GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Spring 1999
Problem Set #7

Assigned: 28 May 99
Due Date: 4 June 99 (FRIDAY)

Final Exam is scheduled for Period #13: 11-June (Friday) at 8:00AM!!! In the ECE Auditorium.

Review Session is planned for Thursday evening of Finals week. More details later.

Reading: In *DSP First*, Chapters 7 and 8 on *Z-Transforms* and *IIR Filters.*

$\Longrightarrow$ The four (4) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM and in old homeworks, especially the "unstarred" problems.

PROBLEM 7.1*:

A linear time-invariant system has system function

$$H(z) = (1 - z^{-2})(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

(a) Determine the impulse response of this system.

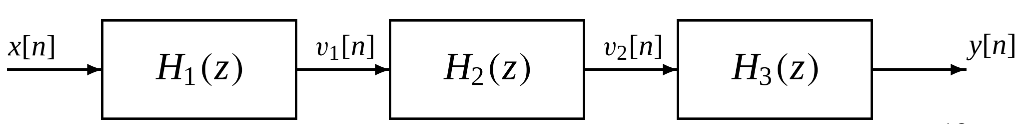
(b) The input to this system is

$$x[n] = 3 + 7\delta[n] + 10\cos(\pi n/2)$$

Determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n. (*Note: This is an easy problem if you approach it correctly!*)

PROBLEM 7.2:

In the following cascade of systems, all of the individual transfer functions are known.



$$H_1(z) = z^{-2} + z^{-3} \qquad H_2(z) = 4 - 3z^{-1} \qquad H_3(z) = \frac{10}{8 - 10z^{-1} + 3z^{-2}}$$

(a) Determine $H(z)$ the z-transform of the cascaded system. Simplify $H(z)$ by cancelling common factors in the numerator and denominator.

(b) Consider the impulse response of the cascaded system, i.e., the response $y[n]$ when the input is $x[n] = \delta[n]$. Prove that the impulse response has the form $h[n] = G\alpha^n$ for $n \geq 3$. Find values for α and G.

PROBLEM 7.3:

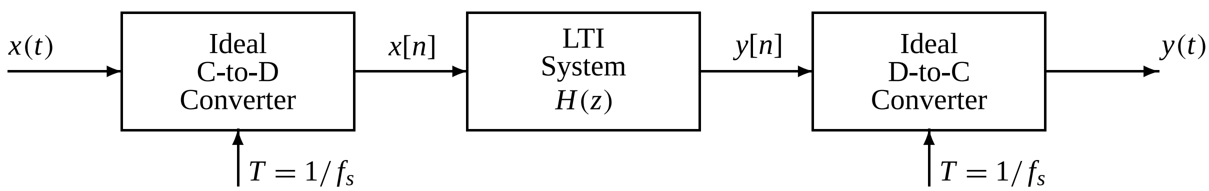
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 2\cos(6000\pi t - \pi/4) + 11\cos(12000\pi t - \pi/3)$$

The system function for the LTI system is

$$H(z) = \frac{1}{4}(1 - z^{-4})$$

If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



PROBLEM 7.4*:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$. [1]

$$S_1: \quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$$

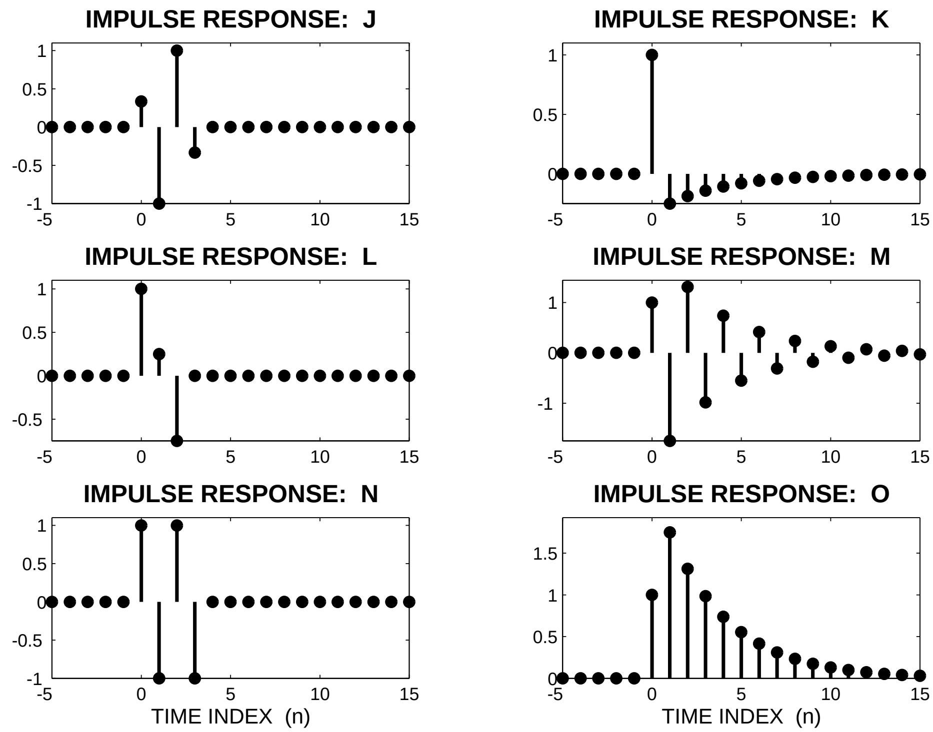
$$S_3: \quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$$

$$S_6: \quad y[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$$

$$S_7: \quad y[n] = x[n] + \tfrac{1}{4}x[n-1] - \tfrac{3}{4}x[n-2]$$

[1] These systems are a subset of those in the following two problems.

PROBLEM 7.5*:



For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems[2] (specified by either an $H(z)$ or a difference equation) matches the impulse response. In addition, derive a formula for the impulse response, $h[n]$, for S_1 and S_4.

S_1 : $\quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

S_2 : $\quad H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

S_3 : $\quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

S_4 : $\quad H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

S_5 : $\quad y[n] = x[n] - x[n-1] + x[n-2]$

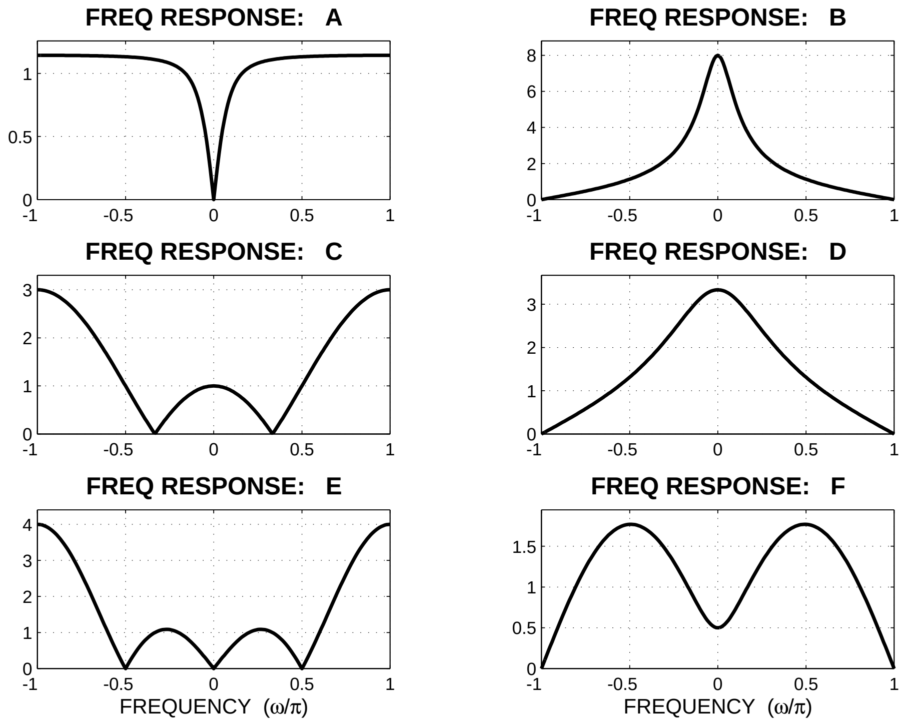
S_6 : $\quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

S_7 : $\quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

S_8 : $\quad H(z) = \frac{1}{3}(1-z^{-1})^3$

[2]These 8 systems are exactly the same as the other matching problems.

PROBLEM 7.6*:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems[3] (specified by either an $H(z)$ or a difference equation) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$. In addition, derive a formula for the magnitude-squared of the frequency response, $|H(e^{j\hat{\omega}})|^2$, for S_3 and S_4.

S_1 : $\quad y[n] = 0.4y[n-1] + x[n] + x[n-1]$

S_2 : $\quad H(z) = \dfrac{1+z^{-1}}{1-0.75z^{-1}}$

S_3 : $\quad y[n] = -0.75y[n-1] + x[n] - x[n-1]$

S_4 : $\quad H(z) = \dfrac{1-z^{-1}}{1-0.75z^{-1}}$

S_5 : $\quad y[n] = x[n] - x[n-1] + x[n-2]$

S_6 : $\quad H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

S_7 : $\quad y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

S_8 : $\quad H(z) = \frac{1}{3}(1-z^{-1})^3$

[3]These 8 systems are exactly the same as the other matching problems.