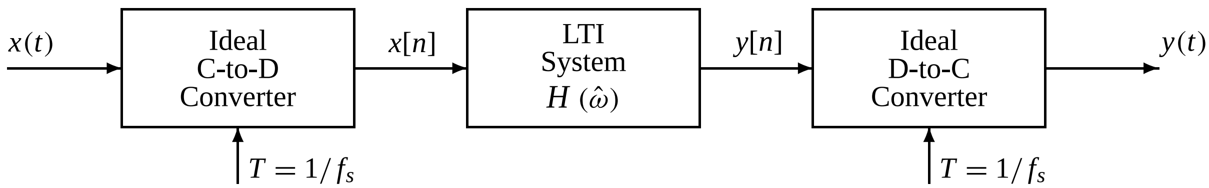
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 4\cos(3000\pi t + \pi/2) + 12\cos(20000\pi t - 2\pi/3)$$

The frequency response for the digital filter (LTI system) is

$$H(\hat{\omega}) = \frac{\sin(4.5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j4\hat{\omega}}$$

If $f_s = 10000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



PROBLEM 6.4*:

The intention of the following MATLAB program is to filter a sinusoid via the `conv` function, but the cosine signal has a starting point at $n = 0$; we assume that it is zero for $n < 0$.

```
omega = pi/2;
nn = [ 0:4000 ];
xn = cos(omega*nn - pi/2);
bb = [ 1 0 0 0 1 ];
yn = conv( bb, xn );
```

(a) Determine $H(\hat{\omega})$ for the FIR filter.

(b) Make a plot of the magnitude of $H(\hat{\omega})$ and label *all* the frequencies where $|H(\hat{\omega})|$ is zero.

(c) Determine a formula for $y[n]$, the signal contained in the vector `yn`. Give the individual values for $n = 0, 1, 2, 3$, and then provide a general formula for $y[n]$ that is correct for $4 \le n \le 4000$. This formula should give numerical values for the amplitude, phase and frequency of $y[n]$.

(d) Give at least one value of `omega` such that the output is guaranteed to be zero, for $n \ge 4$.

PROBLEM 6.5*:

Suppose that three systems are hooked together in "cascade." In other words, the output of S_1 is the input to S_2, and the output of S_2 is the input to S_3. The three systems are specified as follows:

$$\begin{aligned} S_1: &\quad y_1[n] = 3x_1[n] - 3x_1[n-1] \\ S_2: &\quad y_2[n] = 2x_2[n] + 2x_2[n-2] \\ S_3: &\quad H_3(\hat{\omega}) = e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \end{aligned}$$

NOTE: the output of S_i is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into S_1) to the output $y[n]$ which is the output of S_3. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the frequency response $H_i(\hat{\omega})$ for $i = 1, 2$.

(b) Determine the difference equation for S_3.

(c) Determine the z-transform system function $H_i(z)$ for each system.

(d) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

PROBLEM 6.6*:

Suppose that a LTI system has system function equal to

$$H(z) = 1 + z^{-4}$$

(a) Determine the difference equation that relates the output $y[n]$ of the system to the input $x[n]$.

(b) Determine all the zeros of the z-transform system function, $H(z)$. In other words, solve $H(z) = 0$. Express your answer(s) in polar form.

(c) Suppose that the input signal is:

$$x[n] = \delta[n-1] + 2\delta[n-3] + 3\delta[n-5]$$

Determine the output $y[n]$ by using *convolution.*

(d) Demonstrate how the output of the system can also be obtained by multiplying $H(z)$ times the polynomial:

$$X(z) = z^{-1} + 2z^{-3} + 3z^{-5}$$

Describe how the polynomial coefficients of $X(z)$ and $Y(z) = H(z)X(z)$ are related to $x[n]$ and $y[n]$, respectively.