GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 1-Feb-02 COURSE: ECE 2025

NAME: _____ STUDENT #: _____
LAST, FIRST

Recitation Section: **Circle the day & time** when your Recitation Section meets:

L02:Tues-9:30am (Bordelon)	L04:Tues-12:00pm (Yezzi)	L05:Thurs-1:30pm (Williams)
L06:Tues-1:30pm (Bordelon)	L07:Thur-3:00pm (Williams)	L08:Tues-3:00pm (Smith)
L11:Mon-3:00pm (Glytsis)	L14:Mon-4:00pm (McClellan)	RPK: (Abler)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- This exam is closed book. However, one page ($8\frac{1}{2}'' \times 11''$) of HAND-WRITTEN notes (front and back) and a calculator are permitted.
- Justify your reasoning clearly to receive partial credit.
  Explanations are also required to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself. Only these answers will be graded. Circle your answers, or write them in the boxes provided. If space is needed for scratch work, use the backs of previous pages.

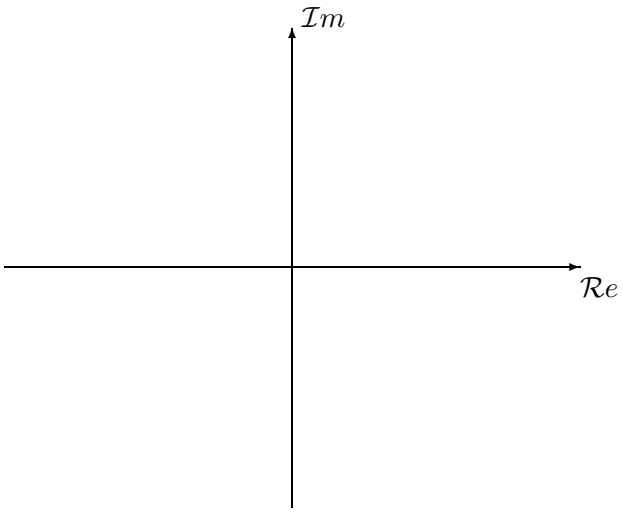
Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	

Problem SPR-02-Q.1.1:
Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form.

Let $Y = \sqrt{2} - j\sqrt{2}$ and $Z = e^{j\pi/3}$.

(a) If $A = Y + Z$, what is its numerical value expressed in rectangular form? **Plot the vectors Y, Z, and A in the complex plane.**

$A =$_____



(b) If $B = ZY^*$, what are the numerical values of the magnitude and phase associated with the polar form representation?

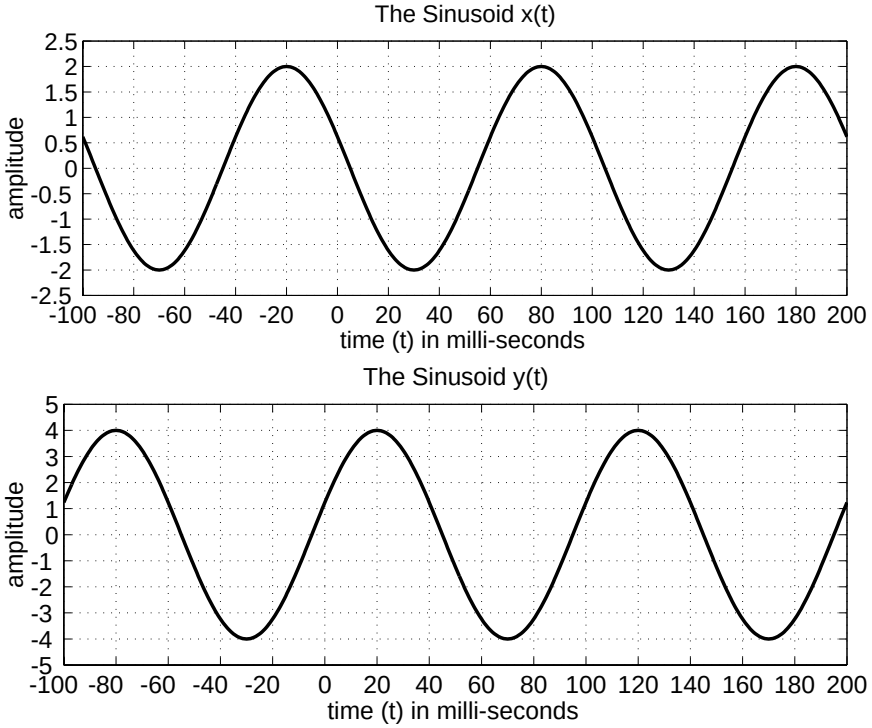
$|B| =$_____, $\angle B =$_____

(c) If $C = (jZ)^{65}$, what is its numerical value expressed in rectangular form?

$C =$_____

Problem SPR-02-Q.1.2:

Consider the sinusoidal signals $x(t)$ and $y(t)$ plotted below.



(a) Determine A, f_0, and ϕ in the representation of $x(t)$ as $x(t) = A\cos(2\pi f_0 t + \phi)$.

$A =$_____. $\phi =$_____. $f_0 =$_____. (in Hz)

(b) Now suppose that $B\cos(\omega_0 t + \psi) = x(t) + 2\cos(\omega_0 t)$. Determine B, ω_0, and ψ.

$B =$_____

$\omega_0 =$_____

$\psi =$_____

(c) The signal $y(t)$ can be expressed in terms of $x(t)$. That is, we can write $x(t) = Cy(t - t_1)$. Determine the numerical values of the scale factor C and the time shift t_1, where $t_1 \geq 0$.

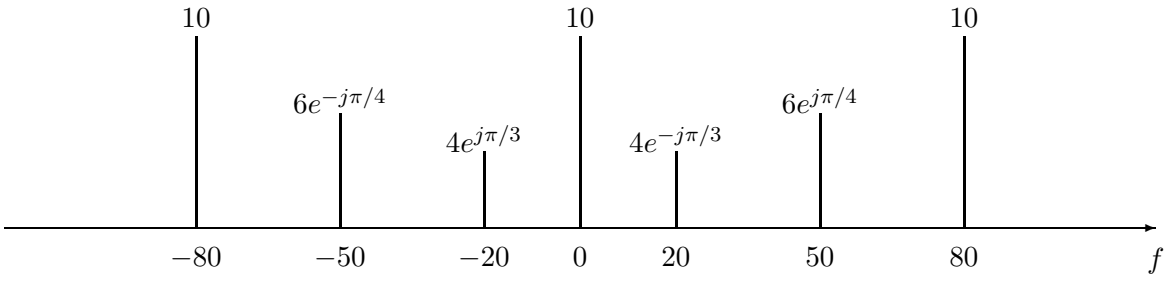
$C =$_____ $t_1 =$_____

Problem SPR-02-Q.1.3:

A real signal

$$x(t) = A\cos(40\pi t + \phi) + B\cos(\omega_1(t-\tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



frequency in Hz

(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$_____

$B =$_____

$C =$_____

$D =$_____

$\phi =$_____

$\omega_1 =$_____

$\omega_2 =$_____

$\tau =$_____

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$_____

Problem SPR-02-Q.1.4:
A signal $x(t)$ is given by the equation

$$x(t) = [A + \cos(40\pi t)]\cos(200\pi t - \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_k's are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).
$N =$ _____

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

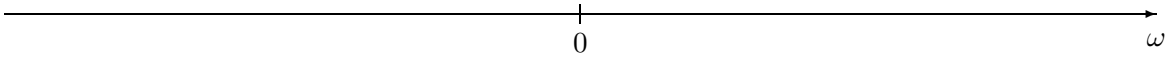
lowest $\omega_k =$ _____

highest $\omega_k =$ _____

(c) The spectrum of $x(t)$ contains a component at frequency 200π rad/sec with complex amplitude $-2j$. What is the numerical value of A?

$A =$_____

(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.



frequency in rad/sec

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QUIZ #1

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NAME: Answer Key STUDENT #: A

LAST, FIRST

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Problem SPR-02-Q.1.1:
Simplify the following complex-valued expressions. In each case reduce the answers to a **simple** numerical form.

Let $Y = \sqrt{2} - j\sqrt{2}$ and $Z = e^{j\pi/3}$.

(a) If $A = Y + Z$, what is its numerical value expressed in rectangular form? **Plot the vectors Y, Z, and A in the complex plane.**

$A =$ 1.914 − j0.5482

Z = 0.5 + j0.866

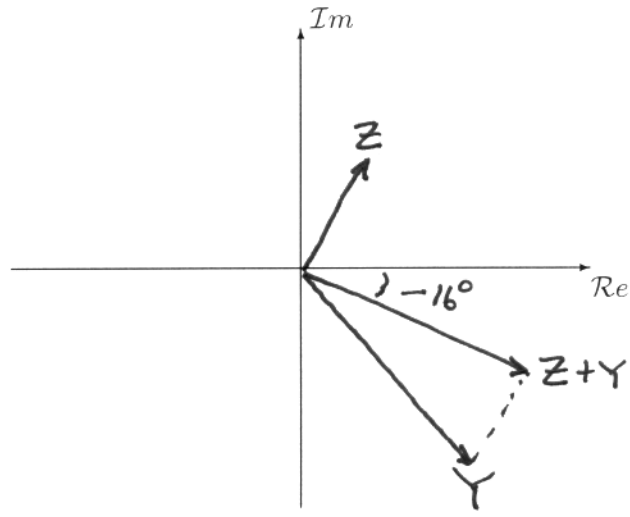
Y = 1.414 − j1.414

1.914 − j0.5482

$= 1.991\, e^{-j0.279}$

-0.09π rads

or $-15.98°$



(b) If $B = ZY^*$, what are the numerical values of the magnitude and phase associated with the polar form representation?

$|B| =$ 2, $\angle B =$ 1.833 rads

$$ZY^* = \left(e^{j\pi/3}\right)\left(2e^{-j\pi/4}\right)^* = 2e^{j(\pi/3 + \pi/4)}$$

$$= 2e^{j7\pi/12}$$

1.833 rads, or 105°

(c) If $C = (jZ)^{65}$, what is its numerical value expressed in rectangular form?

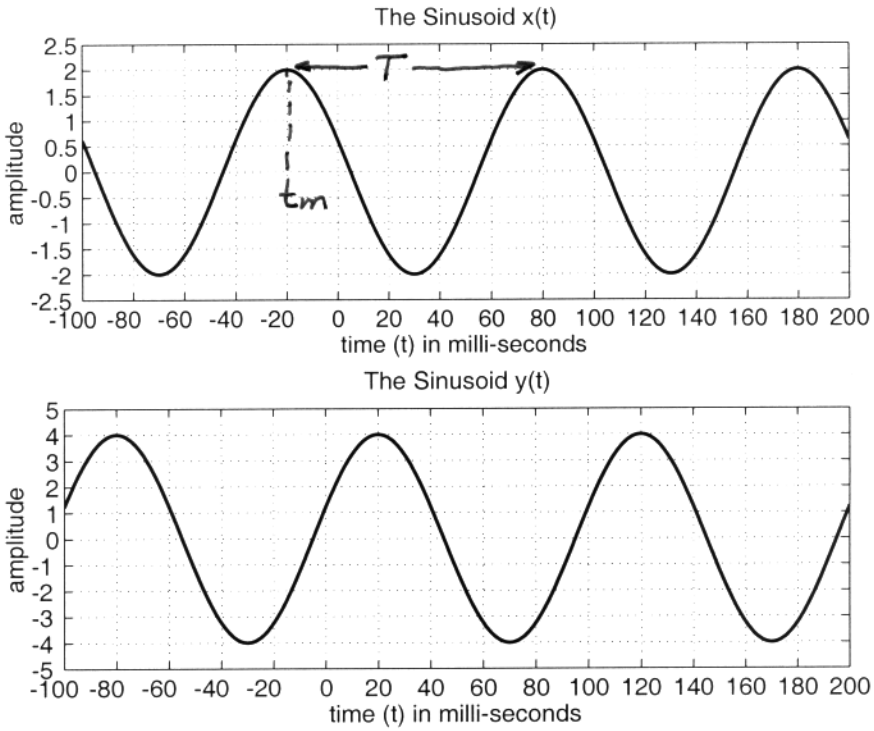
$C =$ $\sqrt{3}/2 + j\,1/2 = 0.866 + j0.5$

$$\left(e^{j\pi/2} e^{j\pi/3}\right)^{65} = \left(e^{j5\pi/6}\right)^{65} = e^{j325\pi/6}$$

$$\frac{325\pi}{6} = 54\pi + \pi/6 \qquad = e^{j\pi/6}$$

Problem SPR-02-Q.1.2:

Consider the sinusoidal signals $x(t)$ and $y(t)$ plotted below.



(a) Determine A, f_0, and ϕ in the representation of $x(t)$ as $x(t) = A\cos(2\pi f_0 t + \phi)$.

$A =$ 2 $\phi =$ 0.4π $f_0 =$ 10 (in Hz)

$t_m = -20$ ms $T = 100$ msec $= \frac{1}{10}$ sec

$\varphi = -(20\pi)(-20/1000)$ $f_0 = 1/T = 10$

(b) Now suppose that $B\cos(\omega_0 t + \psi) = x(t) + 2\cos(\omega_0 t)$. Determine B, ω_0, and ψ.

$B =$ 3.236

$\omega_0 =$ 20π rad/s

$\psi =$ 0.2π

$2e^{j0.4\pi} + 2 = 3.236e^{j0.2\pi}$

0.628 rads 36°

$3.236\cos(20\pi t + 0.2\pi)$

(c) The signal $y(t)$ can be expressed in terms of $x(t)$. That is, we can write $x(t) = Cy(t - t_1)$. Determine the numerical values of the scale factor C and the time shift t_1, where $t_1 \geq 0$.

$C =$ 1/2 $t_1 =$ 60 msec

$C = 3/4$ $t_1 = 80 - 20 = 60$ msec

If $y(t) = Dx(t - t_2)$ then $D = 4/2 = 2$ ✗ 1

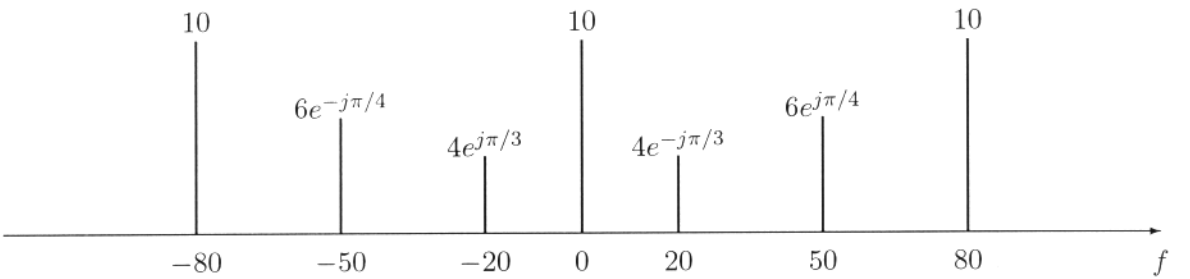
$t_2 = 20 - (-20) = 40$ msec

Problem SPR-02-Q.1.3:

A real signal

$$x(t) = A\cos(40\pi t + \phi) + B\cos(\omega_1(t-\tau)) + C\cos(\omega_2 t) + D$$

has the following two-sided spectrum:



frequency in Hz

(a) Determine A, B, C, D, ω_1, ω_2, ϕ, and τ the signal $x(t)$ with the above spectrum.

$A =$ 8

$B =$ 12

$C =$ 20

$D =$ 10

$\phi =$ $-\pi/3$ rads

$\omega_1 =$ 100π rad/s

$\omega_2 =$ 160π rad/s

$\tau =$ -0.0025 s
$= -1/400$ s

40π rad/s $\Rightarrow$ 20 Hz

$A = 8 \quad \varphi = -\pi/3$

80 Hz has zero phase

$C = 20 \qquad \omega_2 = 2\pi(80)$

$\omega_1 = 2\pi(50) = 100\pi$

$B = 12 \quad$ phase $= \pi/4 = \psi$

$\tau = -\psi/\omega_1 = -\pi/4/100\pi$

$= -\frac{1}{400}$ s $= -0.0025$ s

DC value is D

(b) The signal $x(t)$ is periodic. Determine the fundamental frequency f_0, of the signal $x(t)$.

$f_0 =$ 10 Hz

GCD{0, 20, 50, 80}

Problem SPR-02-Q.1.4:
A signal $x(t)$ is given by the equation

$$x(t) = [A + \cos(40\pi t)]\cos(200\pi t - \pi/2).$$

The signal $x(t)$, which is given above as a *product*, can also be expressed as a *sum* of sinusoids of the form

$$x(t) = \sum_{k=1}^{N} D_k \cos(\omega_k t + \phi_k), \tag{1}$$

where the ω_k's are different freqeuncies.

(a) Determine the number of cosine terms in $x(t)$, i.e. the value of N in Equation (1).
$N =$ 3

(b) What are the lowest and highest frequencies of all the sinusoids in the sum form [Eq. (1)] of $x(t)$?

lowest $\omega_k =$ 160π rad/s

highest $\omega_k =$ 240π rad/s

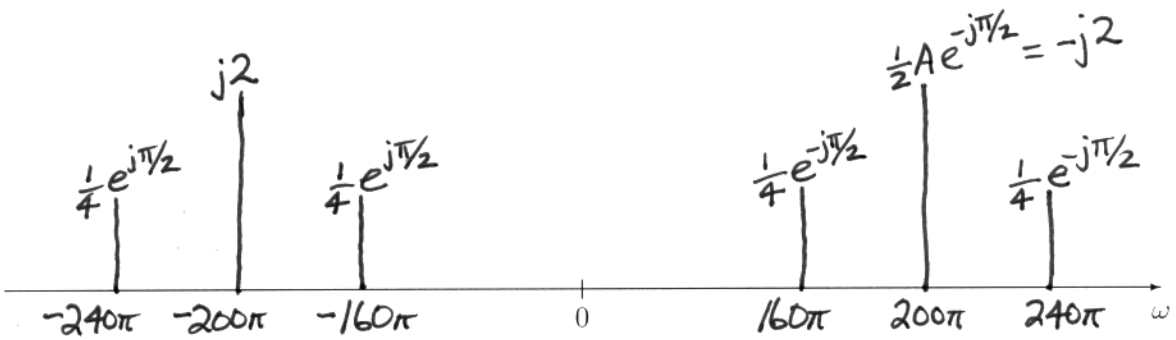
(c) The spectrum of $x(t)$ contains a component at frequency 200π rad/sec with complex amplitude $-2j$. What is the numerical value of A?

$A =$ 4

$$\frac{1}{2}A e^{-j\pi/2} = -j2$$
$$\Rightarrow A = 4$$

(d) Plot the two-sided spectrum of $x(t)$ on the graph below. Be sure to label all components of the spectrum with their frequency (in radians/sec) and their complex amplitude. You may need to use your result from part (c) to label the plot properly.

$$x(t) = \left[A + \frac{1}{2}e^{j40\pi t} + \frac{1}{2}e^{-j40\pi t}\right] \cdot \left[\frac{1}{2}e^{-j\pi/2}e^{j200\pi t} + \frac{1}{2}e^{j\pi/2}e^{-j200\pi t}\right]$$



frequency in rad/sec