GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: February 2, 2001 COURSE: ECE 2025

NAME: ________________________________ STUDENT #: ________________

LAST, FIRST

Recitation Section: Circle the date & time when your Recitation Section meets (not Lab):

Mon-3p (L11:McClellan)	M-4:30p (L13:Frazier)			
Tues-9:30a (L01:Casinovi)	T-Noon (L03:Casinovi)	T-1:30p (L05:Bordelon)	T-3p (L07:Bordelon)	T-4:30p (L09:Casinovi)
Thur-9:30a (L02:Bordelon)	Th-Noon (L04:Bordelon)	Th-1:30p (L06:Smith)	Th-3p (L08:Smith)	Th-4:30p (L09:Casinovi)
Th-6p (L10:Casinovi)				

- Write your name on the front page ONLY.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive any partial credit.
  Explanations are also REQUIRED to receive FULL credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem sp-01-Q.1.1:

For each of the following sinusoidal signals, pick one of the representations below that defines *exactly* the same signal. Write your answer $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, or $x_5(t)$, in the box next to each signal. In addition, write the complex amplitude (phasor) (X_k) of the sinusoid for each case in the space provided.

ANS =	$3\cos(1000\pi t + \pi/3)$
	$X_k =$

ANS =	$3\cos(1000\pi t + 2\pi/3)$
	$X_k =$

ANS =	$\frac{3}{2}e^{j2\pi/3}e^{j1000\pi t} + \frac{3}{2}e^{-j2\pi/3}e^{-j1000\pi t}$
	$X_k =$

ANS =	$3\cos(1000\pi t - 8\pi/3)$
	$X_k =$

ANS =	$\Re e\left\{\frac{3}{2}(1 - j\sqrt{3})e^{j1000\pi t}\right\}$
	$X_k =$

POSSIBLE ANSWERS: Some of these answers can be used <u>more than once.</u>
If one answer is used twice, another one won't be used at all.

1. $x_1(t) = 3\cos(1000\pi t + 8\pi/3)$

2. $x_2(t) = \Re e\left\{\frac{3}{2}e^{-j\pi/3}e^{j1000\pi t}\right\}$

3. $x_3(t) = 3\cos(1000\pi t - \pi/3)$

4. $x_4(t) = \Re e\left\{\frac{3}{2}(1 + j\sqrt{3})e^{j1000\pi t}\right\}$

5. $x_5(t) = \Re e\left\{3e^{-j2\pi/3}e^{j1000\pi t}\right\}$

Problem sp-01-Q.1.2:
Define $x(t)$ as

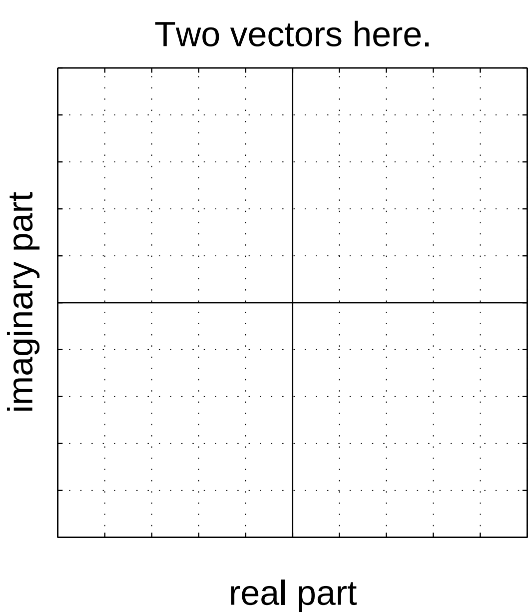
$$x(t) = \Re e\left\{4e^{j2\pi/3}e^{j20\pi t} + 2e^{j20\pi(t-0.2)}\right\} = \Re e\left\{Xe^{j20\pi t}\right\}$$

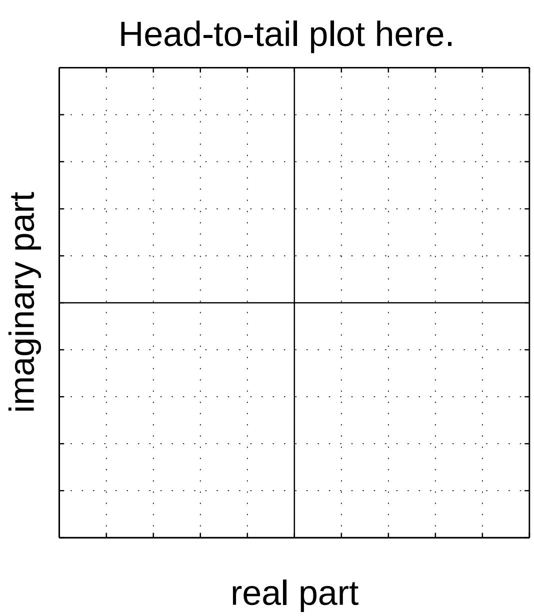
(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

(b) Fill in the MATLAB statements that will compute the complex phasor X from which the numerical values of A and ϕ can be computed.

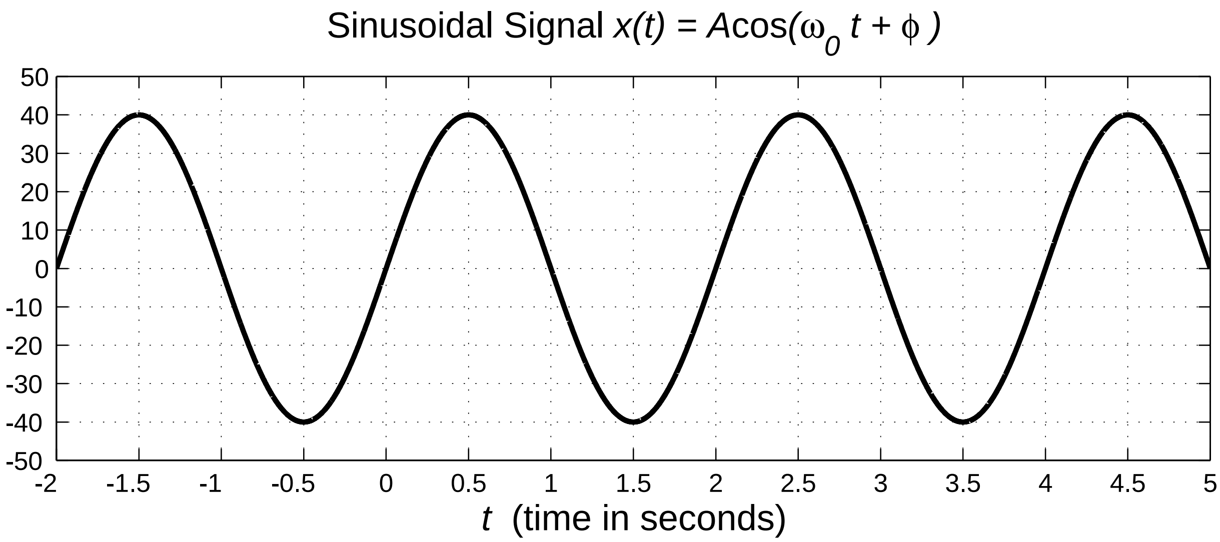
```
X = ________________________________________

A = ________________________________________

phi = ______________________________________
```

(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show only the two complex amplitudes (phasors) that were added to solve part (a); on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail). *Use appropriate scale on the grid below.*





Problem sp-01-Q.1.3:



The above graph is a plot of a sinusoidal signal $x(t) = A\cos(\omega_0 t + \phi)$.

(a) Determine numerical values for A, ω_0 and ϕ with $-\pi < \phi \leq \pi$.

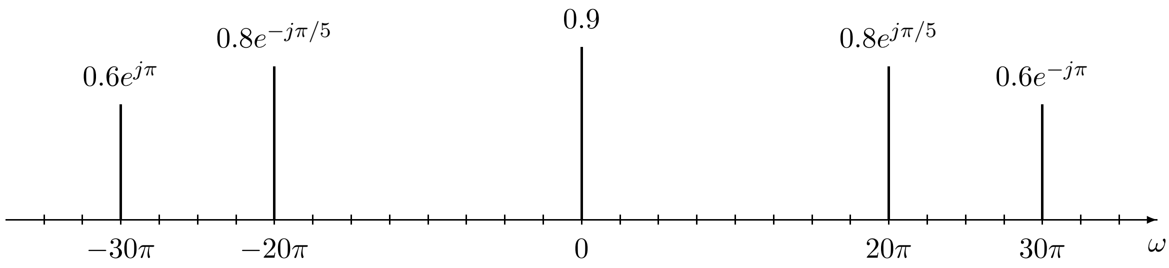
(b) By a suitable choice of delay t_d, we can shift $x(t)$ to obtain the new signal

$$y(t) = x(t - t_d) = A\cos(\omega_0 t + \pi/4) \tag{1}$$

There are an infinite number of values of t_d that satisfy Equation (1). Determine at least **two** different values of t_d that satisfy Equation (1), or give a general formula for all the possible values.

Problem sp-01-Q.1.4:

The spectrum of a signal $x(t)$ is shown in the following figure:



Note carefully that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

(a) Write an equation for $x(t)$ in terms of cosine functions.

(b) Is $x(t)$ periodic? **You must explain this answer. Why or why not?**
If it is periodic, what is the fundamental frequency ω_0 and corresponding period T_0 of $x(t)$?

(c) A new signal is defined as $y(t) = \cos(\beta t - \pi/4) + x(t)$. Choose the radian frequency β so that the fundamental frequency of $y(t)$ is *half* the fundamental frequency of $x(t)$. *Note: There may be more than one possible solution.*

(d) Using the frequency β found in (c), modify the spectrum plot above so that it becomes the spectrum of $y(t)$. *Label the complex amplitude as well as the frequency.*

Problem sp-01-Q.1.1:

For each of the following sinusoidal signals, pick one of the representations below that defines *exactly* the same signal. Write your answer $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, or $x_5(t)$, in the box next to each signal. In addition, write the complex amplitude (phasor) (X_k) of the sinusoid for each case in the space provided.

ANS = 4 $3\cos(1000\pi t + \pi/3)$

$X_k = 3e^{j\pi/3}$

ANS = 1 $3\cos(1000\pi t + 2\pi/3)$

$X_k = 3e^{j2\pi/3}$

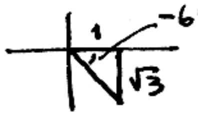
ANS = 1 $\frac{3}{2}e^{j2\pi/3}e^{j1000\pi t} + \frac{3}{2}e^{-j2\pi/3}e^{-j1000\pi t}$

$X_k = 3e^{j2\pi/3}$

ANS = 5 $3\cos(1000\pi t - 8\pi/3)$

$X_k = 3e^{-j8\pi/3} = 3e^{-j2\pi/3}$

ANS = 3 $\Re e\left\{\frac{3}{2}(1 - j\sqrt{3})e^{j1000\pi t}\right\}$



$X_k = 3e^{-j\pi/3}$

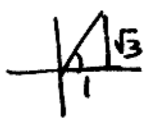
POSSIBLE ANSWERS: Some of these answers can be used **<u>more than once.</u>**
If one answer is used twice, another one won't be used at all.

1. $x_1(t) = 3\cos(1000\pi t + 8\pi/3)$ $X_1 = 3e^{j8\pi/3} = 3e^{j2\pi/3}$

2. $x_2(t) = \Re e\left\{\frac{3}{2}e^{-j\pi/3}e^{j1000\pi t}\right\}$ $X_2 = \frac{3}{2}e^{-j\pi/3}$

3. $x_3(t) = 3\cos(1000\pi t - \pi/3)$ $X_3 = 3e^{-j\pi/3}$

4. $x_4(t) = \Re e\left\{\frac{3}{2}(1 + j\sqrt{3})e^{j1000\pi t}\right\}$



60° $X_4 = 3e^{j\pi/3}$

5. $x_5(t) = \Re e\left\{3e^{-j2\pi/3}e^{j1000\pi t}\right\}$ $X_5 = 3e^{-j2\pi/3}$

Problem sp-01-Q.1.2:
Define $x(t)$ as

$$x(t) = \Re e\left\{4e^{j2\pi/3}e^{j20\pi t} + 2e^{j20\pi(t-0.2)}\right\} = \Re e\left\{Xe^{j20\pi t}\right\}$$

(a) Use phasor addition to express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of A and ϕ, as well as ω_0.

$$X = 4e^{j2\pi/3} + 2e^{-j20\pi(0.2)}$$
$$= 4e^{j2\pi/3} + 2e^{-j4\pi}$$
$$= 4e^{j2\pi/3} + 2e^{j0} = 3.464\,e^{j\pi/2}$$

1.571 rads

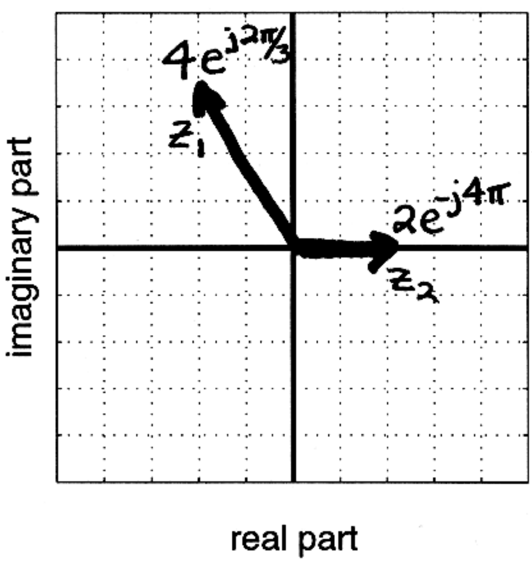
$$x(t) = 3.464\cos(20\pi t + \pi/2)$$

(b) Fill in the MATLAB statements that will compute the complex phasor X from which the numerical values of A and ϕ can be computed.

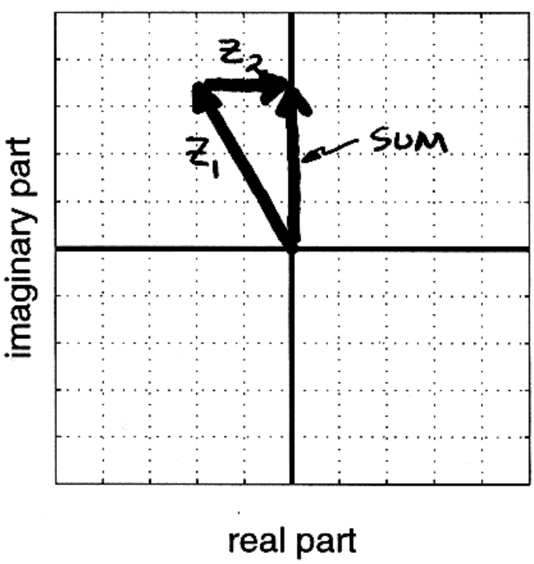
```
X = 4*exp(j*2*pi/3) + 2*exp(-j*4*pi)
A = abs(X)
phi = angle(X)
```

(c) Make two complex plane plots to illustrate how complex amplitudes (phasors) were used to solve part (a). On the first plot, show only the two complex amplitudes (phasors) that were added to solve part (a); on the second plot, show your solution as a vector and the addition of the two complex amplitudes as vectors (head-to-tail). *Use appropriate scale on the grid below.*

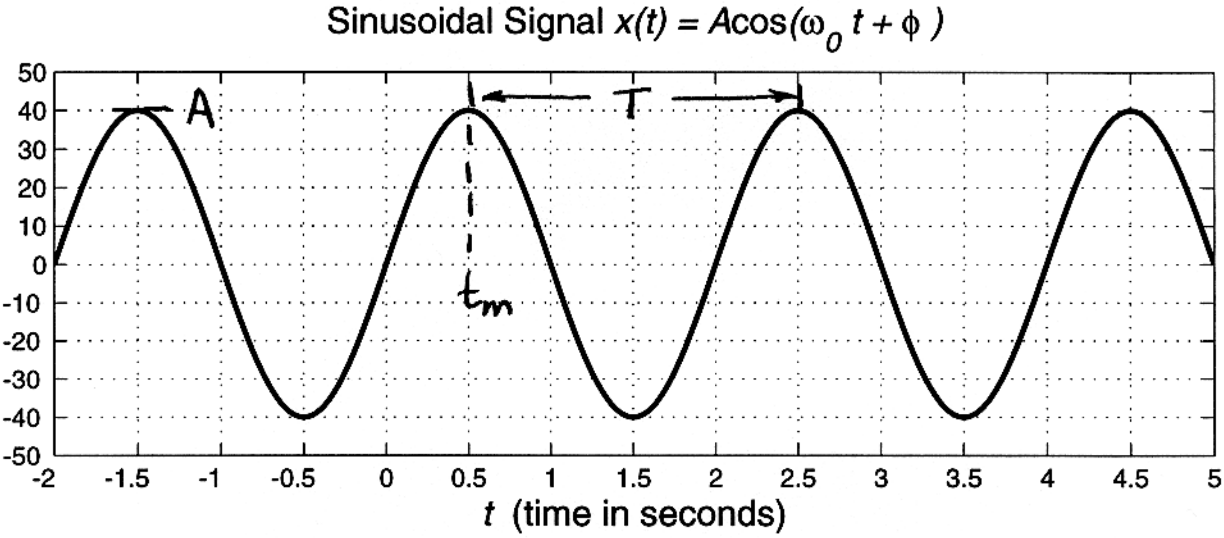
Two vectors here.



Head-to-tail plot here.



Problem sp-01-Q.1.3:



The above graph is a plot of a sinusoidal signal $x(t) = A\cos(\omega_0 t + \phi)$.

(a) Determine numerical values for A, ω_0 and ϕ with $-\pi < \phi \leq \pi$.

$A = 40$

$T = 2.5 - 0.5 = 2$ secs $\Rightarrow$ $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

$t_m = 0.5$ s

$\varphi = -\omega_0 t_m = -\pi(\frac{1}{2}) = -\frac{\pi}{2}$ rads.

$x(t) = 40\cos(\pi t - \pi/2)$

(b) By a suitable choice of delay t_d, we can shift $x(t)$ to obtain the new signal

$$y(t) = x(t - t_d) = A\cos(\omega_0 t + \pi/4) \quad (1)$$

There are an infinite number of values of t_d that satisfy Equation (1). Determine at least **two** different values of t_d that satisfy Equation (1), or give a general formula for all the possible values.

$x(t - t_d) = 40\cos(\pi(t - t_d) - \pi/2) \overset{?}{=} A\cos(\omega_0 t + \pi/4)$

Match the phases, but there can be a difference of $2\pi k$, k = integer.

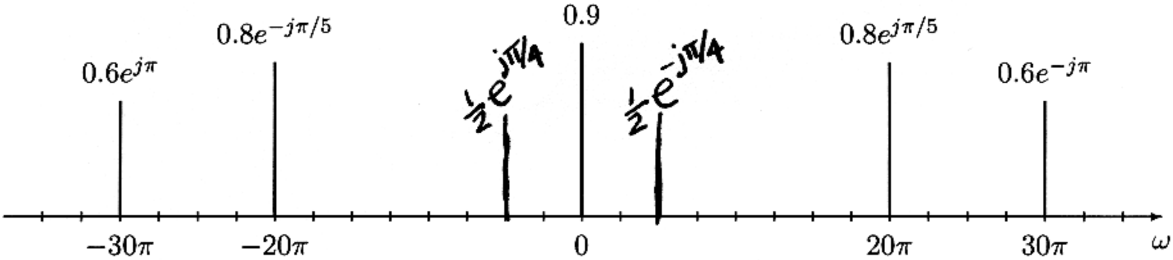
$-\pi t_d - \pi/2 = \frac{\pi}{4} + 2\pi k$

$t_d = \frac{3\pi/4 + 2\pi k}{-\pi} = -\frac{3}{4} - 2k$ sec.

$k = 0 \Rightarrow t_d = -\frac{3}{4}$ sec., $k = -1 \Rightarrow t_d = \frac{5}{4}$ sec., etc.

Problem sp-01-Q.1.4:

The spectrum of a signal $x(t)$ is shown in the following figure:



Note carefully that the frequency axis is radian frequency (ω) *not* cyclic frequency (f).

(a) Write an equation for $x(t)$ in terms of cosine functions.

$$x(t) = 0.9 + 1.6\cos(20\pi t + \pi/5) + 1.2\cos(30\pi t - \pi)$$

(b) Is $x(t)$ periodic? **You must explain this answer. Why or why not?**
If it is periodic, what is the fundamental frequency ω_0 and corresponding period T_0 of $x(t)$?

Yes, it is periodic because the freq 10π divides 20π and 30π to give an integer quotient.

$\omega_0 = 10\pi$ rad/s

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10\pi} = \frac{1}{5} \text{ sec.}$$

(c) A new signal is defined as $y(t) = \cos(\beta t - \pi/4) + x(t)$. Choose the radian frequency β so that the fundamental frequency of $y(t)$ is *half* the fundamental frequency of $x(t)$. *Note: There may be more than one possible solution.*

Want ω_0 to be 5π rad/sec. Choose β to be an ODD multiple of 5π rad/s.

$\beta = 5\pi$, or 15π, or 25π, etc.

$\boxed{\beta = 5\pi \text{ rad/s}}$

(d) Using the frequency β found in (c), modify the spectrum plot above so that it becomes the spectrum of $y(t)$. *Label the complex amplitude as well as the frequency.*

$$\cos(5\pi t - \pi/4) = \tfrac{1}{2}e^{-j\pi/4}e^{j5\pi t} + \tfrac{1}{2}e^{+j\pi/4}e^{-j5\pi t}$$